Study of local thermal nonequilibrium in porous media due to temperature sudden change and heat generation

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1. Introduction

Much effort has been devoted recently to determine conditions which guarantee that the assumption of local thermal equilibrium (LTE) is accurate when modeling of heat transfer in porous media. When it is accurate, then the thermal field is well-approximated by a single thermal energy equation. An excellent review of conductive effects in a stagnant porous medium may be found in Cheng and Hsu [1]. In their chapter these authors consider periodic media and their aim is to determine the effective thermal conductivity of the combined medium in the terms of the conductivities of the constituent phases. Therefore Cheng and Hsu provide important information for those wishing to use a single temperature field to model a two-phase saturated porous medium, or equivalently a composite solid consisting of two different constituents. In other circumstances, local thermal nonequilibrium (LTNE) prevails and it is necessary to employ two energy equations, one for each phase. The first papers which used two different temperature fields presented by Anzelius [2] and Schumann [3], and they were both published about eighty years ago. In their presented energy equations, we see that diffusion and advective terms have been neglected in the work of Anzelius. The numerical study by Combarnous [4] predated by a couple of decades further work on fully nonlinear convection using this model. Nakayama et al. [5] have proposed the nonthermal equilibrium two-energy equations model for conduction and convection, in which the two-energy equations for the individual phases at constant porosity are combined together and solved analytically. Neild and Bejan [6] stated the simplest equations which are generally regarded as modeling unsteady heat transfer in a saturated porous medium where LTE does not apply.

Great heat generation is one of the efficacious reasons to create LTNE condition between phases, (e.g., in the fluid phase this factor is appeared as chemical reaction). In the absent of fluid flow, Rees [7] determines both analytical and numerical formulae for interfacial heat transfer coefficient in the porous media, when a uniform heat generation in fluid phase is produced and uphold LTNE condition. Nonthermal energy heat transfer in the stagnant porous medium with variable porosity is analyzed by Nazari and Kowsari [8], where heat generation takes place within the solid phase. They use from energy equations of the solid and fluid phases with the assumption of steady-state and one-dimensional heat conduction.

Temperature sudden change is another effective factor for LTNE condition. When the temperature at the bounding surface changes significantly with respect to time in each phase, the local volumes of the solid and fluid phase can not react quickly and therupon two equations are used to model the fluid and solid phases separately. In the two-field model, the energy equations are coupled by means of terms which account for the heat lost or gained from the other phase. Nouri-Borujerdi et al. [9] inspect the effect of LTNE on the evolution of the stagnant temperature field in a semi-infinite porous medium and then conduction takes place more rapidly in one phase than in the other, although local thermal equilibrium is always approached as time increases and in continuance Kayhani et al. [10] studied the effect of LTNE on a two-dimensional porous medium under a step temperature change on the boundaries.

In summarize, special situations for LTNE to occur in the porous media are: 1) the great heat generation is happened in each phases (Chemical Reactions), 2) boundary temperature change suddenly along time, 3) hot fluid is injected in the cold porous media and 4) phases have different specific heat capacities and thermal conductivities.

In some of the LTNE applications such as fruit drying technology [11], heat transfer in biological tissues [12] and thermal analysis of the porous burner that is reviewed by Mujeebu et al. [13], temperature sudden change is governed and in some others, such as chemical catalyst [14] and nuclear reactors [15], heat generation affect on heat transfer process as well as previous condition.

In the present paper, we assume a two-temperature model for conduction in a stagnant porous medium that is saturated with the incompressible fluid which temperature change suddenly in boundary \( x = 0 \) and simultaneously uniform significant heat generation takes place within the solid phase. Also we consider how the heat generation in solid alters the behavior of temperature gradients in the different values of conductivity and thermal diffusivity ratio. The present paper is in continuance performed research projects by Nouri-Borujerdi et al. [9] and Kayhani et al. [10]. These projects followed from the similarity solution with complicated calculations and without presence of heat generation term. We simplify the solution method using direct numerical method and inspect the effect of heat generation as well as temperature sudden change in porous media.
2. Model development

As shown in Fig. 1, consider semiinfinite porous media that is saturated with a stagnant incompressible fluid. Using one-dimensional heat conduction in the porous media, the nonthermal equilibrium energy equations of the fluid and solid phases are as follows [6, 16]:

\[
\varepsilon (\rho c) \frac{\partial T_f}{\partial t} = \varepsilon \nabla (k_f \nabla T_f) + h(T_s - T_f) \\
(1 - \varepsilon) (\rho c) \frac{\partial T_s}{\partial t} = (1 - \varepsilon) \left( \nabla (k_s \nabla T_s) + q'' \right) - h(T_s - T_f)
\]

The subscripts \( f \) and \( s \) denote fluid and solid phases respectively. The quantities \( \varepsilon, \rho, \) and \( c \) are the porosity, density and specific heat capacity and \( q'' \) is the uniform heat generation per unit solid volume. The last term in energy equations represent the coupled heat transfer between the two phases because of the existing temperature difference. Many of scientists attempted to determine suitable values of \( h \) have generally relied upon averaging methods, and various assumptions then need to be made about closure; (see Rees [17]). Some of these formulations for determination of \( h \) yield a zero value for \( h \) when \( Re = 0 \), which implies that there is no transfer of heat between the separate phases when the porous medium is stagnant. However, some of others yield nonzero values for \( h \) in the absence of flow, but the resulting expressions are independent of the conductivity of the solid phase. Based on presented models for \( h \), we assume interfacial heat transfer coefficient as follows:

\[
h = \frac{\varepsilon k_f}{L}
\]

(3)

By introducing the following dimensionless parameters:

\[
\hat{x} = \frac{x}{L} \\
\hat{t} = \frac{t }{L^2} \\
\theta_{\text{fluid}} = \frac{T_f - T_{\text{ref}}}{q''L^2/k_f} \\
\theta_{\text{solid}} = \frac{T_s - T_{\text{ref}}}{q''L^2/k_f}
\]

The governing Eqs. (1) and (2) can be nondimensionalized as:

\[
\text{Fluid} \rightarrow \theta = \theta_s + H (\phi - \theta) \\
\text{Solid} \rightarrow \alpha \phi = \phi_s + H \gamma (\theta - \phi) + 1
\]

(5)

(6)

Fig. 1 Semiinfinite media

Where that the nondimension media

\[
\alpha = \frac{\alpha_f}{\alpha_s} \quad \text{diffusivity ratio} \\
H = \frac{hL^2}{\varepsilon k_f} \quad \text{interfacial heat transfer coefficient} \\
\gamma = \frac{q''}{(1 - \varepsilon) k_s} \quad \text{conductivity ratio}
\]

The above parameters are constant and do not vary with temperature. This assumption helps us to verify the changes of phases together as well as simplification. Nouri-Borujerdi et al. [9] eliminate parameter \( H \) from energy equations by using natural coordinates provided by Carslaw and Jaeger [18]. But in this paper, by using the natural length scale in relation (3), parameter \( H \) will be equal to unit.

Assuming a high temperature sudden change at the boundary \( x = 0 \), LTNE condition between fluid and solid phases is possible and heat generation term amplify it. According to the Fig. 1, initial and boundary conditions can be simply defined as the following form:

\[
t = 0 \rightarrow \theta(x = 0) = \phi(x = 0) = 1 \\
x = 0 \rightarrow T_f = T_s = 0 \rightarrow \theta(x) = \phi(x) = 1 \\
x = \infty \rightarrow T_f = T_s = 0 \rightarrow \theta(x) = \phi(x) = 0
\]

(10a)

(10b)

Boundary conditions (10b) showed a sudden change of temperature in the boundary of semiinfinite domain which induced LTNE between the phases.

3. Solution method

There are many numerical methods for solving ordinary differential equations which each of these methods have certain accuracy. In order to solve Eqs. (5) and (6), two different numerical methods have been used. For the calculation of second order derivatives, we use from compact finite difference and about time from forth order Runge-Kutta methods. Compact finite difference is the useful method to discrete domain with high accuracy. Basic of this method is very simple and similar to finite difference method but with the less error. For example in derivation, we use from backward, forward and central operators but in the compact finite difference method we mix them and use from an operator for derivation. This method was completed by Hirsh [19] and Lele [20] generalized it.

At early and late times we check the results using perturbation method. We determine the power series solution of Eqs. (5) and (6). At the suitable order it is possible to proceed easily analytically. We have to use from the
other numerical solution (Shooting Method) at the end of analytical procedure.

4. Solution at early and late times

In this section we are going to examine results of numerical method in the special case at early and late times. In this case we assume no heat generation occurred in the solid phase. Fig. 2 shows the result of numerical method without heat generation when just temperature sudden change is the reason of heat transfer in porous media.

![Graph showing temperature gradient](image)

**Fig. 2** