

# A simple-to-integrate formula of stress as a function of strain in concrete and its description procedure

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## 1. Introduction

When developing and researching structures it is often necessary to calculate the stress strain state of reinforced concrete members at sections perpendicular to the axis (at normal sections). The calculations have to be performed at different stages of loading:

- before cracking, (calculation of the stress of prestressed concrete, calculation of the losses of prestressing of reinforced concrete members);
- at cracking stage (defining the cracking moment);
- in concrete members with cracks at service stage (forecasting the width of cracks, the curvature and stiffness of the structures with cracks);
- at element failure stage (defining bearing strength of the element).

In the regulations [1], for the calculations of the parameters of these states simple but very conditional diagrams of the stresses are used:

- to calculate stresses before cracking triangular diagrams of elastic materials are taken;
- the cracking moments are calculated according to a very inaccurate pattern: in tensile zone – *rectangular* diagram, in compressed zone – *curvilinear* diagram that is not clearly defined;
- the empirical formulae for calculating crack width curvature and stiffness and the equations of the element strength were obtained having assumed a *rectangular* compressed concrete stresses diagram; the tensioned concrete above the crack tensioning is absolutely disregarded here.

In the missing of a uniform calculation method and when calculation patterns are so grossly distorted it is assumed:

- numerous correction coefficients need to be used;
- the empiric formulae of these correction coefficients are sometimes very bulky and cumbersome and devising of such formulae is costly;
- there is no possibility to calculate the height of the cracks and the height of tensile zones of the concrete above them;
- the height of compressive zone of the concrete is defined with a very low degree of accuracy;
- it is impossible to calculate stress strain state of reinforced members according to the measured parameters for the stages before cracking and before failure;
- it is impossible to define stress strain state in plastic hinges of the continuity beams, i.e. when the strength of the member decreases due to the fact that the stress of the reinforcement exceeds yield limit or/and the stresses of the compressive zone of concrete exceed the strength limit of concrete;
- there is no method of defining stress strain state of

reinforced members according to the measured parameters of the cracks (the height and the width of the cracks and the distance between the cracks) in stages when their loads exceed the service loads.

In the absence of a general calculation method and when so grossly distorted calculation patterns are assumed, the calculation has a lot of shortcomings.

It is possible to solve many of these problems using more realistic curvilinear diagrams of interrelation between strain and stress of concrete. Such a curvilinear diagram is presented in the regulations [1, 2]. But its use is aggravated by the integral calculation problems.

Unfortunately, there exists no such universal but user-friendly method that would directly use nonlinear stress strain relation without replacing it, for instance, by broken lines comprised of segments of a straight line [3] or would take into account the possible deviations of the strains of the materials from the plane sections hypothesis [4].

In the calculation methods proposed by the author [5, 6] simple-to-integrate *curvilinear* diagrams of materials are used and it is possible to take into account the deviations of the strains from the *plane sections hypothesis*.

The relevance of the use of curvilinear diagrams  $\sigma - \varepsilon$  is proved by research papers of numerous authors [7–15].

**The objective of the present paper** is to describe the most general curvilinear diagram of interrelation between strain and stress of concrete presented in the the regulations [1, 2] Eq. (1) so that it would be easy to integrate and would be applicable to the concrete of strength classes C8/10-C90/105.

## 2. Object of the work

The present paper deals with the normal, heavy and fine-grained concrete of classes C8/10-C90/105.

The key designations are the same as in the regulations [1]:  $\sigma_c$  is normal stresses of compression of concrete;  $f_{cm}$  is average compression stress of concrete;  $f_{ck}$  is characteristic concrete crushing stress;  $E_{cm}$  is concrete secant modulus of elasticity;  $\varepsilon_c$  is strain of concrete;  $\varepsilon_{c1}$  is strain of concrete corresponding to  $f_{cm}$ ;  $\varepsilon_{cu1}$  is limiting strain/deformation of concrete.

SI units used in the paper:

- of stress – MPa;
- of elastic modulus – GPa;
- of strain – %.

Fig. 1 shows curvilinear interrelation between strain and stress  $\sigma - \varepsilon$  of concrete specified in the concrete and reinforced concrete design regulations [1] adopt-

ed for the eurocode area [2].

