

Investigation of the probabilistic low cycle fatigue design curves at stress cycling

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1. Introduction

In working conditions some equipment's components are affected by high loads, which results in elastic plastic cyclic deformation. These type of loads may cause fatigue damage over time and results in crack propagation and fracture of the equipment. Due cyclic overloads, the cyclic varying stresses exceeds the proportional limit of the materials and after thousands cycles the fatigue crack appears which will result in part's failure with hardly predictable corollary [1, 2].

Aiming to improve the reliability and safety of the structures, the probabilistic methods for low cycle fatigue data, are applied in many manuscripts [3, 4]. These methods are based on statistically collected cyclic loading data and their probabilistic characteristics.

Probabilistic methods have been used for the assessment of statistical and fatigue damage properties [5].

Considerable contribution to the investigation of probabilistic methods, probabilistic substantiation of allowable stresses and strength safety and proposed calculation in the static and cyclic strength are used by many investigators [6-8].

The aim of this paper is to design the probabilistic estimation curves of low cycle fatigue. Also, a comparison of the probabilistic curves with the experimental data have been made [9].

2. Experimental setup for monotonic and low cycle loading

Experimental setup consists of 50kN testing machine YMM-5 and an electronic part, which is designed to record the stress-strain diagrams, cycles and control the gear reversal. The 4 mm/min speed of deformation was used for tensile experiments. The cycle frequency of 4-10 cycles/min and 20 mm/min speed of deformation were used for low cycle tension-compression testing. The force and deformation measurement range varies not more than $\pm 5\%$. GOST 25.502-79 standard was used for metal fatigue tests, whereas GOST 22015-76 - for probabilistic calculation of metallic parts. All the testing set-up with mechanical and electrical schemes is described in literature [10].

Experimental monotonic tension and low cycle tension-compression were carried out under ambient temperature. The specimens were tested under symmetric loading ($R = -1$) and experimental data was registered up to of the crack initiation.

The specimens of circular cross-section with $d = 10$ mm were used for low cycle fatigue tension-compression experiments. The specimens were made of grade steel 15Cr2MoVA and 45 rods according the dimensions presented in the Fig. 1.

For determination of mechanical properties the material for monotonic tension specimens was taken from the part of cyclic test specimens which has not been subjected to plastic deformation. The specimens of circular cross-section with $d = 5$ mm and length $5d$ were used for these tests.

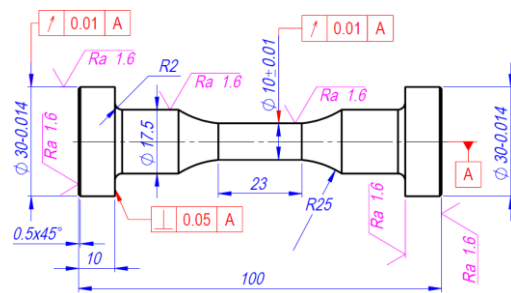


Fig. 1 Low cycle test specimen

In this paper, the low cycle fatigue tests were carried out under stress limited loading with two materials: cyclically softening steel 15Cr2MoVA (Fig. 2, a) and cyclically stable steel 45 (Fig. 2, b). The chemical composition and mechanical properties of the tested materials are presented in Table 1 and Table 2. As it seen from the Table 2, the materials are very ductile, especially 15Cr2MoVA ($\psi = 80\%$) [7-9]. The monotonous tension curve has plasticity of yielding, whereas 15Cr2MoVA doesn't have [9].

Table 1

Chemical compositions of tested materials

	C	Si	Mn	Cr	Ni	Mo
Steel 15Cr2MoVA	0.18	0.27	0.43	2.7	0.17	0.67
	V	S	P	Mg	Cu	Al
	0.30	0.019	0.013	-	-	-
Steel 45	C	Si	Mn	Cr	Ni	Mo
	0.46	0.28	0.63	0.18	0.22	-
	V	S	P	Mg	Cu	Al
-	0.038	0.035	-	-	-	-

Three stress levels at stress cycling are evenly distributed over the range. Number of the tested specimens at each loading level are presented in the Table 3. In the

middle levels number of the specimens were increased because the increased scatter was observed in the intermediate zone.

