Dynamic behaviours of coupling with gas

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1. Introduction

Pneumatic drives together with mechanical, electrical, and hydraulic drives have been widely used in mechanical engineering, transport, power engineering etc. Pneumatic drives have one essential disadvantage: the influence of transient processes on its main work. Density of gas in a pneumatic drive has the greatest influence on transient processes. The main parameters in pneumatic drives are selected according to the calculation results of transient processes. Therefore, there is a necessity to carry out the calculation of the transient processes in pneumatic drive very precisely. Most often pneumatic drives are investigated as the systems with concentrated parameters. Taking this assumption into consideration, it is not possible to evaluate pressure wave propagation in pneumatic drive. Therefore, it is not possible to study the impact of shock wave and hydraulic impact on the pneumatic drive elements connected with simple and complicated pneumatic signal transmission lines.

The accuracy of pneumatic mechanisms depends not only on gas parameters and management factors but also on mechanical factors. The most important factors influencing the accuracy of pneumatic drive are frictional and inertial forces. Therefore, when studying transient processes in pneumatic drive, the impact of these factors should be evaluated accurately.

In this article the dynamic processes in the drive together with the asynchronous motor, coupling with gas, and mechanical drive are considered. The distinguishing property of this coupling is that semi couplings interact with each other through gas. The coupling of this type consists of separate segments, where additional masses are used. Every segment consists of three pipelines.

Many authors investigated the pneumatic system as the system with lumped values and thus did not take into consideration wave processes going on in pneumatic lines. The present work aims to show wave processes going on in the coupling system as well investigated dynamic processes in the drive as a general system.

2. Dynamic model of the drive

The drive consists of asynchronous motor, the coupling with gas and mechanical drive (Fig. 1). The coupling of this type consists of separate segments, where additional masses are input (Fig. 1, b). Every segment consists of three pipelines. Semi couplings of the coupling are separated by gas. The pressure wave propagation in gas, the interaction between separate coupling bodies and gas are evaluated in the presented dynamic model of the drive.

One of the most progressive variants of automated electrical driver is an alternating current electrical driver with asynchronous motor (AE). The foundation of the AE mathematical model consists of differential equations of electrical and mechanical balance and the equations of electromagnetic energy transformation to mechanical energy.
where \( \{ \psi' \} = [\psi_1, \psi_2, \psi_3, \psi_4]^\top \) is the bound flow vector; \( I_s \) is the rotor inertia moment; \( \phi_e \) is the rotor turning angle, \( I_e \) is the torque.

\[
M_e = \frac{3}{2} \mu_0 L_s \mu_p \rho (\psi_4 - \psi_4')
\]

The system of the equations of drive is the following:

\[
\begin{align*}
I_\phi_1 \ddot{\phi}_1 &= -k_{1i} (\phi_i - \phi_i) - h_{1i} (\phi_i - \phi_i) - \\
&- M_{\phi-i1} (q_{0i} - q_{0i}) + M_{\rho_{10}} \rho (\phi_i, \phi_i) \\
I_\phi_2 \ddot{\phi}_2 &= M_{\phi-i2} (q_{0i} - q_{0i}) - k_{2i} (\phi_i - \phi_i) - \\
&- h_{2i} (\phi_i - \phi_i) + M_{\rho_{20}} \rho (\phi_i, \phi_i) \\
I_\phi_3 \ddot{\phi}_3 &= -k_{3i} (\phi_i - \phi_i) - h_{3i} (\phi_i - \phi_i) - M_e(t) + \\
&+ M_{\rho_{30}} (\phi_i, \phi_i) \\
m_i \ddot{q}_3 &= S_{i3} p_{3i} - k_{3i} (A_i + q_{0i}) - h_{3i} q_{3i} - \\
&- m_{3i} (q_{0i} - q_{0i}) + f_{3i} \text{sign}(\dot{q}_{3i})
\end{align*}
\]

(4)

where \( M_{\phi-i1} (q_{0i} - q_{0i}) = \sum_{i=1}^{N_{\text{seg}}} r_{ii} \left[ S_{i1} p_{1i} - k_{1i} (A_i + q_{0i} - q_{1}) - h_{1i} (q_{1i} - q_{0i}) \right] \)