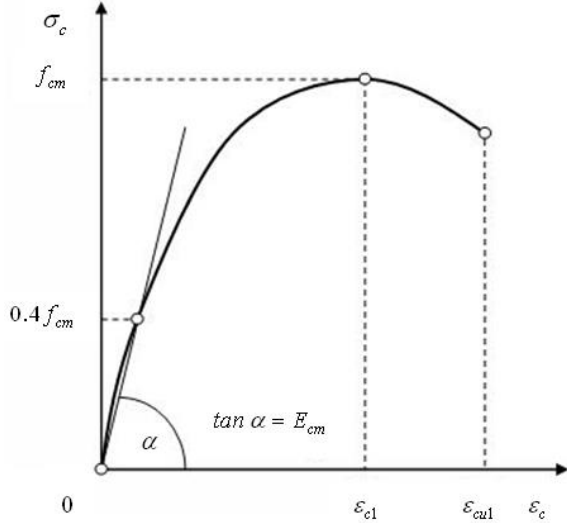


Fig. 1 Stress strain interrelation of concrete (Eq. 1)

This  $\sigma$ - $\varepsilon$  interrelation is described by expressions

(1-3)

$$\frac{\sigma_c}{f_{cm}} = \frac{k\eta - \eta^2}{1 + (k - 2)\eta} \quad (1)$$

$$\eta = \varepsilon_c / \varepsilon_{c1} \quad (2)$$

$$k = \frac{1.1E_{cm}\varepsilon_{c1}}{f_{cm}} \quad (3)$$

Here and hereafter the values of  $\varepsilon_{c1}$  and  $\sigma_c$  are considered to be positive. If  $E_c = 1.1E_{cm}$  and  $\nu_{c1} = f_{cm} / E_c \varepsilon_{c1}$ , then  $k = 1/\nu_{c1}$  and

$$\sigma_c = \frac{\eta / \nu_{c1} - \eta^2}{1 + (1/\nu_{c1} - 2)\eta} f_{cm}.$$

Eq. (1) is simple, but is unhandy when we have to

Values of coefficients  $c_i$

$$c_1 = \{2(\eta_r + 1)(1 - \eta_r)^3 - (\eta_r - 1)e_r + [(5\eta_r - 3)n_r + (3\eta_r - 5)\eta_r^3]\nu_{c1}\} / [\eta_r(\eta_r - 1)^3] \quad (6)$$

$$c_2 = \{(\eta_r^2 + 4\eta_r + 1)(\eta_r - 1)^3 + (2\eta_r^2 - \eta_r - 1)e_r + 2[(1 + \eta_r - 5\eta_r^2)n_r + (5 - \eta_r - \eta_r^2)\eta_r^3]\nu_{c1}\} / [\eta_r^2(\eta_r - 1)^3] \quad (7)$$

$$c_3 = \{2(\eta_r + 1)(1 - \eta_r)^3 - (\eta_r^2 + \eta_r - 2)e_r + [(5\eta_r^2 + 5\eta_r - 4)n_r + (4\eta_r^2 - 5\eta_r - 5)\eta_r^2]\nu_{c1}\} / [\eta_r^2(\eta_r - 1)^3] \quad (8)$$

$$c_4 = \{(\eta_r - 1)^3 + (\eta_r - 1)e_r + 2[(1 - 2\eta_r)n_r + (2 - \eta_r)\eta_r^2]\nu_{c1}\} / [\eta_r^2(\eta_r - 1)^3] \quad (9)$$

### 3. Calculation procedure of stresses $\sigma_c$ employing the proposed method

Assuming that  $k_2 = k - 2$ ,  $\eta = \varepsilon_c / \varepsilon_{c1} = \eta_{11}\varepsilon_c$ ,  $\eta_{11} = 1/\varepsilon_{c1}$ ,  $\beta_c = \sigma_c / f_{cm}$ , from Eq. (1) we get

$$\sigma_c = \frac{f_{cm}(k - \eta)\eta}{1 + k_2\eta} \quad (10)$$

integrate curvilinear diagrams of stresses.

In the methodology proposed by us [5, 6] for the calculation of the parameters of stress of strain of reinforced concrete at normal sections a curvilinear  $\sigma$ - $\varepsilon$  diagram is used which is described by the following formula

$$\sigma_c = \nu_c E_c \varepsilon_c = \nu_c \sigma_{ce} \quad (4)$$

In Eqs. (4-9)

$$\nu_c = 1 + c_1\eta + c_2\eta^2 + c_3\eta^3 + c_4\eta^4 \quad (5)$$

$E_c = \tan \beta$ ;  $E_r = \tan(-\gamma)$ ;  $e_r = E_r / E_c$ ;  $\sigma_{ce} = E_c \varepsilon_c$ ;  $\sigma_{ce1} = E_c \varepsilon_{c1}$ ;  $\sigma_{c1} = f_{cm}$ ;  $\nu_{c1} = \sigma_{c1} / \sigma_{ce1}$ ;  $\nu_r = \sigma_{cr} / \sigma_{cre}$ ;  $n_r = \nu_r / \nu_{c1} = \beta_r / \eta_r$ ;  $\beta_r = \sigma_{cr} / \sigma_{c1}$ ;  $\eta_r = \varepsilon_{cr} / \varepsilon_{c1}$ . For values  $\beta$ ,  $\gamma$  and for values of indexes see Fig. 2.