Table 2

Mechanical properties of tested materials

Grade	σ_{pr}	$\sigma_{0.2}$	σ_u	S_k	ϵ_{pr}	ψ
	MPa				%	
Steel 15Cr2MoVA	280	400	580	1560	0.2	80
Steel 45	340	340	800	1150	0.26	39

Table 3

Statistical low cycle testing program

Grade	Stress limited loading	
	Loading level	Number of specimen
Steel 15Cr2MoVA	1.00	20
	1.12	40
	1.25	20
Steel 45	1.00	20
	1.25	40
	1.50	20

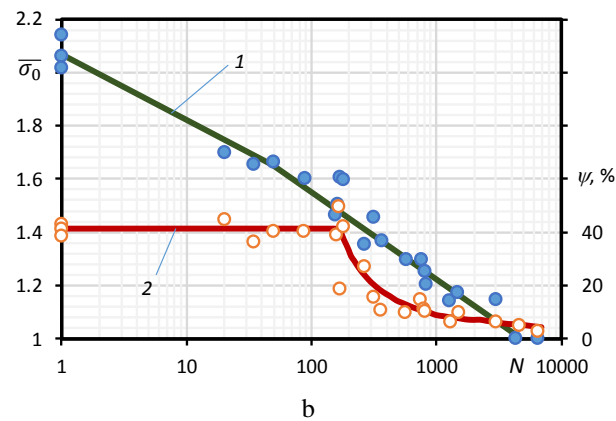
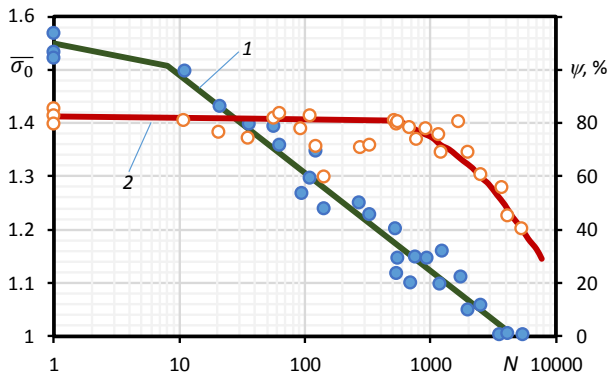


Fig. 2 Low cycle fatigue curves (1) and reduction of the area of cross-section ψ (2) under stress limited loading of the materials: a - steel 15Cr2MoVA; b - steel 45

3. Mathematical analysis of probabilistic low cycle fatigue design curves

The purpose of stress limited tests is to define material fracture characteristics at elastic-plastic cyclic loading. For the assessment of a material strength the analytical

relation between the static damage and accumulative plastic deformation was introduced in the Eq. (1) [1]:

$$\sigma_a N_p^m = \sigma_u N_u^m = const, \quad (1)$$

where σ_a is stress amplitude; N_p is fatigue durability; N_u is number of cycles to fracture; m is a constant.

For analytical calculation, the fatigue curve of stress limited loading can be expressed as follow [11]:

$$e_0 = \frac{1}{N^{m_\sigma}} \ln \frac{100}{100 - \psi} + \frac{\sigma_{-1}}{E}, \quad (2)$$

where e_0 - initial strain; m_σ - a constant dependent of mechanical properties of the material.

Under stress limited loading the part fractures due to quasi-static damage d_K , caused by the accumulated plastic strain \bar{e}_{pk} , and due to fatigue damage d_N , resulted by the cyclic plastic strain, which is characterized by the hysteresis loop's width $\bar{\delta}_k$. The total damage d may be written (up to fracture, $d = 1$) [10]:

$$d = d_K^q + d_N^l, \quad (3)$$

where d_K^q , d_N^l - quasi-static and fatigue damage; q and l - material constants [9-10].

Fatigue damage is calculated using the following Eq. (4):

$$d_N = \frac{\sum_1^k \bar{\delta}_k}{\sum_1^{k_0} \bar{\delta}_k}, \quad (4)$$

where $\sum_1^k \bar{\delta}_k$ - fatigue damage accumulated during k loading semi-cycles, $\sum_1^{k_0} \bar{\delta}_k$ - fatigue damage accumulated up to crack initiation (when $d_K = 0$).

Quasi-static damage is calculated using the following Eq. (5):

$$d_K = \frac{\bar{e}_{pk}}{\bar{e}_{u_2}}, \quad (5)$$

where \bar{e}_{pk} - accumulated plastic strain after k loading semi-cycles; \bar{e}_{u_2} - maximum uniform strain under monotonic loading which corresponds σ_u .