(5)

\[
M_{\phi-i2} (q_{0i} - q_{0i}) = \sum_{i=1}^{N_{\text{seg}}} r_{ii} \left[ S_{i2} p_{2i} - k_{2i} (A_i + q_{0i} - q_{1}) - h_{2i} (q_{3i} - q_{0i}) \right]
\]

(6)

where \( I_1, I_2 \) are semi coupling inertia moments; \( I_3 \) is third mass inertia moment; \( I_{0i} \) is the inertia moment of \( i \) th segment; \( m_{3i} \) is the mass of additional body in \( i \) th segment; \( k_i \) are the coefficients of stiffness; \( h_i \) are the coefficients of damping; \( R_i \) \((i=1, 2, 3)\) are radii; \( S_i (i=1, 2, 3) \) are cross-section areas; \( p_{0i} \) is initial pressure in the \( i \) th segment; \( p_{1i}, p_{2i}, p_{3i} \) is gas pressure of segment \( i \) on semi coupling one and two; \( p_{hi} \) is gas pressure on \( m_s \) mass surface; \( \phi_{ei} \) is the engine turning angle; \( \phi_i, \phi_i \) are turning angles of semi coupling one and two; \( \phi_{0i} \) is \( i \) segment turning angle; \( q_i \) is the displacement of \( i \) th mass; \( f_{10i}, f_{20i}, f_{30i} \) are friction forces between semi couplings and \( i \) th segment, corresponding; \( f_{hi} \) is friction force between additional mass and \( i \) th segment.

### 3. Equations of gas movement

Equations of gas movement are described by differential equations with partial derivatives which express the law of momentum. This differential equation is supplemented by the equation of gas state which relates thermodynamic variables.

The one-dimensional isothermal gas movement in the variable cross-section of a pipeline is studied, where the gas velocity vector is directed along the axis of a pipeline, and thermodynamical variables change in time and along the axis of the pipeline (coordinate \( x \)). The cross-section area of a pipeline depends on coordinate \( x \). Equation of the gas continuity can be written in a differential form as follows [1-6]

\[
\frac{\partial}{\partial t} [S(x) \rho(x)] + \frac{\partial}{\partial x} [S(x) \rho v] = F_i (x)
\]

(7)

where \( \rho, v \) are density and velocity of gas; \( S(x) \) is cross section area of the pipeline; \( F_i (x) \) is discharge of gas mass to the unit of the length, in the pipeline.

The equation of liquid flow impulse (momentum)

\[
\frac{\partial}{\partial t} [S(x) \rho(x)] + \frac{\partial}{\partial x} [S(x) \rho v(x)] + \Pi(x) \tau + S(x) \rho a_x = F_i (x) + \rho \frac{\partial S}{\partial x}
\]

(8)

where \( \rho \) pressure; \( \Pi(x) \) is the perimeter of cross-section of the pipeline; \( \tau \) is tangential gas stress in the inner surface of the pipeline; \( a_x \) is acceleration along \( x \) axis; \( F_i (x) \) is kinetic energy of the gas flow in the pipeline to the unit of area.

Semi couplings of the coupling are separated by real gas. The equation of state of real gas is the following
\[ p = Z(\rho, T)\rho RT \]  

(9)

Bogomolov and Mayer equation of the state of real gas is used [1]

\[ Z(\rho, T) = 1 + \sum_{i=1}^{N_i} \sum_{j=0}^{N_j} b_{ij} \rho_i^{a_j} \rho \frac{\partial f_i}{\partial \rho} \frac{\partial f_j}{\partial \rho} \]  

(10)

where \( \omega = \frac{\rho}{\rho_{cr}} \), \( \rho_{cr} \) is critical density; \( N_i, N_j \) are variation limits; \( b_{ij} \) are coefficients; \( \tau = \frac{T}{T_{cr}} \), \( T_{cr} \) is critical temperature.

The system of equations of real gas movement with variables \((\rho, v)\) can be written as the system of second order quasilinear differential equations

\[ [A] \left[ \frac{\partial u}{\partial t} \right] + [B] \left[ \frac{\partial u}{\partial x} \right] = \{f\} \]  

(11)

where

\[
[A] = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad [B] = \begin{bmatrix} \frac{v}{\rho} & \rho v \\ \rho \frac{\partial f_i}{\partial \rho} & \frac{\partial f_i}{\partial \rho} \end{bmatrix}, \quad \{u\} = [\rho, v]
\]

\[
\{f\} = [\psi_1, \psi_2]; \quad \psi_1 = \frac{F_1(x)}{S(x)} - \rho \frac{dS}{dt}, \quad \psi_2 = \frac{1}{\rho S(x)} \left[ F_2(x) - vF_1(x) - \frac{\partial f_i}{\partial \rho} \right] \left[ \frac{\partial f_i}{\partial \rho} \right] \\
\frac{\partial f_i}{\partial \rho} = \frac{\partial f_i}{\partial \rho} = ZRT + \rho RT \frac{\partial Z}{\partial \rho}
\]

(12)

4. Characteristic method

Consider that on plane \( t, x \), domain \( G \) the solution of (11) system’s \([u]\) and given curve \( \Gamma \). Then knowing \([u]\) along \( \Gamma \), a solution can be obtained in the environment of curve \( \Gamma \), or all partial derivatives \( \frac{\partial u}{\partial t} \) and \( \frac{\partial u}{\partial x} \) can be determined along curve \( \Gamma \). If the determinant of the matrix

\[ [B] dt - [A] dx = 0 \]  

(13)

in every point of curve \( \Gamma \) is not equal to zero, all derivatives are determined unambiguously. If the determinant along curve \( \Gamma \) equates to zero, then from the assumption that the solution of (11) system exists, we shall receive that derivatives \( \frac{\partial u}{\partial t} \) and \( \frac{\partial u}{\partial x} \) are determined unambiguously. In this case curve \( \Gamma \) is called characteristics.

Equating the determinant of matrix (13) to zero, we shall receive the equation

\[ \left| [B] - [A] \frac{dx}{dt} \right| = 0 \]  

(14)

which allows to determine \( \frac{dx}{dt} \) derivative, which determines characteristic direction. If this equation has \( n \) various real roots \( dx/dt = \lambda_i \) \( (i = 1, 2, \ldots) \), the initial system of the differential equations is referred to as hyperbolic. The inclination tangent \( \lambda_i \) to the characteristic depends not only on coordinates but also on solution \([u]\).

Inserting expressions \([A]\) and \([B]\) from matrices (12) into equation (14) and having solved it, we receive three equations of characteristics

\[
C_+ : \frac{dx}{dt} = v + a \\
C_- : \frac{dx}{dt} = v - a
\]

(15)

where \( a \) is sound velocity in real gas,

\[
da^2 = c_1 - c_2 / b_1 \\
c_2 = \frac{\partial p}{\partial T} = Z\rho R + \rho RT \frac{\partial Z}{\partial T} = \]

(16)

\[
b_1 = \frac{b_2 \rho}{b_2 \rho - ZRT} \\
b_2 = \frac{\partial h_0}{\partial T} + \frac{\partial h_0}{\partial T} - \rho \left( Z + T \frac{\partial Z}{\partial T} \right)
\]

(17)

To ensure the stability of the solution, the Currant condition shall be fulfilled

\[ C_r = \frac{\Delta X + d}{\Delta X} < 1 \]

The enthalpy of real gas can be expressed like this

\[ h(\rho, T) = h_0(T) + h_1(\rho, T) \]  

(18)

where \( h_0(T) \) is enthalpy of ideal gas, \( h_1(\rho, T) \) is correction of enthalpy, evaluating the difference between real and ideal gases. Therefore characteristics can be drawn on plane \( t, x \) only for concrete solution of the system of differential equations. If (11) system is hyperbolic, the range of matrix (13) is equal to 1. On the other hand, if a solution exists, the range of matrix

\[ \left( [B] - [A] \frac{dX}{dt} \right) \]  

(19)

is equal to 1.

Equating the determinant of matrix (19) which consists of 1 column and vector \( \{f\} dt - [A] d[u] \) in matrix (14), i.e.
received an equation which is called a condition of compatibility or differential conjugation on characteristics. Characteristics are lines which separate domains with slight excitations. There can be weak interruptions of fluid parameters on the characteristics. Equation (14) is called the equation of characteristic directions.

Compatibility conditions on characteristics are equal to

\[ C^+ : \frac{d \rho}{dt} + \left( \frac{\rho a}{c_1} \right) \frac{dv}{dt} = \frac{a^2 f_1}{c_1} + \frac{\rho a f_2}{c_1} \]  \hspace{1cm} (21)

\[ C^- : \frac{d \rho}{dt} + \left( \frac{\rho a}{c_1} \right) \frac{dv}{dt} = \frac{a^2 f_1}{c_1} + \frac{\rho a f_2}{c_1} \]  \hspace{1cm} (22)

The total length of a gas pipeline is divided into elements with the length \( \Delta x \). At the moment of time \( t + \Delta t \) the unknown variables of the task: \( \rho \) is density, \( v \) is velocity are determined by the known values at the moment of time \( t \) (Fig. 2).

Substituting differentials by finite differences in the system of equations (21) and (22) the system of two non-linear algebraic equations is received

\[ C^+ : \Phi_1 = \rho_L - \rho_D + \frac{1}{2} \left( \frac{v_L - v_D}{\Delta x} \right) \left[ \left( r_2 \right)_L + \left( r_1 \right)_D \right] - \frac{\Delta t}{2} \left[ \left( r_2 \right)_L + \left( r_1 \right)_D \right] = 0 \]  \hspace{1cm} (23)

\[ C^- : \Phi_2 = \rho_D - \rho_R + \frac{1}{2} \left( \frac{v_R - v_D}{\Delta x} \right) \left[ \left( r_2 \right)_R + \left( r_1 \right)_D \right] - \frac{\Delta t}{2} \left[ \left( r_2 \right)_R + \left( r_1 \right)_D \right] = 0 \]  \hspace{1cm} (24)

where

\[ r_1 = \frac{\rho a}{c_1}; \quad r_2 = \frac{a^2 f_1}{c_1}; \quad r_3 = \frac{\rho a f_2}{c_1} \]  \hspace{1cm} (25)

At point \( L \) the gas flow parameters \( (\rho, v) \) are determined from the system of equations

\[ \Phi_3 = \rho_L - \rho_D + \theta \left( v_L + a_i \right) \left( \rho_B - \rho_D \right) = 0 \]

\[ \Phi_4 = v_L - v_D + \theta \left( v_L + a_i \right) \left( v_C - v_D \right) = 0 \]  \hspace{1cm} (27)

where \( \theta = \Delta t / \Delta x \).

At point \( R \) the gas flow parameters are determined from the system of equations

\[ \Phi_5 = \rho_R - \rho_C - \theta \left( v_R - a_R \right) \left( \rho_C - \rho_B \right) = 0 \]

\[ \Phi_6 = v_R - v_C - \theta \left( v_R - a_R \right) \left( v_C - v_R \right) = 0 \]  \hspace{1cm} (28)

The system of non-linear algebraic equations is solved by the method of Newton.

In each segment of the coupling potential and kinetic energy of gas is transferred to the mechanical system, which transforms this energy into kinetic and vice versa. Solving the system of equations of gas movement by the method of characteristics, there are four cases of interaction between the gas flow and the solid body (Fig. 3).

