### Investigation of physically nonlinear behaviour of polystyrene packages and their elements

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#### 1. Introduction

One of the most important mechanical characteristics of packaging is its resistance to vertical compression load. This load increases significantly when the packed goods are loaded on pallets which, in their turn, are stacked one upon another. In such cases the greatest loads are affecting the packages at the bottom and the forces are directed vertically downwards. The packed product can sometimes overtake part of the loads and thus diminish the danger of damage.

In terms of their construction, the design of plastic packages meant for liquid and paste-type products of different consistencies may be different even when packing the same type of production; however, there exist certain major, globally accepted solution groups. The most popular plastic packages are truncated – cone – shape containers, tapered downwards, often thermally sealed with laminated multilayered foil lids or pressed plastic closures, etc. Recently, the application of plastics in packaging has been successfully competing with the industry of traditional packaging materials (paper, cardboard, glass and metal). Huge amounts of plastics as relatively cheap and easily recyclable material are used in packaging.

The data obtained when testing plastic containers subjected to static forces are important in developing the packaging more suitable for industry and usage in accordance with environmental EU requirements which define that further development of packaging is related to the manufacture of durable packages, minimizing the amount of packaging materials [1, 2, 3].

After performing the analytical investigation of available research papers it was found that there are some experimental investigations of vertical compression load of various surfaces and the results of performed investigations are presented in. In the research papers [4, 5] the effects of the types of material and of the shape of the plastic packages to the mechanical properties are analyzed. The authors of the paper consider that there are insufficient investigations in which the mechanical characteristics of polymeric packages and their behaviour under the vertical compression load are analyzed. Because of the mentioned reasons this research is considered important.

The model for the analysis of compression of a polystyrene package is based on the analysis of an axisymmetric physically nonlinear elastic structure. Nonlinear elasticity is taken into account using the hyperbolic model and considered as an approximation to the plastic behaviour which can be analyzed by the theory of deformational plasticity valid for monotonic loading [6-9]. The force is increased by small steps and thus the graphical relationship of axial load – axial deformation is calculated.

The model for the analysis of bending of a strip of polystyrene is presented. The beam model by taking cubic nonlinearity of Duffing type into account is used. The analysis is performed on the basis of the models for the analysis of beam bending described in [10, 11]. It is shown that the bending behaviour is substantially influenced by the physical nonlinearity.

Experimental investigations using a specially developed setup have been performed for a number of types of polystyrene packages. The main graphical relationships and results of the experimental investigations are presented. It is determined that the results of experimental investigations in the initial stage of deformations correspond with the numerical ones.

The obtained results are used in the process of design of the elements of packages.

# 2. Model for the analysis of compression of polystyrene packages

Axi-symmetric model is used for the investigation of physically nonlinear compression of polystyrene packages. Thus the structure is analyzed in the cylindrical system of coordinates and there is no dependence on the angular coordinate. Further x denotes the radial coordinate (sometimes denoted as r) and y denotes the axial coordinate of the cylindrical system of coordinates. The element has two nodal degrees of freedom: the displacements u and v in the directions of the axes x and y. The force is increased by small steps.

The equivalent strain on the basis of the second invariant of the strain deviator is calculated as

$$\overline{\varepsilon} = \sqrt{\left(\varepsilon_x - \varepsilon_y\right)^2 + \left(\varepsilon_y - \varepsilon_z\right)^2 + \left(\varepsilon_z - \varepsilon_x\right)^2 + \frac{3}{2}\gamma_{xy}^2}$$
(1)

where  $\varepsilon_x$ ,  $\varepsilon_y$ ,  $\varepsilon_z$ ,  $\gamma_{xy}$  are the components of strain of the axisymmetric problem with *z* corresponding to the circumferential (angular) direction of the cylindrical system of coordinates.