The accumulated plastic strain after k loading semi-cycles, can be expressed as follows:

$$\begin{aligned} \bar{e}_{pk} = & \bar{e}_0 - \bar{\sigma}_0 - \left(\bar{e}_0 - \frac{\bar{S}_T}{2} \right) \sum_1^k \left[\frac{A_1}{2} - (-1)^k \frac{A_1}{2} \right] \times \\ & \times \exp \beta(k-1) + \left(\bar{e}_0 - \frac{\bar{S}_T}{2} \right) \sum_1^k \left[\frac{A_2}{2} + (-1)^k \frac{A_2}{2} \right] \times \\ & \times \exp \beta(k-1). \end{aligned} \quad (6)$$

For cyclically softening and stable materials the accumulated plastic strain can be expressed as follows:

$$\begin{aligned} \bar{e}_{pk} = & \bar{e}_0 - \bar{\sigma}_0 - \left(\bar{e}_0 - \frac{\bar{S}_r}{2} \right) \sum_1^k \left[\frac{A_1}{2} - (-1)^k \frac{A_2}{2} \right] \frac{1}{k^0} + \\ & + \left(\bar{e}_0 - \frac{\bar{S}_r}{2} \right) \sum_1^k \left[\frac{A_2}{2} + (-1)^k \frac{A_1}{2} \right] \frac{1}{k^0}, \end{aligned} \quad (7)$$

where A_1, A_2, β, α are material constants.

For strain cycling, the Eq. 4 can be written as:

$$d_N = \frac{\bar{\delta}_k}{\sum_1^{k_0} \bar{\delta}_k}. \quad (8)$$

For k loading semi-cycles the Eq. 4 can be expressed as:

$$d_N = \frac{\sum_1^k \bar{\delta}_k}{\sum_1^{k_0} \bar{\delta}_k}, \quad (9)$$

where $\bar{\delta}_k$ - width of the hysteresis loop of initial semi-cycle.

When $k = k_0$ the accumulated fatigue damage is equal $d_N = 1$.

Durability of low cycle stress limited loading till the fatigue fracture can be expressed:

$$\frac{\sum_1^k \bar{\delta}_k}{\sum_1^{k_0} \bar{\delta}_k} = 1. \quad (10)$$

Quasi-static damage can be calculated applying Eq. 5 and for quasi-static and intermediate mode $d_K < 1$. For quasi-static mode $d_N \ll d_K$ and for practical calculations it is assumed that $d_K = 1$, then:

$$\bar{e}_{pk} = \bar{e}_u = \ln \frac{100}{100 - \psi}. \quad (11)$$

For the intermediate mode quasi-static and fatigue damages can be calculated from Eqs. (3)-(5):

$$\left(\frac{\sum_1^{k_0} \bar{\delta}_k}{\sum_1^{k_N} \bar{\delta}_k} \right)^q + \left(\frac{\bar{e}_{pk_0}}{e_{pk_0}^{av}} \right)^l = 1, \quad (12)$$

for tested materials it is assumed that $q = l$.

Calculated curve of low cycle stress limited loading only fatigue damage is evaluated then can be written:

$$\frac{\bar{\delta}_1}{\sum_1^{k_{01}} \bar{\delta}_k} + \frac{\bar{\delta}_2}{\sum_1^{k_{02}} \bar{\delta}_k} + \dots + \frac{\bar{\delta}_{k_0}}{\sum_1^{k_{0k}} \bar{\delta}_k} = 1, \quad (13)$$

where $\sum_1^{k_{01}} \bar{\delta}_k$ - accumulated fatigue damage till crack initiation which is characterized by the hysteresis loop's width $\bar{\delta}_1$; $\sum_1^{k_{02}} \bar{\delta}_k$ - accumulated fatigue damage till crack initiation which is characterized by the hysteresis loop's width $\bar{\delta}_2$, etc.

The fatigue curve in logarithmic coordinates $\lg \bar{\delta}_{av} - \lg k_0$ has a shape of the straight line, then:

$$\bar{\delta}_{av} = C_2 k_0^{-\alpha_2}, \quad (14)$$

where $\bar{\delta}_{av} = \frac{1}{k_0} \sum_1^{k_0} \bar{\delta}_k$ then:

$$\sum_1^{k_0} \bar{\delta}_k = C_2 k_0^{1-\alpha_2}, \quad (15)$$

where $\bar{\delta}_{av}$ - average value of the hysteresis loop's width; k_0 - semi-cycle of initial loading; α_2 and C_2 - material constants.