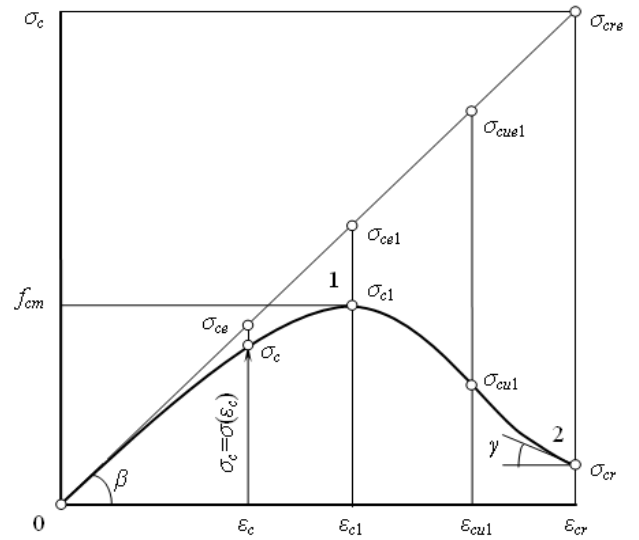


Fig. 2 Interrelation between strain and stress of concrete (Eq. 4)

$$\eta = \frac{k - \beta_c k_2}{2} \pm \sqrt{\frac{(k - \beta_c k_2)^2}{4} - \beta_c} \quad (11)$$

$$E_t = \frac{d\sigma_c}{d\varepsilon_c} = \frac{f_{cm}\eta_{11}(k - 2\eta - k_2\eta^2)}{(1 + k_2\eta)^2} \quad (12)$$

The values of  $E_t$  calculated using Eq. (12) in the interval  $\varepsilon_c < \varepsilon_{c1}$  are positive, and in the interval  $\varepsilon_c > \varepsilon_{c1}$  they are negative.

When  $\beta_c = 0$ , i.e.  $\sigma_c = 0$ , then  $\eta = k$ .

Using the formulae given in the regulations [1] we get the following values of the parameters

$$f_{cm} = f_{ck} + 8 \quad (13)$$

$$E_{cm} = 22(f_{cm}/10)^{0.3} \quad (14)$$

$$E_c = 1.1E_{cm} \quad (15)$$

$$\varepsilon_{c1} = 0.7 f_{cm}^{0.31} \quad (16)$$

$$\varepsilon_{cu1} = 3.5, \text{ if } f_{ck} \leq 50 \text{ MPa} \quad (17)$$

$$\varepsilon_{cu1} = 2.8 + 27[(98 - f_{cm})/100]^4, \text{ if } f_{ck} \geq 50 \text{ MPa}$$

The next step is as follows:

- it is assumed that  $\varepsilon_{cr} = \varepsilon_{cu1}$ ; then  $\eta_r = \varepsilon_{cr} / \varepsilon_{c1} = \varepsilon_{cu1} / \varepsilon_{c1}$ ; when  $\eta_r < 1,1$ , then instead of the actual value of  $\eta_r$  we assume that  $\eta_r = 0,5$ ;
- according to  $\eta = \eta_r$  from Eq. (10) we get  $\sigma_c = \sigma_{cr}$  and using Eq. (12) we get the value  $E_t = E_r$  of the tangential modulus;
- we calculate  $\beta = \beta_r = \sigma_{cr} / \sigma_{c1} = \sigma_{cr} / f_{cm}$ ,  $n_r = \nu_r / \nu_{c1} = \beta_r / \eta_r$ ,  $e_r = E_r / E_c$  and  $\nu_{c1} = f_{cm} / E_c \varepsilon_{c1}$ ;
- using Eq. (4) we calculate the values of  $\sigma_c$ .

For the calculation of the values of stress  $\sigma_c$  using Eq. (1) and Eq. (4), the author of the present paper has developed respective programmes. When calculating  $\sigma_c$  the value of only one parameter  $f_{ck}$  needs to be entered in the programs.