In case number two the following conditions shall be fulfilled

\[ x_{i-1} < q_H < x_i \]

\[ x_{i-1} < q_H + \Delta q_H \leq x_i \]

\[ X_i \leq q_H + \Delta t \left( v_H + a_H \right) < X_{i+1} \]  \hspace{1cm} (29)

The system of equations in this case of interaction between the gas and solid body is equal

\[ \Phi_7 = q_G - x_F - \left( C_F - a \left( p_F \right) \right) = 0 \]

\[ \Phi_8 = v_G - v_F - \frac{1}{2} \left( p_G - p_F \right) \left( \frac{1}{\rho a} \right) + \frac{1}{\rho a} \left( G_G \right) = 0 \]

\[ \Phi_H = \rho S_C \left( q_G - q_H \right) - \frac{4}{\Delta x} \left( \frac{\Delta q_H - \Delta H}{\Delta t} \right) \]

\[ \Phi_H = \frac{1}{r_i} \left( 4 \frac{G_G}{\Delta x} \left( q_G - q_H \right) - \frac{4}{\Delta t} \left( \frac{\Delta q_H - \Delta H}{\Delta t} \right) \right) \]

The system of equations of interaction of gas with a rigid body for other parts of segments of the coupling similarly enters the name.

The systems of equations of motion of asynchronous engine and the first mass with equations of interactions are solved by trapezoidal method and the obtained system of nonlinear algebraic equations is solved by Newton method

\[ \left[ J_k \right] \left\{ \Delta X_k \right\}_k = - \left\{ \Phi_k \right\}_k \]  \hspace{1cm} (31)

where

\[ \left\{ \Delta X_k \right\} = \left[ \left( \psi_{k}, \psi_{k+1}, \psi_{k+2}, \psi_{k+3}, \psi_{k+4}, \psi_{k+5}, X_{k+1}, P_{G1}, \right) \right] \]

\[ v_{G1, X_{Fausg}, P_{Gausg}, V_{Gausg}} \]  \hspace{1cm} (32)
Fig. 3 Cases of body and gas flow interaction: a - one; b - two; c - three; d - four

The system of equations of motion of the second and third mass with the equation of interactions are solved by trapezoid method and the obtained system of nonlinear algebraic equations is solved by Newton method

\[
\begin{bmatrix}
J_{12} \Delta X_{12} = -\Phi_{12}
\end{bmatrix}
\]

where

\[
\begin{bmatrix}
\Delta X_{12} = [X_{F1}, P_{G1}, V_{G1}, X_{Fr1}, P_{G01}, V_{G01}, \Phi_{2}, \Phi_{3}]
\end{bmatrix}
\]

5. Theoretical results

As an example of mechanical drive with asynchronous engine A-100S4Y3, coefficients of stiffness: 
\[ k_{1e} = 10^6 \text{ Nm/rad }; \]
\[ k_{23} = k_{2}, \quad k_{1} = k_{2} = k_{3} = 10^5 \text{ N/m }; \]
coefficients of damping: 
\[ h_{10} = h_{23} = 10^{-7} \text{ Ns/rad }; \]
\[ h_{1} = h_{2} = h_{3} = 10^{-3} \text{ Ns/m }; \]
Geometrical parameters of the coupling with gas are: 
\[ r_{0} = 0.12 \text{ m }; \]
\[ r_{1} = r_{2} = 0.12 \text{ m }; \]
\[ r_{3} = 0.045 \text{ m }; \]
\[ n_{seg} = 2 \]. Inertia moment of masses: 
\[ I_{1} = I_{2} = I_{3} = 0.01 \text{ kgm}^2; \]
\[ I_{10} = I_{20} = 0.01 \text{ kgm}^2; \]
\[ m_{3} = 0.050 \text{ kg }; \]

A numerical study of periodic solutions of the system of equations of the drive has been performed using direct time integration when resistance moment is harmonic and random quantity distributed by the normal law [6].

The random moment of resistance is equal to (Fig. 4)

\[
M_{r}(t) = \begin{cases}
M_{r0} \frac{t}{t_{1}}, & \text{when } t \leq t_{1} \\
M_{r0} + M_{r1}(\sigma_{1}, \mu_{1}) \sin \left[ \omega(\sigma_{2}, \mu_{2})(t-t_{1}) \right], & \text{when } t > t_{1}
\end{cases}
\]

The dynamic processes of mechanical drive have been investigated with different initial pressures 
\[ p(t = 0) = 0.20 \text{ MPa } \] and 
\[ p(t = 0) = 0.50 \text{ MPa } \] in segments of the coupling (Fig. 6 and Fig. 10).