The matrix of elastic constants is expressed as

$$[D] = \begin{bmatrix} K + \frac{4}{3} \frac{G}{1 + aG\overline{\varepsilon}} & K - \frac{2}{3} \frac{G}{1 + aG\overline{\varepsilon}} & K - \frac{2}{3} \frac{G}{1 + aG\overline{\varepsilon}} & 0 \\ K - \frac{2}{3} \frac{G}{1 + aG\overline{\varepsilon}} & K + \frac{4}{3} \frac{G}{1 + aG\overline{\varepsilon}} & K - \frac{2}{3} \frac{G}{1 + aG\overline{\varepsilon}} & 0 \\ K - \frac{2}{3} \frac{G}{1 + aG\overline{\varepsilon}} & K - \frac{2}{3} \frac{G}{1 + aG\overline{\varepsilon}} & K + \frac{4}{3} \frac{G}{1 + aG\overline{\varepsilon}} & 0 \\ 0 & 0 & 0 & \frac{G}{1 + aG\overline{\varepsilon}} \end{bmatrix}$$
(2)

where  $K = \frac{E}{3(1-2\nu)}$  and  $G = \frac{E}{2(1+\nu)}$ , here E is the

modulus of elasticity and v is the Poisson's ratio and also a is the parameter of the hyperbolic model determining the physical nonlinearity.

Thus the stiffness matrix has the following form

$$[K] = \int [B]^T [D] [B] 2\pi x dx dy$$
(3)

here the integration includes the direct stiffness procedure, that is the integrations are performed over finite elements and then added to the corresponding place of the system matrix, also where

$$[B] = \begin{bmatrix} \frac{\partial N_1}{\partial x} & 0 & \dots \\ 0 & \frac{\partial N_1}{\partial y} & \dots \\ \frac{N_1}{x} & 0 & \dots \\ \frac{\partial N_1}{\partial y} & \frac{\partial N_1}{\partial x} & \dots \end{bmatrix}$$
(4)

where  $N_i$  are the shape functions of the analyzed finite element and one is to remember that x is the radial coordinate of the cylindrical system of coordinates.

#### 3. Results of analysis of compression of polystyrene packages

A thin axi-symmetric structure rectangular in the xOy plane parallel to the axial coordinate is analyzed.



Fig. 1 The axial load – axial deformation graphical relationship

On the lower boundary all the displacements are assumed equal to zero. The node at the centre of the upper boundary is loaded by a force in the direction of the axial coordinate, the value of the force is negative (it is acting in the opposite direction than the *y* axis).

The following values of physical parameters are assumed:  $E = 8 \text{ N/m}^2$ , v = 0.3,  $a = 6 \text{ m}^2/\text{N}$  (Eq. (2)) and the expressions of the bulk modulus *K* and the shear modulus *G* presented after this equation).

The graphical relationship is obtained and presented in Fig. 1. The axial load is represented in vertical direction, while the axial deformation of the same degree of freedom is represented in horizontal direction. From the presented results it is seen that the obtained deformation compression load dependence is non-linear.

# 4. Model for the analysis of nonlinear bending of a strip of polystyrene

The problem of nonlinear bending of a strip of polystyrene is analyzed using the orthogonal system of coordinates. This is another problem of analysis of physically nonlinear behaviour of polystyrene and one is to have in mind that though some quantities are denoted by the same letters as in the previous problem, they have a different meaning here. Further x, y and z denote the axes of the orthogonal system of coordinates. Bending of the strip of polystyrene as of a beam with physical nonlinearity is analyzed. The beam bending element has two nodal degrees of freedom: the displacement w in the direction of the z axis and the rotation  $\Theta_y$  about the y axis. The displacement u in the direction of the x axis is expressed as  $u=z\Theta_y$ .

Longitudinal strain is expressed as

$$\varepsilon_x = z[B]\{\delta\} \tag{5}$$

here z is the distance from the middle plane of the strip of polystyrene in the status of equilibrium, also where

$$\begin{bmatrix} B \end{bmatrix} = \begin{bmatrix} 0 & \frac{dN_1}{dx} & \dots \end{bmatrix}$$
(6)

where  $N_i$  are the shape functions of the finite element and  $\{\delta\}$  is the vector of generalized displacements.

