Tolerance design of multistage radial flow submersible pumps

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1. Introduction

The functional quality of a product assembly is heavily dependent upon one or two critical dimensions of the assembly. For example, the quality of a ball bearing is based on the clearance between the balls and the inner or outer racings. These critical dimensions result from the cumulative effect of two or more functional dimensions. A tolerance analysis has to be conducted for identifying the functional dimensions that are affecting a critical dimension. Once the functional dimensions are identified, then the tolerances for these dimensions are to be allocated from the critical dimension tolerance. Thus the tolerance design of a product consists of tolerance analysis and tolerance allocation [1]. Dimensional tolerancing are designed to ensure that products produced will meet the designed requirements, e.g. functionality, minimum cost and maximum interchangeability [2]. In the present study, tolerance design is carried out for a multistage radial flow submersible pump assembly. One of the critical dimensions of the pump assembly is the axial play between the impeller and the volute casing (Fig. 1). The impeller has to rotate inside the casing without rubbing the wall. An axial play of 2±0.5mm is required to be maintained for preventing the rubbing action. As the number of functional dimensions controlling this axial play is more than fifteen, an assigned tolerance of ±0.5mm on the critical dimension is very difficult to achieve. The method adopted for achieving the required axial play is discussed in details in this paper.

2. Problem definition

The total pump assembly consisting of motor and pump subassemblies are shown in Fig. 2. The coupling connects the shafts of both the subassemblies. The length of the coupling is computed based on the measured gap \( L \) as shown in Fig. 3. In order to maintain an axial play of 2 mm between the impellers and their casings, the length of the coupling is found out by adding 2 mm with the measured gap. The gap \( L \) is the sum of the assembly dimensions \( M \) and \( P \). The industry in which the problem was analyzed could standardize the assembly dimension \( M \) on the motor subassembly. The dimension \( P \) is measured after assembling all the parts of the pump subassembly (Fig. 3). In the assembled condition, the impellers (Fig. 4, a) interspaced with sleeves (Fig. 4, b) mounted over the shaft (Fig. 4, c) occupies the bottom position in their respective casings (Fig. 4, d) as shown in (Fig. 5). This is due to the fact that the shaft containing these parts is pulled to the bottom due to their total weight.

3. Present scenario of assembling the subassemblies

The method followed at present by the industry for overcoming the unreasonable length variation of the gap \( L \) is explained in the following lines. As already discussed, the industry could standardize the assembly dimension \( M \) on the motor subassembly. The dimension \( P \) is measured after assembling all the parts of the pump subassembly (Fig. 3). In the assembled condition, the impellers (Fig. 4, a) interspaced with sleeves (Fig. 4, b) mounted over the shaft (Fig. 4, c) occupies the bottom position in their respective casings (Fig. 4, d) as shown in (Fig. 5). This is due to the fact that the shaft containing these parts is pulled to the bottom due to their total weight.

Fig. 1 Axial play to be maintained between impeller and casing

Fig. 2 Motor and pump subassemblies of the submersible pump
by 2 mm in order to centre the impellers. This is accomplished by choosing the coupling length 2 mm in excess of the measured gap $L$. For ease of manufacture, the coupling of varying length is split into two, namely a coupling of standard length $85 \pm 0.2$ and a sand guard of varying thickness (Fig. 7). The sand guard is machined to the required size only during the final stage of the assembly after measuring the gap in which it is to be fitted. It serves the purpose of preventing sand and other impurities from entering into the motor subassembly.

In the working condition of the subassembly, all the impellers have to be centered within the space available in the casings (Fig. 6). The shaft has to be pushed upwards...
4. Tolerance analysis for a five stage pump

The functional dimensions affecting a critical dimension can be identified by conducting tolerance analysis on the product. The product shown in Fig. 8 has totally ten dimensions. The critical dimension is horizontal distance between the hole centre. Functional dimensions affecting the critical dimension are identified by the tolerance loop as shown in Fig. 8. Starting from the first hole, moving along the dimension line towards right is considered positive and vice versa.

![Critical Dimension Diagram](image)

For the loop shown in (Fig. 8), the loop equation is

\[ \text{Critical Dimension} + X_5 - X_3 + X_4 = 0 \]  \hspace{1cm} (1)

where Critical Dimension = -X_5 + X_3 - X_4.