Fig. 4 Dependence of the random moment of resistance on time 
\[ M_{r0} = 10.0 \text{ Nm}; \]
\[ \mu_{1} = 5.0 \text{ Nm}; \]
\[ \sigma_{1} = 0.10 \text{ Nm}; \]
\[ \mu_{2} = 90 \text{ rad/s}; \]
\[ \sigma_{2} = 0.10 \text{ rad/s}; \]
\[ t_{1} = 0 \text{ s} \]

Phase plots of additional mass in the first segment when pressures in the coupling are different are shown in Figs. 5 and 9.

Dependences of semi coupling with gas angular velocity 
\[ dw_{21} = \phi_{3} - \phi_{1} \] on time, when the moment of resistance is random quantity, distributed according to the normal law are shown in Figs. 7 and 11.

Dependences of angular velocity 
\[ \omega_{3} = \phi_{3} \] on time, when pressure is 
\[ p(t = 0) = 0.20 \text{ MPa } \] and 
\[ p(t = 0) = 0.50 \text{ MPa } \] are shown in Figs. 8 and 12.
Fig. 6 Dependence of pressure in the centre of the first segment on time when $p(t = 0) = 0.20$ MPa

Fig. 7 Dependence of angular velocities difference of semi-couplings $\Delta w_{21}$ on time when $p(t = 0) = 0.20$ MPa

Fig. 8 Dependence angular velocity $w_3$ when $p(t = 0) = 0.20$ MPa

Fig. 9 Phase plot $(q_{31}, q_{31})$ for the solution when $p(t = 0) = 0.50$ MPa

Fig. 10 Dependence of pressure in the centre of the first segment on time when $p(t = 0) = 0.50$ MPa

Fig. 11 Dependence of difference angular velocities of semi couplings $\Delta w_{21}$ on time when $p(t = 0) = 0.50$ MPa

Fig. 12 Dependence angular velocity $w_3$ when $p = 0.50$ MPa

6. Conclusions

1. There is mathematical model of a drive with asynchronous motor, coupling with gas and mechanical driver composed. In the coupling with gas into wave motion of a gas complex drive is taken into account.

2. The transients us mechanical drive are determined when resistance moment is harmonic and random quantity is distributed by the normal law.

3. Mathematical model of the interaction of gas with a rigid body is designed.

4. Damping properties of coupling depend from initial pressure, and parameters of each segments of coupling. The stiffness of coupling increases when pressure in the coupling also increases. By finding these parameters of coupling can damp vibrations in the mechanical driver with asynchronous engine.

5. The designed mathematical model of mechanical driver is possible for using for optimization of dynamic characteristics of this driver.
References


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DYNAMIC BEHAVIOURS OF COUPLING WITH GAS

Summary

This paper presents the investigation of dynamic processes of mechanical drive with asynchronous motor. The driver consists of a complex coupling which has few segments with real gas. Dynamic model of complex coupling is made. Semi couplings are separated by gas. The pressure wave propagation in gas, the interaction between separate coupling bodies and the gas, and asynchronous engine dynamics are evaluated in the presented dynamic model of the drive. The system of equations of gas is solved by characteristics method. Numerical simulation results when resistance torque is random quantity distributed by the normal law are presented.

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ДИНАМИЧЕСКИЕ РЕЖИМЫ МУФТЫ СОДЕРЖАЩЕЙ ГАЗ

Резюме

В статье приведены результаты исследования динамических процессов механической передачи с асинхронным электродвигателем. Механическая передача содержит муфту, в которой полумуфты разделены реальным газом. Разработана динамическая модель муфты и механической передачи, в которой учитывается распространение волн давления в реальном газе, взаимодействие газа с полумуфтами и динамические свойства асинхронного электродвигателя. Уравнения движения газа решаются методом характеристик. Получены численные результаты, когда момент сопротивления является стохастическим и распределен по нормальному закону.

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