The comparison of the results of stress calculation  $\sigma_c$  according to Eq. (1) of the regulations [1] and Eq. (4) proposed by the author of the present paper is illustrated in Fig. 3 and in the Table. The results of the calculation of strain  $\varepsilon_c$  in the interval from zero till  $\varepsilon_{cu1}$ , (in Fig. 3 the limit  $\varepsilon_{cu1}$  is marked by vertical lines in the graph, and by bold numbers in the Table) according to both methods are very close: the maximum difference is 1.5 %. The results are also close when  $\varepsilon_c > \varepsilon_{cu1}$ .

We have also analysed the values of  $\sigma_c$  for the concretes whose  $f_{ck} = (5-8)$  MPa and  $f_{ck} = (90-130)$  MPa, that have been similarly calculated using Eq. (1) and Eq. (4). Good results have been obtained.

#### 4. Examples of the curvilinear diagram use

We illustrate the possibility of using the curvilinear diagram by an example of the calculation of the reinforcement and bearing strength of a reinforced concrete beam.

The cross-section to be calculated and the forces

acting in it are shown in Fig. 4. The stresses of the concrete in tensile zone are disregarded. The stresses of the concrete in compressive zone are calculated from Eq. (4). The resultant force  $N_c$  of these stresses and its moment  $M_c$  in respect of the neutral axis 0-0 is calculated from the Eqs. (18) and (19)

$$N_c = \int_{x_w}^0 \sigma_c b(dx_c) = \omega \varepsilon_w E_c b x_w = \omega \varepsilon_w E_c b d \xi_w \quad (18)$$

$$M_c = \int_{x_w}^0 \sigma_c b x_c (dx_c) = \varpi \varepsilon_w E_c b x_w^2 = \varpi \varepsilon_w E_c b d^2 \xi_w^2 \quad (19)$$

In Eqs. (18), (19) (see [5 or 6])

$$\omega = \frac{1}{2} + \frac{c_1}{3} \eta + \frac{c_2}{4} \eta^2 + \frac{c_3}{5} \eta^3 + \frac{c_4}{6} \eta^4 \quad (20)$$

$$\varpi = \frac{1}{3} + \frac{c_1}{4} \eta + \frac{c_2}{5} \eta^2 + \frac{c_3}{6} \eta^3 + \frac{c_4}{7} \eta^4 \quad (21)$$

$$\eta = \frac{\varepsilon_w}{\varepsilon_{c1}}, \quad c_i - \text{from Eqs. (6)-(9)}, \quad \xi_w = \frac{x_w}{d}.$$

Eccentricity  $e_c$  of force  $N_c$  in respect of the neutral axis 0-0 is as follows

$$e_c = \frac{M_c}{N_c} = \frac{\varpi}{\omega} x_w = \frac{\varpi}{\omega} \xi_w d \quad (22)$$

When the plane-sections hypothesis is assumed, then

$$\frac{\varepsilon_w}{\varepsilon_s} = \frac{x_w}{d - x_w} = \frac{\xi_w}{1 - \xi_w} \quad (23)$$

and the force of the reinforcement

$$\begin{aligned} N_s &= \sigma_s A_s = \varepsilon_s \nu_s E_s A_s = \nu_s E_s A_s \frac{d - x_w}{x_w} \varepsilon_w = \\ &= \nu_s E_s A_s \frac{1 - \xi_w}{\xi_w} \varepsilon_w \end{aligned} \quad (24)$$

In Eq. (24)  $E_s$  is the module of elasticity of reinforcement and  $\nu_s = \sigma_s / \varepsilon_s E_s$  is the coefficient of elasticity of reinforcement.

Interval from  $N_c$  to  $N_s$

$$\begin{aligned} z_c &= d - a_c = d - \left(1 - \frac{\varpi}{\omega}\right) d \xi_w = \\ &= \left[1 - \left(1 - \frac{\varpi}{\omega}\right) \xi_w\right] d \end{aligned} \quad (25)$$

where

$$a_c = x_w - e_c = d \xi_w - \frac{\varpi}{\omega} d \xi_w = \left(1 - \frac{\varpi}{\omega}\right) d \xi_w \quad (26)$$