If the expressions $\lg \bar{e} - \lg k_0, \bar{e} k_0^{\alpha_1} = C_3$ are used, where $C_3 = 2^{\alpha_1} C_1$, then:

$$k_0 = \frac{C_3^{\alpha_1}}{\bar{e}^{\alpha_1}}. \quad (16)$$

Substituting Eq. (16) into (15) it is obtained

$$\sum_1^{k_0} \bar{\delta}_k = C_2 \frac{C_3^{\alpha_1}}{\bar{e}^{\alpha_1}}.$$

By introducing the $\frac{1-\alpha_2}{\alpha_1} = \alpha_3$, the dependence

can be written:

$$\sum_1^{k_0} \bar{\delta}_k = C_2 \frac{C_3^{\alpha_3}}{\bar{e}^{\alpha_3}}. \quad (17)$$

The analytical curves as only fatigue damage were calculated applying the following Eq. (18):

$$\frac{\bar{\delta}_1 \bar{e}_1^{\alpha_3}}{C_2 C_3^{\alpha_3}} + \frac{\bar{\delta}_2 \bar{e}_2^{\alpha_3}}{C_2 C_3^{\alpha_3}} + \dots + \frac{\bar{\delta}_{k_C} \bar{e}_{k_C}^{\alpha_3}}{C_2 C_3^{\alpha_3}} = 1. \quad (18)$$

For calculation of the monotonous tension, the linear or step-function approximation can be applied. Using the step-function approximation, the Eq. (18) will look:

$$\begin{aligned}
& A_1 \left(\frac{1}{\bar{\sigma}_0^{m_0}} - \frac{\bar{S}_T}{2} \right) \left\{ F(1) \left[2\bar{\sigma}_0 + A_1 \left(\frac{1}{\bar{\sigma}_0^{m_0}} - \frac{\bar{S}_T}{2} \right) F(1) \right]^{\alpha_3} + F(3) \left[2\bar{\sigma}_0 + A_1 \left(\frac{1}{\bar{\sigma}_0^{m_0}} - \frac{\bar{S}_T}{2} \right) F(3) \right]^{\alpha_3} + \dots + F(2N_0 + 1) \right\} \times \\
& \times \left[2\bar{\sigma}_0 + A_1 \left(\frac{1}{\bar{\sigma}_0^{m_0}} - \frac{\bar{S}_T}{2} \right) F(2N_0 + 1) \right]^{\alpha_3} \left\{ + A_2 \left(\frac{1}{\bar{\sigma}_0^{m_0}} - \frac{\bar{S}_T}{2} \right) \left\{ F(2) \left[2\bar{\sigma}_0 + A_2 \left(\frac{1}{\bar{\sigma}_0^{m_0}} - \frac{\bar{S}_T}{2} \right) F(2) \right]^{\alpha_3} + \right. \right. \\
& \left. \left. + F(4) \left[2\bar{\sigma}_0 + A_2 \left(\frac{1}{\bar{\sigma}_0^{m_0}} - \frac{\bar{S}_T}{2} \right) F(4) \right]^{\alpha_3} + \dots + F(2N_0) \left[2\bar{\sigma}_0 + A_2 \left(\frac{1}{\bar{\sigma}_0^{m_0}} - \frac{\bar{S}_T}{2} \right) F(2N_0) \right]^{\alpha_3} \right\} \right\} = C_2 C_3^{\alpha_3}, \quad (19)
\end{aligned}$$

where $N_0 = \frac{k_0}{2}$; m_0 - material intensification coefficient for step-function approximation.

For cyclically hardening, softening and stable materials:

$$F(k) = \exp \beta(k-1); \quad (20)$$

$$F(k) = \frac{1}{k^0}; \quad (21)$$

$$F(k) = 1, \quad (22)$$

where $\alpha = \beta = 0$.