From the loop equation, it can be identified that the functional dimensions X_5, X_3 and X_4 affect the critical dimension while the remaining seven dimensions do not affect it. A similar analysis can be conducted on the pump subassembly to identify functional dimensions affecting the dimension P.

4.1. Assembly parameter affecting tolerance loop

The pump subassembly shown in Fig. 5 has all the five impellers occupying the bottom positions in their respective casings. But in real conditions, all the impellers will not touch their respective casings simultaneously. Only one impeller will touch the casing while all the other impellers will maintain some gap between them and their respective casings. Deciding which impeller will touch the casing is based on the tolerance built up of parts of the assembly. Thus the tolerance loop varies according to the impeller that touches the casing. For a pump with five stages, five different tolerance loops are therefore possible for the dimension P.

4.2. Tolerance loop for five different conditions of assembly

The first condition of the assembly is shown in Fig. 9. The first impeller is in contact with its casing while the other four impellers maintain some gap between them and their respective casings. The loop equation for this condition of assembly is

\[ P = X_1 + X_27 - X_29 - X_17 \]  \hspace{1cm} (2)

![Fig. 9 First impeller in contact with its casing](image)

![Fig. 10 Second impeller in contact with its casing](image)
Fig. 10 shows the second impeller in contact with its casing. The loop equation for this condition is

$$P = X_1 + X_2 + X_{24} - X_{25} - X_{26} - X_{15} - X_{16} - X_{17} \quad (3)$$

Figs. 11 - 13 show respectively the third, fourth and fifth impellers in contact with their casings. The corresponding equations are

$$P = X_1 + X_2 + X_3 + X_{21} - X_{22} - X_{23} - X_{13} - X_{14} - X_{15} - X_{16} - X_{17} \quad (4)$$

$$P = X_1 + X_2 + X_3 + X_4 + X_{18} - X_{19} - X_{20} - X_{11} - X_{12} - X_{13} - X_{14} - X_{15} - X_{16} - X_{17} \quad (5)$$

$$P = X_1 + X_2 + X_3 + X_4 + X_5 + X_6 - X_7 - X_8 - X_9 - X_{10} - X_{11} - X_{12} - X_{13} - X_{14} - X_{15} - X_{16} - X_{17} \quad (6)$$

Comparing the Eqs. (2) to (6), it can be found that the largest tolerance loop occurs with the last impeller making contact with its casing. Finding the critical dimension $P$ using Eq. (6) therefore will yield the largest tolerance and it gives the worst condition of assembly. Hence tolerance design is accomplished by considering the last impeller in contact with its casing.
4.3. Tolerance summation

The value of the critical dimension $P$ can be found out from Eq. (6) using three different methods namely worst case analysis, normal law and Monte Carlo simulation. The worst case analysis yields the following results.

$$P = X_1 + X_2 + X_3 + X_4 + X_5 + X_6 - X_7 - X_8 - X_9 - X_{10} - X_{11} - X_{12} - X_{13} - X_{14} - X_{15} - X_{16} - X_{17} = 125 + 40 + 40 + 40 + 4 + 5 - 7 - 20 - 20 - 20 - 20 - 20 - 40 = 77$$

$$p = 0.2 + 0.2 + 0.2 + 0.2 + 0.1 + 0.1 + 0.1 + 0.1 + 0.1 + 0.1 + 0.05 = 2.15$$

$$P \pm p = 77 \pm 2.15$$

The worst case approach is based on the assumption that all the functional dimensions affecting critical dimension simultaneously assume their extreme values. But in real practice, the probability of all the dimensions assuming their extreme values is very low. Hence the resulting dimension value arrived using this approach has wider tolerance than what really results in the actual condition.

The normal law can be applied to a product assembly if all the functional dimensions in a tolerance loop follow normal frequency distribution pattern. If a functional dimension on a part is arrived by machining the corresponding surfaces, it can be expected that it will follow normal frequency distribution pattern. In the submersible pump assembly under the study, all the functional dimensions have been arrived by machining in lathe. Therefore normal law can be applied to find the critical dimension. Applying normal law,

$$p = \sqrt{0.2^2 + 0.2^2 + 0.2^2 + 0.2^2 + 0.1^2 + 0.1^2 + 0.1^2 + 0.1^2 + 0.1^2 + 0.1^2 + 0.05^2} = 0.56$$

$$P \pm p = 77 \pm 0.56$$

The Monte Carlo simulation technique simulates the assembly of different parts of the pump to make the final product. If an assembly is made of two components having functional dimensions $7 \pm 0.1$ and $5 \pm 0.08$, the assembly dimension can lie between 11.82 and 12.18 as per the worst case analysis. When the assembly is mass produced, the exact location of each assembly dimension in the range 11.82 – 12.18 depends on the locations occupied by the part dimensions within their respective ranges. The Monte Carlo simulation randomly chooses one location within the range 6.9 – 7.1 for the first dimension and another random location within the range 4.92 – 5.08 for the second dimension. The assembly dimension is found out by summing these part dimension values. Thus the value arrived in the first simulation is registered. A number of simulations is run by this method and simulations results are shown in the form of frequency chart and statistics. The simulation software Crystal Ball \[3\] is used for this purpose. The resulting dimension and its tolerance can be found out from the statistical output of the software simulation.

The tolerance loop for the five stage submersible assembly has totally seventeen functional dimensions as shown in Fig. 13. The nominal value, frequency distribution and the standard deviation of all the seventeen dimensions are the inputs given to the Crystal Ball software as shown in Fig. 14. The outputs of the software are shown in Fig. 15 and Fig. 16. The standard deviation of the resulting frequency distribution pattern is 0.19. The resulting dimension tolerance $p$ is therefore three times the standard deviation i.e., 0.57 mm.

The frequency chart for the resulting dimension $P$ for a five stage pump is shown in Fig. 15. The statistical output of the simulation is shown in Table 3.
The resulting dimension $P$ obtained by the three methods namely worst case approach, normal law and Monte Carlo simulation are compared in Table 1. Among the three methods, the Monte Carlo simulation technique is found to give more accurate results since it simulates the real condition of the assembly.

**Table 1**

<table>
<thead>
<tr>
<th>Summation method</th>
<th>Worst case analysis</th>
<th>Normal law</th>
<th>Monte Carlo simulation using Crystal ball software</th>
</tr>
</thead>
<tbody>
<tr>
<td>Resulting Dimension $P$</td>
<td>$77 \pm 2.15$</td>
<td>$77 \pm 0.56$</td>
<td>$77 \pm 0.57$</td>
</tr>
</tbody>
</table>

5. Simulation of results for different number of stages

The number of stages of a submersible pump depends on the depth at which the pump works in a borewell. In places where water is available only at greater depths, a submersible pump with higher number of stages is required to lift water from the borewell [4]. Thus the number of stages in a pump varies from five up to a maximum of forty. The tolerance loop and the value of dimension $P$ also varies accordingly. Monte Carlo simulation technique was applied using Crystal Ball software to find the resulting tolerance $p$ for different number of stages. Table 2 lists the resulting tolerance for $P$ for different number of stages. As the number of stages increases, the resulting tolerance increases nonlinearly as shown in Fig. 17.

**Table 2**

<table>
<thead>
<tr>
<th>S.No.</th>
<th>Number of stages</th>
<th>Number of functional dimensions</th>
<th>Resulting tolerance $p$, mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5</td>
<td>17</td>
<td>0.57</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
<td>32</td>
<td>0.78</td>
</tr>
<tr>
<td>3</td>
<td>15</td>
<td>47</td>
<td>0.96</td>
</tr>
<tr>
<td>4</td>
<td>20</td>
<td>61</td>
<td>1.11</td>
</tr>
<tr>
<td>5</td>
<td>25</td>
<td>77</td>
<td>1.23</td>
</tr>
<tr>
<td>6</td>
<td>30</td>
<td>92</td>
<td>1.35</td>
</tr>
<tr>
<td>7</td>
<td>35</td>
<td>107</td>
<td>1.44</td>
</tr>
<tr>
<td>8</td>
<td>40</td>
<td>122</td>
<td>1.56</td>
</tr>
</tbody>
</table>

6. Selective assembly

In order to maintain the functional quality of the submersible pump assembly, axial plays between the impellers and their respective casings are to be controlled between $2 \pm 0.5$ as already discussed in the introduction. The axial play is introduced in the pump subassembly by machining the sand guard to the required size. Since the final assembly of the product involves machining operation, the assembly time is unnecessarily elongated resulting in cost increase. Two approaches are suggested to avoid machining operation in final stage of the assembly. In the first approach, tolerances of the part dimensions can be reallocated [5] so that the play between the impeller and casing does not exceed $2 \pm 0.5$ mm. This approach calls for tightening the part tolerances which definitely results in the increase in total assembly cost. Moreover, the submersible pump with forty stages has totally 122 numbers of functional dimensions. It is infeasible to allocate tolerances to all the 122 dimensions satisfying the assembly constraint. Therefore this approach is not recommended.

The concept of selective assembly is used in the second approach. The complete assembly of entire parts is made except for the last component. In the submersible pump assembly, the sand guard is the last component to be assembled. The sand guard is manufactured with different thicknesses. The total pump assembly is made without the sand guard. The gap wherein the sand guard is to be assembled is measured. Then suitable sand guard is chosen from the available range. The two subassemblies are dismantled for the purpose of putting the sand guard in place and reassembled again. In this approach, the dismantling and reassembling of the subassemblies might result in some delay in the final stage. This delay can be eliminated by arriving at the sand guard gap based on the measurements $P$, $M$ and coupling length using the equation

$$\text{Sand guard gap} = P + M - \text{coupling length} \quad (7)$$

The Table 3 shows the sand guard to be chosen for different ranges of sand guard gap.

**Table 3**

<table>
<thead>
<tr>
<th>Sl.No.</th>
<th>Sand guard gap, mm</th>
<th>Sand guard thickness, mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6.41 to 6.65</td>
<td>8.2 ± 0.05</td>
</tr>
<tr>
<td>2</td>
<td>6.65 to 7.55</td>
<td>9.1 ± 0.05</td>
</tr>
<tr>
<td>3</td>
<td>7.55 to 8.45</td>
<td>10 ± 0.05</td>
</tr>
<tr>
<td>4</td>
<td>8.45 to 9.35</td>
<td>10.9 ± 0.05</td>
</tr>
<tr>
<td>5</td>
<td>9.35 to 10.25</td>
<td>11.8 ± 0.05</td>
</tr>
</tbody>
</table>

7. Conclusions

A five stage submersible pump assembly is analyzed in the present study. A need was felt by the industry producing this product to standardize the coupling that transmits power from motor shaft to pump shaft. Tolerance analysis was conducted to identify the causes of variation of coupling length. Tolerance loops were drawn to identify functional dimensions affecting the coupling length. It was found that five different tolerance loops are possible for a...
five stage pump. Thus a pump with \( n \) stages will have \( n \) possible tolerance loops. Since tolerance design has to be conducted for the worst condition of assembly, tolerance loop having maximum number of functional dimensions has to be considered for the analysis. Thus for the five stage pump, the assembly condition involving the fifth impeller in contact with its casing has resulted in the largest tolerance loop. The critical dimension value was found out based on this largest tolerance loop using three different methods. The concept of selective assembly was used to overcome the problem of nonstandardized coupling length.

8. References


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Summary

Axial play between the impeller and its casing is one of the critical dimensions in a multistage submersible pump assembly. The axial play is introduced in the assembly using a coupling that connects the motor and pump sub-assemblies. The length of the coupling could not be standardized by the industry where this analysis was conducted. Tolerance analysis is conducted on the pump sub-assembly to identify the causes of variation of coupling length. It has been found that there are \( n \) possible tolerance loops in a \( n \) stage pump. Since the tolerance design is to be conducted for the worst case, the assembly condition that yields the largest tolerance loop has been considered for the analysis. The methodology currently adopted by industry to overcome the problem of variation of coupling length involves machining operation during the assembly stage. It results in unreasonably longer assembly time. The concept of selective assembly is suggested in this paper to overcome the problem of coupling length variation.

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Resumé


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