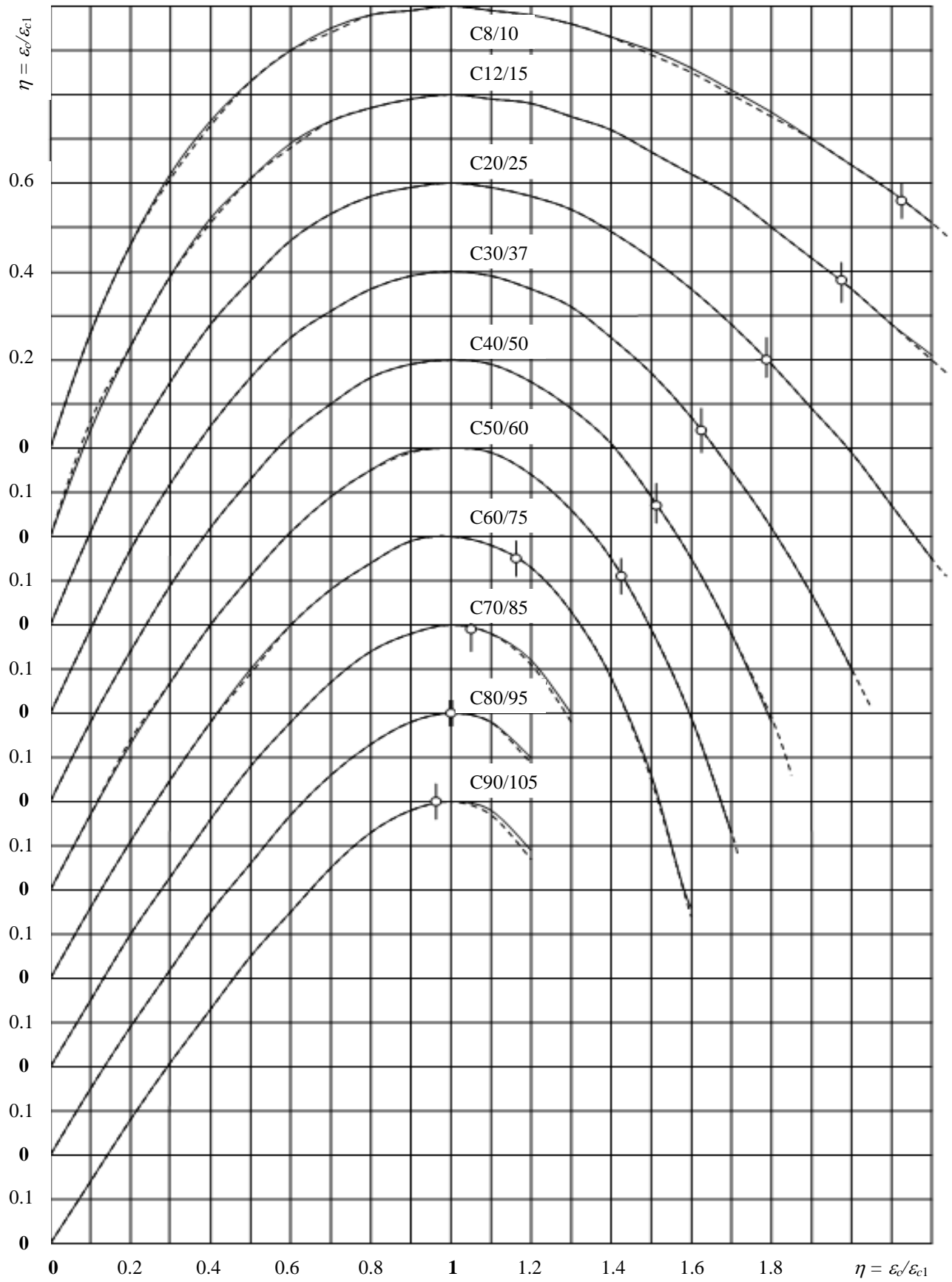


Fig. 3 Graphs of the relative values  $\beta = \sigma_c / f_{cm}$  of stresses  $\sigma_c$ : -----  $\beta_{STR} = \sigma_c / f_{cm}$ , when  $\sigma_c$  calculated using Eq. (1);  
 —  $\beta_{ZI} = \sigma_c / f_{cm}$ , when  $\sigma_c$  calculated using Eq. (4)