For cyclically stable materials:

$$\frac{\sum_1^k \bar{\delta}_k}{\sum_1^{k_0} \bar{\delta}_k} = \frac{k}{k_0}. \quad (23)$$

Then sum of relative cyclic deformations (Eq. 13) can be replaced by sum of relative durability, then:

$$\sum_i \frac{k_i}{k_{0i}} = 1. \quad (24)$$

Then $\bar{\delta}_k = \bar{\delta}_{av} = \text{const}$ and substituting into Eqs. (14), (24) becomes:

$$\sum_i k_i \bar{\delta}_k^{\frac{1}{\alpha_2}} = C_2^{\alpha_2} \quad (25)$$

or

$$\left(\sum_i k_i \bar{\delta}_k^{\frac{1}{\alpha_2}} \right)^{\alpha_2} = \left(\bar{\delta}_1^{\frac{1}{\alpha_2}} + \bar{\delta}_2^{\frac{1}{\alpha_2}} + \dots + \bar{\delta}_k^{\frac{1}{\alpha_2}} \right)^{\alpha_2} = C_2. \quad (26)$$

For the step-function approximation using Eq. 26 it can be:

$$\bar{\sigma}_0 = \left\{ \frac{C_2}{\left[\left(A_1^{\alpha_2} + A_2^{\alpha_2} \right) N \right]^{\alpha_2} - \frac{\bar{S}_T}{2}} \right\}^{m_0}. \quad (27)$$

4. Verification of low cycle fatigue design curves

The probabilistic low cycle design curves were calculated for steel 15Cr2MoVA and 45 using the Eqs. (19) and (27). The parameters for calculation of these curves are presented in Table 4.

Table 4

Probabilistic values of the cyclic characteristics

Parameters	Probability, %						
	1	10	30	50	70	90	99
Steel 15Cr2MoVA							
A_1	0.23	0.87	1.32	1.60	1.94	2.40	3.04
A_2	0.30	0.94	1.34	1.64	2.00	2.43	3.06
\bar{S}_T	1.15	1.20	1.25	1.28	1.30	1.35	1.40
m_0	0.15	0.17	0.19	0.21	0.23	0.26	0.30
$\beta \times 10^{-3}$	0.04	0.16	0.42	0.85	1.80	4.80	19.0
$C \times 10^{-4}$	1.10	3.00	4.80	7.40	11.0	21.0	50.0
Steel 45							
A_1	0.60	0.76	0.87	0.95	1.04	1.15	1.29
A_2	0.64	0.80	0.89	0.96	1.05	1.16	1.30
\bar{S}_T	0.92	1.02	1.07	1.11	1.15	1.21	1.29
m_0	0.14	0.16	0.18	0.19	0.20	0.22	0.25

The probabilistic constants of the low cycle strain limited loading of steel 15Cr2MoVA and 45 were used for calculation are shown in Table 5.

Probabilistic constants of C_2 and α_2 were determined using logarithmic coordinates $\lg \bar{\delta}_{av} - \lg k_0$ (Fig. 3). Width of the hysteresis loop was calculated using Eq. (28):

$$\bar{\delta}_1 = A_1 \left(\bar{\epsilon}_0 - \frac{\bar{S}_T}{2} \right). \quad (28)$$

The probabilistic curves of low cycle stress limited loading for steel 15Cr2MoVA was calculated using Eq. (19) and for steel 45 – using Eq. (27). The parameter of cyclical deformation was determined by Eq. (29):