The following notation is introduced

$$\overline{\varepsilon} = [B]\{\delta\} \tag{7}$$

Physical nonlinearity is assumed in the expression of longitudinal stress

$$\sigma_x = \frac{E}{1 - \nu^2} \left( \varepsilon_x + b \varepsilon_x^3 \right) \tag{8}$$

where E is modulus of elasticity, v is Poisson's ratio and b

is the Duffing parameter. The nonlinearity of this type is extensively used in the theory of nonlinear vibrating systems and it is used in the problems of nonlinear bending of beams [12], because it is one of the simplest representations of nonlinear behaviour and at the same time enables to analyze basic nonlinear effects in mechanical systems.

From the previous equations it is obtained

$$\sigma_x = \frac{E}{1 - \nu^2} \left( z + z^3 b \overline{\varepsilon}^2 \right) [B] \{\delta\}$$
<sup>(9)</sup>

Shear strain is expressed as

$$\gamma_{xz} = \left[\overline{B}\right] \{\delta\} \tag{10}$$

where

$$\begin{bmatrix} \overline{B} \end{bmatrix} = \begin{bmatrix} \frac{dN_1}{dx} & N_1 & \dots \end{bmatrix}$$
(11)

Shear stress is expressed as

$$\tau_{xz} = \frac{E}{2(1+\nu)}\gamma_{xz} \tag{12}$$

On the basis of the expressions of strains and stresses presented above the stiffness matrix has the form

$$\begin{bmatrix} K \end{bmatrix} = \int \begin{bmatrix} B \end{bmatrix}^T \left[ \frac{E}{1 - \nu^2} \left( \frac{h^3}{12} + b\overline{\varepsilon}^2 \frac{h^5}{80} \right) \right] \begin{bmatrix} B \end{bmatrix} + \left[ + \left[ \overline{B} \right]^T \left[ \frac{E}{2(1 + \nu) 1.2} h \right] \begin{bmatrix} \overline{B} \end{bmatrix} \right] dx \quad (13)$$

where h is the thickness of the polystyrene and 1.2 is the shear correction factor. Also in obtaining the expression of the stiffness matrix the following integrals have been taken into account

$$\int_{\frac{h}{2}}^{\frac{h}{2}} z^2 dz = \frac{h^3}{12}$$
(14)

and

$$\int_{\frac{h}{2}}^{\frac{h}{2}} z^4 dz = \frac{h^5}{80}.$$
(15)

# 5. Results of analysis of nonlinear bending of a strip of polystyrene

Here physically nonlinear behaviour of a strip of polystyrene is analyzed using the model of a straight beam in the status of equilibrium coinciding with the x axis of the orthogonal system of coordinates. At both ends of a

strip of polystyrene both generalized displacements are assumed equal to zero. The node at the centre of the strip of polystyrene is loaded by a force ( $F = 8 \times 10^{-7}$  N). The following values of parameters are assumed: E = 8 N/m<sup>2</sup>, v = 0.3, h = 0.1 mm,  $b = 10^8$  (Eq. (13)).

Graphical representation of deflection of the strip of polystyrene when the force is increased by small steps (for  $F \times (17-i)/16$ , when i = 1, 2, ..., 16) is obtained and presented in Fig. 2 for a linear problem. The same result for a nonlinear problem is presented in Fig. 3.



Fig. 2 Deflection of the strip of polystyrene for a linear problem



Fig. 3 Deflection of the strip of polystyrene for a nonlinear problem

From the presented results it is seen that physical nonlinearity substantially reduces the deflections of the strip of polystyrene.

#### 6. Method of experimental investigations

The experimental investigations were carried out by using different size samples (plastic containers) made from polystyrene (PS), whose exterior view and technical characteristics are presented in Table 1. The selection of the samples was determined by the fact, that this type of packaging is widely used in Lithuanian food industry. When analyzing geometric variety of the package exterior shapes, three characteristic types of containers were distinguished. They differed in the configuration of the top and bottom parts (Table 1). The load and deformation dependence curves were obtained by performing empty (not filled) polystyrene container tests under the action of static vertical axial load. During the experimental investigation the highest value of the axial compression load, at which the structure of the package looses stability and starts to buckle, was registered.