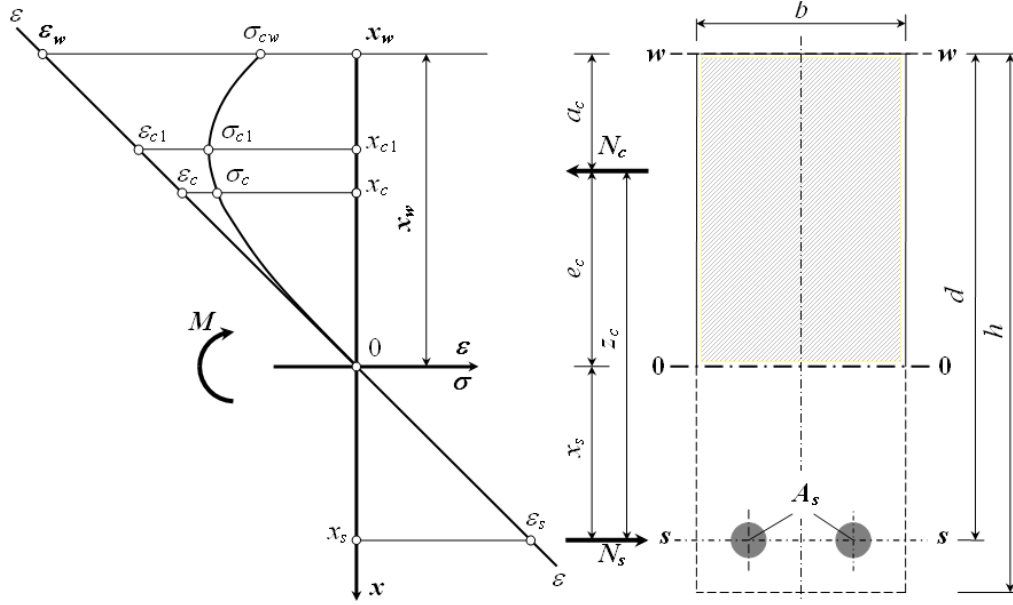


Fig. 4 The cross-section to be calculated and the acting forces

The moment of force  $N_c$  in respect of axis  $s-s$

$$M_s = N_c z_s = \omega \varepsilon_w E_c b d \xi_w \left[ 1 - \left( 1 - \frac{\varpi}{\omega} \right) \xi_w \right] d =$$

$$= [\omega \xi_w - (\omega - \varpi) \xi_w^2] \varepsilon_w E_c b d^2 \quad (27)$$

The conditions of static equilibrium of axial forces and bending moments in respect of the axis  $s-s$  of the centre (point) of gravity of the reinforcement

$$\omega \varepsilon_w E_c b d \xi_w - \nu_s E_s A_s \frac{1 - \xi_w}{\xi_w} \varepsilon_w = 0 \quad (28)$$

$$\omega \varepsilon_w E_c b d \xi_w^2 + \nu_s \varepsilon_w E_s A_s \xi_w - \nu_s \varepsilon_w E_s A_s = 0 \quad (28a)$$

$$M - [\omega \xi_w - (\omega - \varpi) \xi_w^2] \varepsilon_w E_c b d^2 = 0 \quad (29)$$

$$(\omega - \varpi) \varepsilon_w E_c b d^2 \xi_w^2 - \omega \varepsilon_w E_c b d^2 \xi_w + M = 0 \quad (29a)$$

When the plane-sections hypothesis is disregarded and it is assumed that  $\varepsilon_w = \varepsilon_{cu1}$  and  $\sigma_s = f_s$  (here  $f_s$  is stress of reinforcement), simpler equations are received

$$\omega \varepsilon_w E_c b d \xi_{cu1} - f_s A_s = 0 \quad (30)$$

$$M = M_s = N_c z_c = [\omega \xi_w - (\omega - \varpi) \xi_w^2] \varepsilon_{cu1} E_c b d^2 \quad (31)$$

**For example**, dimensions of the rectangular cross-section shown in Fig. 4:  $b = 0.20$  m;  $h = 0.50$  m;  $d = 0.46$  m; strength class of concrete C25/30  $\sigma_{c1} = f_{cm} = 33$  MPa;  $E_{cm} = 31.476$  GPa;  $\varepsilon_{c1} = 2.0694$  ‰;  $\varepsilon_{cu1} = 3.5$  ‰; reinforcement S400:  $f_s = f_{sk} = 400$  MPa;  $E_s = 200$  GPa.

$$E_c = 1.1 E_{cm} = 1.1 \cdot 31.476 = 34.623 \text{ GPa,}$$

$$\nu_{c1} = \frac{\sigma_{c1}}{E_c \varepsilon_{c1}} = \frac{33}{31.476 \cdot 2.0694} = 0.5066 ,$$

$$\alpha_{es} = E_s / E_c = 200 / 34.623 = 5.7765 .$$

When  $\varepsilon_w = \varepsilon_{cu1} = 3.5$  ‰, then

$$\eta = \frac{\varepsilon_{cw}}{\varepsilon_{c1}} = \frac{3.5}{2.0694} = 1.6913 , \text{ and from Eqs. (6-9):}$$

$$c_1 = -0.6311 , \quad c_2 = 0.1059 , \quad c_3 = -0.01559 \quad \text{and}$$

$$c_4 = 0.001389 .$$

From Eqs. (20) and (21)  $\omega = 0.20674$  and  $\varpi = 0.11622$  .

4.1. Calculation of the area  $A_s$  of the reinforcement, when the bending moment  $M = 240$  kN·m

$$c1 = \varepsilon_w E_c b d = 3.5 \cdot 34.623 \cdot 0.20 \cdot 0.46 = 11.1486 \text{ MN;}$$

$$c = \varepsilon_w E_c b d^2 = 3.5 \cdot 34.623 \cdot 0.20 \cdot 0.46^2 = 5.1284 \text{ MN}\cdot\text{m;}$$

$$\omega - \varpi = 0.20674 - 0.11622 = 0.09052 .$$

From Eq. (29a) we calculate the value of  $\xi_w$

$$(0.20674 - 0.11622) \cdot 3.5 \cdot 34.623 \cdot 0.20 \cdot 0.46^2 \cdot \xi_w^2 +$$

$$-0.20674 \cdot 3.5 \cdot 34.623 \cdot 0.20 \cdot 0.46^2 \cdot \xi_w + 240 \cdot 10^{-3} = 0;$$

$$0.09052 \cdot 5.1284 \cdot \xi_w^2 - 0.20674 \cdot 5.1284 \cdot \xi_w + 240 \cdot 10^{-3} = 0;$$

$$0.46422 \cdot \xi_w^2 - 1.06025 \cdot \xi_w + 240 \cdot 10^{-3} = 0 ;$$

$$\xi_w^2 - 2.28394 \cdot \xi_w + 0.51700 = 0 ;$$

$$\xi_w = \frac{2.28393}{2} \pm \sqrt{\left( \frac{2.28393}{2} \right)^2 - 0.51700} =$$

$$= 1.14197 \pm 0.88718 = 0.25479$$

$$x_w = \xi_w d = 0.25479 \cdot 0.46 = 0.11720 \text{ m}$$

From Eq. (30) we calculate the area  $A_s$  of cross-section of the reinforcement

$$\begin{aligned} A_s &= \omega \varepsilon_{cu1} E_c b d \xi_w / f_s = \\ &= 0.20674 \cdot 3.5 \cdot 34.623 \cdot 0.20 \cdot 0.46 \cdot 0.25479 / 400 = \\ &= 14.681 \cdot 10^{-4} \text{ m}^2 \end{aligned}$$

The reinforcement rate

$$\rho_l = 14.681 \cdot 100 / (20 \cdot 46) = 1.596 \%$$

4.2. Calculation of the bending moment  $M$  when the area of the reinforcement  $A_s = 14.681 \cdot \text{cm}^2$ .

From Eq. (30) we calculate  $\xi_w$

$$\xi_w = \frac{f_s A_s}{\omega \varepsilon_{cu1} E_c b d} = \frac{400 \cdot 14.681 \cdot 10^{-4}}{0.20674 \cdot 11.1486} = 0.25478$$

From Eq. (31) we calculate the bending moment:

$$\begin{aligned} M &= [\omega \xi_w - (\omega - \varpi) \xi_w^2] \varepsilon_w E_c b d^2 = \\ &= [0.20674 \cdot 0.25478 - 0.09052 \cdot 0.25478^2] \cdot 5.1284 = \\ &= 0.2399952 \text{ MN} \cdot \text{m} \approx 240 \text{ kN} \cdot \text{m}. \end{aligned}$$

## 5. Conclusions

For the calculation of stresses  $\sigma_c$  of heavy and fine-grained concrete of strength classes C8/10-C90/105 within the strain interval  $\varepsilon_c$  the interval from zero till  $\varepsilon_{cu1}$  instead of the equation adopted in the regulations [1] (in our paper it is marked by number (1)) Eq. (4) can be used. It is reasonable to do this when there is a need to perform integral calculation curvilinear diagram of stresses.

The results of calculation of stresses  $\sigma_c$  using Eqs. (1) and (4) are also close for concretes e  $f_{ck} = (5-8)$  MPa and  $f_{ck} = (90-130)$  MPa.

When the stress strain relation is described in an easy-to-integrate form such as Eq. (4), a lot of things may be calculated using the method proposed, for instance, in paper [6].

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LENGVAI INTEGRUOJAMA BETONO  
DEFORMACIJŲ IR ĮTEMPIŲ TARPUSAVIO  
PRIKLAUSOMYBĖ IR JOS APRAŠYMO METODIKA

R e z i u m ė

Dažnai tenka apskaičiuoti gelžbetoninių elementų deformacijų įtempių būvį normaliniuose pjūviniuose. Tai daroma įvairiose elemento apkrovimo stadijose: iki plyšių susidarymo, plyšimo, konstrukcijų su plyšiais eksploataavimo, elemento irimo stadijose.