$$\beta = C \left(\bar{\epsilon}_0 - \frac{\bar{S}_T}{2} \right). \quad (29)$$

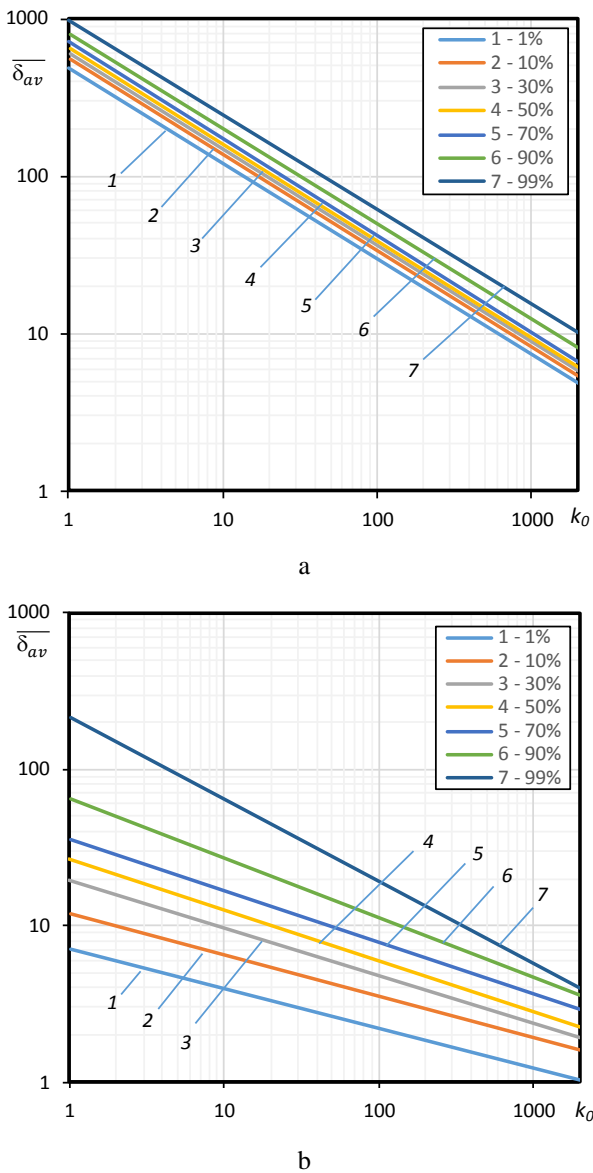


Fig. 3 Probabilistic calculation of hysteresis loop dependence on semi-cycles of stress limited curves: a - steel 15Cr2MoVA; b - steel 45

Fig. 4, a shows the probabilistic curves of stress limited loading where curve of 99% probability shows lowest lifetime, whereas 1% - highest. This is due to high reduction of e_{pr} , which was used in this type of calculation and also high reduction of deformation parameters compare to reduction of the area of cross-section ψ . This type of calculation showed contrariety with experimental data. To eliminate this type of discrepancy, mean arithmetic parameters of cyclic deformation was used for the next step.

For determination of the probabilistic curves, the probabilistic coefficients' C_2 , C_3 , a_3 values were used (Table 5). Using this type of calculation methodology, the low cycle curves got appropriate order, i. e. curve of 1% showed lowest lifetime, whereas 99% - highest (Fig. 4, b).

The probabilistic calculation of low cycle stress limited curves for steel 15Cr2MoVA is shown in Fig. 4, a, whereas for steel 45 in Fig. 5, a. The comparison of calculation and experimental data of these graphs is shown in

Fig. 4, b, and Fig. 5, b, respectively.

Low cycle stress limited calculation for anisotropic materials showed a satisfactory agreement with experimental data when loading level is $\bar{\sigma}_0 = 1$. When loading level $\bar{\sigma}_0 > 1$, the calculation is not adequate, because of quasi-static damage which gives error, but doesn't count to fatigue lifetime calculation.

Table 5

Probabilistic constants

Parameters	Probability, %						
	1	10	30	50	70	90	99
Steel 15Cr2MoVA							
C_1	205	215	235	240	260	285	350
α_1	0.48	0.49	0.49	0.50	0.50	0.50	0.50
C_2	500	535	590	630	680	790	940
α_2	0.53	0.53	0.53	0.54	0.54	0.54	0.54
C_3	286	302	330	339	368	403	494
α_3	0.98	0.96	0.96	0.92	0.92	0.92	0.92
Steel 45							
C_2	7.60	12.0	19.8	27.5	37.0	67.0	230
α_2	0.24	0.25	0.28	0.31	0.32	0.38	0.52