Continuing the research presented in the previous paper [5], a compression experimental setup was used for testing, and special computer equipment was used for processing the measurement results and visualization of the data. The diagram and external view of the experimental setup are presented in [5, 12].

Table 1

Geometric parameters of tested packaging and maximum values of vertical compression and deformation under maximum loads

Geometry of the top and bottom parts			Wall	Package	Maximum	Deformation
of the package	code	parameters of the package,	thickness,	volume,	compres-	under maxi-
		mm	mm	ml	sion load	mum com-
					$F_{imax}$ , N	pression
						load, $\Delta H_i$ ,
						mm
Type I	7000/L	$D_V = 95; D_{V1} = 90; D_{V2} = 84;$	0.34	350	459.74	3.15
$\triangleleft D_{\nu}$		<i>D</i> <sub>A</sub> =67; <i>H</i> =90; <i>H</i> <sub>V1</sub> =12				
	7067/L	$D_V = 95; D_{V1} = 90; D_{V2} = 84;$	0.34	150	416.56	2.8
$H$ $D_{\nu_2}$		$D_A$ =80; H=43; $H_{V1}$ =13				
	7068/L	$D_V=95; D_{V1}=89; D_{V2}=84;$	0.29	250	391.16	2.8
		$D_A$ =78; H=63; $H_{V1}$ =13				
H	7069/L	$D_V=95; D_{V1}=90; D_{V2}=84;$	0.33	200	365.76	3.15
		$D_A=79; H=53; H_{V1}=13$				
δ						
¥						
Type II	7001/L	$D_V = 95; D_{V1} = 89; D_{V2} = 83;$	0.23	500	233.68	3.5
$ = \frac{D_v}{1 + \frac{D_v}$		$D_A=67; D_{A1}=73; H=120;$				
		$H_{V1}=13; H_{A1}=4; H_{A2}=17$				
$H$ $D_{n}$	7072/L	$D_V = 95; D_{V1} = 88; D_{V2} = 83;$	0.23	350	259.08	2.8
	101211	$D_A = 64; D_{A1} = 72; H = 91;$	0.25	550	237.00	2.0
		$H_{V1}=9; H_{A1}=8; H_{A2}=12$				
H		$11_{V_1}$ $3, 11_{A_1}$ $3, 11_{A_2}$ $12$				
8 1 Da						
Type III	7060/L	$D_V=95; D_{V1}=89; D_{V2}=76;$	0.33	130	332.74	3.15
$  = \frac{D_v}{D_v}$		$D_A$ =60; $D_{A1}$ =65; $H$ =50;				
		$H_{V1}=5; H_{V2}=10; H_{A1}=6;$				
$H_{I_1}$		$H_{A2}=8$				
*						
	7062/L	$D_V = 95; D_{V1} = 88; D_{V2} = 78;$	0.28	250	213.36	2.45
Star In Dat		$D_A=58; D_{A1}=65; H=75;$				
		$H_{V1}=6; H_{V2}=11; H_{A1}=10;$				
		$H_{A2}=14$				

During the testing, the packaging under investigation was placed on the bottom slab which is the component of the compression load measurement unit. The bottom plane of base element is parallel to the bottom base slab. The dependence of compression force and package deformation was displayed on the computer monitor. For processing of the testing data and visualization of the obtained dependencies, a PC with oscilloscope and the software Pico Log for Windows Release 5.14.6 were used. During the test, the bottom stand slab with the package moves vertically upwards at the regular speed of  $V = 3.5 \times 10^{-4}$  m/s. When the upper part of the package touches the fixed upper base element, the process of package compression is started. The personal computer receives concrete test data, expressing the dependence of electric signal from the load measuring unit upon the time. Then it is transformed into the dependence of compression force upon package deformation in vertical direction.

From the estimates of the values of the axial compression force obtained in the course of experimental investigations, the maximum value, at which the polystyrene package loses stability and starts to buckle, is determined. The relationships of package deformation due to axial load are presented graphically in Figs. 4 - 6, when the package deforms by the interval of 0.35 mm.