Šių stadijų deformacijų įtempių būvio parametrus apskaičiuoti naudojamos labai sąlygiškos įtempių diagramos. Nėra kaip apskaičiuoti plyšių bei tempiamo betono aukščio virš jų. Labai netiksliai nustatomas gniuždomos betono zonos aukštis. Negalima apskaičiuoti minėtų parametrų stadijose prieš plyšių susidarymą bei prieš elementų irimą. Neturime galimybės nustatyti deformacijų įtempių būklę nekarpytų elementų plastiškumo šarnyruose. Nėra metodo nustatyti gelžbetoninių elementų deformacijų įtempių būklę pagal išmatuotus plyšių parametrus (plyšių plotį, aukštį ir atstumą tarp jų) stadijose, artimose elemento irimui.

Daug problemų galima išspręsti, naudojant realnes, kreivines betono deformacijų įtempių diagramas. Bet jas naudoti sunku dėl integravimo problemų.

Normaliojo, sunkiojo bei smulkiagrūdžio C8/10-C90/105 stiprio klasių betono įtempiams  $\sigma_c$  apskaičiuoti straipsnio autorius siūlo formulę. Ją tikslinga naudoti tuomet, kai reikia integruoti kreivinę įtempių diagramą, pavyzdžiui, sudarant poveikių statinės pusiausvyros lygtis.

I. Židonis

A SIMPLE-TO-INTEGRATE FORMULA  
OF STRESS AS FUNCTION OF STRAIN  
IN CONCRETE AND ITS DESCRIPTION PROCEDURE

S u m m a r y

It is often necessary to calculate the stress strain state of reinforced concrete members at normal sections. The calculations have to be performed at different stages of loading: before cracking, at the cracking moment, in sections on the crack under service and breaking load.

For the calculations, very inaccurate diagrams of the stresses are used. There is no possibility to calculate height of the cracks and height of tensile zones of the concrete above them. The height of compressive zone of the concrete is especially inaccurate. It is impossible to calculate the mentioned parameters for the stages before cracking and before fracture. It is impossible to define these parameters in plastic hinges of the continuity beams. There is no method of stress strain state calculation of reinforced concrete members according to the measured parameters of normal cracks (height and width of the cracks and distance between the cracks) in stages close to fracture load.

It is possible to solve many of these problems using more realistic curvilinear diagrams of interrelation between strain and stress of concrete. But its use is aggravated by the integration problem.

The author of the article offers his equation for the calculation of normal stress of heavy and fine-grained concrete of strength classes C8/10-90/105. It is reasonable to use this equation in cases when there is a need to integrate the curvilinear diagram of the stress when making equations of the static equilibrium of stresses.

И. Жидонис

ЛЕГКО ИНТЕГРИРУЕМАЯ ЗАВИСИМОСТЬ  
МЕЖДУ ДЕФОРМАЦИЯМИ И НАПРЯЖЕНИЯМИ  
БЕТОНА И МЕТОДИКА ЕЕ ОПИСАНИЯ

Р е з ю м е

Часто приходится рассчитывать напряженно-деформированное состояние по нормальным сечениям железобетонных элементов. Это делается в различных стадиях нагружения элемента: до образования трещин, на стадии трещинообразования, на стадиях работы элемента с трещинами, на стадии разрушения.

При расчете параметров напряженно-деформированного состояния на этих стадиях применяются сугубо условные эпюры напряжений. Нет возможности рассчитать высоту трещин и высоту растянутой зоны бетона над ними. Очень условно определяется высота сжатой зоны бетона. Нет возможности рассчитать упомянутые параметры на стадиях, близких к образованию трещин или разрушению элементов. Нет возможности определить напряженно-деформированное состояние в пластических шарнирах неразрезных балок. Нет метода для определения напряженно-деформированного состояния железобетонных элементов по параметрам трещин (высоте, раскрытию и расстоянию между трещинами) на стадиях, близких к разрушению.

Много проблем можно решить с применением более реальных, криволинейных диаграмм напряжений деформаций бетона. Но их применение затруднено сложностью интегрирования.

Для расчета напряжений обычного, тяжелого и мелкозернистого бетона классов прочности C8/10-C90/105 автор статьи предлагает зависимость для определения напряжений в бетоне  $\sigma_c$ . Эту формулу целесообразно применять в тех случаях, когда придется интегрировать криволинейную эпюру напряжений, например, при составлении уравнений статического равновесия усилий.

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