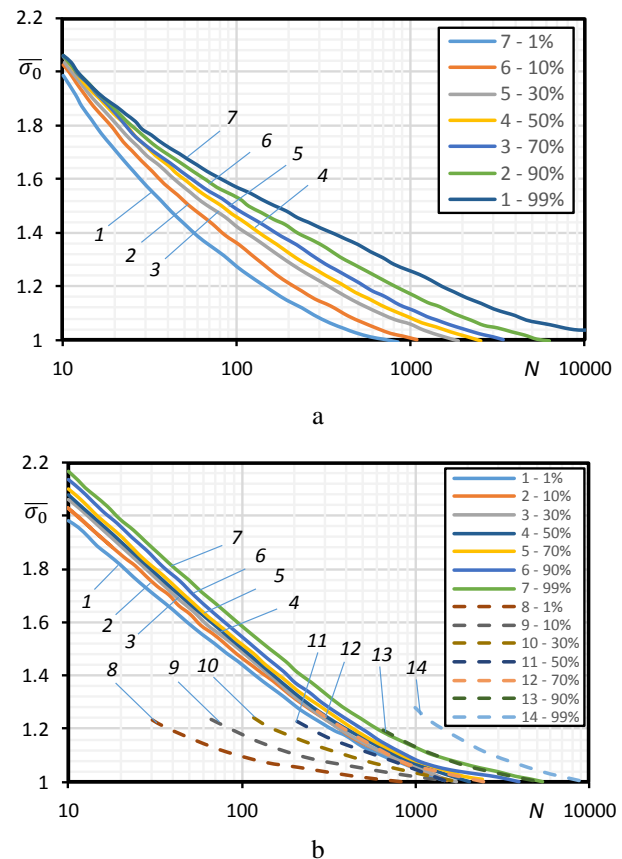


Fig. 4 Probabilistic calculation of low cycle stress limited curves for steel 15Cr2MoVA (1 – 7 analytical probability 1% - 99%; 8 – 14 experimental probability 1% - 99%): a - calculation; b - comparison with experiment

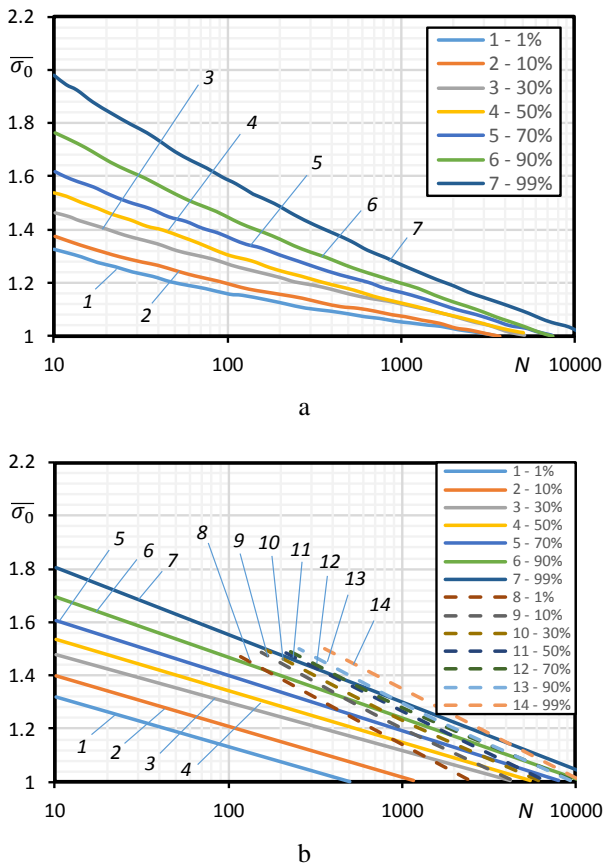


Fig. 5 Probabilistic calculation of low cycle stress limited curves for steel 45 (1 – 7 analytical probability 1% - 99%; 8 – 14 experimental probability 1% - 99%): a - calculation; b - comparison with experiment

5. Conclusions

The following conclusions can be drawn from this study:

1. Design curves of fracture probability for steels 15Cr2MoVA and 45 at low cycle stress limited loading were calculated and the probabilistic curves were compared with experimental ones.

2. Low cycle stress limited calculation for anisotropic materials showed a satisfactory agreement with experimental data when loading level is $\bar{\sigma}_0 = 1$. When loading level $\bar{\sigma}_0 > 1$, the calculation is not adequate, because of quasi-static damage which gives error, but doesn't count to fatigue lifetime calculation.

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INVESTIGATION OF THE PROBABILISTIC LOW CYCLE FATIGUE DESIGN CURVES AT STRESS CYCLING

S u m m a r y

This paper presents the investigation of probabilistic calculation of low cycle fatigue curves. The experiment of low cycle fatigue was made and cyclic constants were defined. Based on data obtained, design curves of equal fracture probability 15Cr2MoVA and 45 at low cycle stress limited loading were drawn. The design curves were compared with experimental data and the conclusions were made. Low cycle stress limited calculation for anisotropic materials showed a satisfactory agreement with experimental data and can be used in lifetime assessment of various power plant facilities.

Keywords: probabilistic calculation, low cycle fatigue.

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