Tests with the packages were carried out deforming the containers up to 5 mm, since during the initial deformation stage the critical deformations occur, which means that under working conditions the packaging filled with grainy or liquid products is already not suitable for usage.

In order to compare the behaviour of the geometrical shape of the experimental samples during the process of compression, the samples were photographed by digital camera with a fixed interval of photographing.

The tests were carried out at the ambient temperature  $20 \pm 2$  °C and air humidity  $65 \pm 2$  %.

#### 7. Results of the analysis of compression of packages

Figures 4 - 6 and Table 1 present the findings of compression resistance tests of all the main types of packaging samples. When analyzing the dependences (Figs. 4 - 6) obtained during the tests, it can be noted that at the initial compression stage the resistance of all the containers to vertical load is the largest, and the obtained deformation - compression load dependence can be considered as nonlinear.

When maximum load is reached (e.g., in Fig. 4, curve 4 would reach it when  $F_{1max} = 459.74$  N, deformation  $\Delta H_1$  at 3.15 mm), the packaging resistance to compression starts falling rapidly. Such fall of the container's resistance is typical for all the tested packaging. It can be stated that at this initial stage of packaging resistance decrease the deformation - load dependence is non-linear in all cases.

I type packages start to buckle under the vertical axial load at about 365 - 460 N (Fig. 4), while II and III type packages start to buckle under the axial compression of about 215 - 330 N (Figs. 5 - 6). The latter package starts to buckle under the axial compression force which is about 20 - 25 % lower.

In further deformation stages, it is not possible to express the clear and regular deformation - load dependence: in some cases, compression resistance clearly decreases with increasing deformation, while in other cases the changing increase - decrease tendencies are observed (e.g., Fig. 6, curve 2). However, this last stage of container deformation is not important for further study since, judging by the test findings, during usage some of the tested containers can be deformed up to approximately 3 mm, later the container walls get irreversible plastic deformations and they are no longer suitable for usage. During the testing other regularities were also observed.



Fig. 4 Graphic dependence of axial deformation and vertical load of I type empty polystyrene containers: 1 – 7068/L; 2 – 7067/L; 3 – 7069/L and 4 – 7000/L



Fig. 5 Graphic dependence of axial deformation and vertical load of II type empty polystyrene containers: l - 7072/L and 2 - 7001/L



Fig. 6 Graphic dependence of axial deformation and vertical load of III type empty polystyrene containers: l - 7060/L and 2 - 7062/L

For example, under the load when the container's resistance to deformation is the highest (e.g.,  $F_{1max} = 459.74$  N, Fig. 4, curve 4), its walls start deforming, bending in different directions, in some places cracks or folds appear, and so on. These maximum values of loads and deformations that cause cracks in the walls of containers during the compression process are presented in Table 1.

Comparison of the results obtained numerically and experimentally (Fig. 1 and Figs. 4 - 6) shows that this three-dimensional polymer package compression FEM model qualitatively reflected the findings of the experimental tests in the initial stage of deformation.

In Figs. 7, 8 and 9 the photos of the polystyrene packages are presented, which show the behaviour of the package during its deformation.



Fig. 7 External view before (a) and after (b) compression of I type (6069/L) of PS package



Fig. 8 External view before (a) and after (b) compression of II type (7072/L) of PS package



Fig. 9 External view before (a) and after (b) compression of III type (7060/L) of PS package

#### 8. Conclusions

Compression of an axi-symmetric polystyrene package is analyzed. Physical nonlinearity is taken into account assuming the hyperbolic model of nonlinear elasticity. The graphical relationship of axial load – axial deformation is determined.

The proposed model for the analysis of bending of a strip of polystyrene is based on the assumption of physically nonlinear behaviour by assuming the model of Duffing type. From the presented results it is seen that the physical nonlinearity substantially reduces the deflections of the strip of polystyrene.

Experimental investigations using a specially developed setup have been performed for a number of types of polystyrene packages. The main graphical relationships and results of the experimental investigations are presented. It is determined that the results of experimental investigations in the initial stage of deformations correspond with the numerical ones.

The obtained results are used in the process of design of the elements of packages.

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#### POLISTIRENINIŲ PAKUOČIŲ IR JŲ ELEMENTŲ FIZIŠKAI NETIESINĖS ELGSENOS TYRIMAS

#### Reziumė

Polistireninės pakuotės gniuždymo analizės modelis remiasi ašiai simetriškos fiziškai netiesinės tamprios konstrukcijos tyrimu. Netiesinis tamprumas įvertinamas taikant hiperbolinį modelį ir yra laikomas plastinės elgsenos aproksimacija, kuri gali būti tiriama remiantis deformacine plastiškumo teorija, galiojančia monotoniškam apkrovimui.

Pateiktas polistireno juostos lenkimo analizės modelis. Ji analizuojama kaip strypas, esant kubiniam Diufingo netiesiškumui. Parodyta, kad lenkiamos polistireno juostos elgsena iš esmės priklauso nuo fizinio netiesiškumo.

Naudojant specialiai sukurtą eksperimentinį stendą buvo atlikti keleto tipų polistireno pakuočių eksperimentiniai tyrimai. Pateikti eksperimentinių tyrimų metu gauti pagrindiniai grafiniai sąryšiai ir rezultatai. Nustatyta, kad eksperimentinių tyrimų rezultatai pradinėje deformavimo stadijoje atitinka skaičiavimų rezultatus. Tyrimų duomenys taikomi pakuočių elementams projektuoti. K. Ragulskis, E. Kibirkštis, A. Dabkevičius, V. Bivainis, V. Miliūnas, L. Ragulskis

#### INVESTIGATION OF PHYSICALLY NONLINEAR BEHAVIOUR OF POLYSTYRENE PACKAGES AND THEIR ELEMENTS

#### Summary

The model for the analysis of compression of a polystyrene package is based on the analysis of an axisymmetric physically nonlinear elastic structure. Nonlinear elasticity is taken into account using the hyperbolic model and it is considered as an approximation to the plastic behaviour which can be analyzed by the theory of deformational plasticity valid for monotonic loading.

The model for the analysis of bending of a strip of polystyrene is presented. The beam model by taking cubic nonlinearity of Duffing type into account is used. It is shown that the bending behaviour is substantially influenced by the physical nonlinearity.

Experimental investigations using a specially developed setup have been performed for a number of types of polystyrene packages. The main graphical relationships and results of the experimental investigations are presented. It is determined that the results of experimental investigations in the initial stage of deformations correspond with the numerical ones. The obtained results of investigations are used in the process of design of the elements of packages. К. Рагульскис, Э. Кибиркштис, А. Дабкявичюс, В. Бивайнис, В. Милюнас, Л. Рагульскис

#### ИССЛЕДОВАНИЕ ФИЗИЧЕСКИ НЕЛИНЕЙНОГО ПОВЕДЕНИЯ ПОЛИСТИРОЛОВЫХ УПАКОВОК И ИХ ЭЛЕМЕНТОВ

#### Резюме

Модель анализа сжатия полистироловых упаковок основана на исследовании осесимметричной физически нелинейной упругой конструкции. Нелинейная упругость учитывается используя гиперболическую модель, которая является аппроксимацией пластического поведения, которое может быть исследовано применяя деформационную теорию пластичности, являющуюся справедливой при монотонном нагружении.

Приведена модель анализа изгиба полистироловой ленты. Её исследования основаны на модели балки принимая кубическую нелинейность типа Дуффинга. Показано, что поведение полистироловой ленты при изгибе находится под существенным влиянием физической нелинейности.

Проведены экспериментальные исследования используя специально созданную экспериментальную установку для ряда типов полистироловых упаковок. Приведены основные графические закономерности и результаты, полученные во время экспериментальных исследований. Установлено, что данные экспериментальных исследований в начальной стадии деформирования соответствуют численным результатам. Полученные результаты исследований применяются при проектировании элементов упаковок.

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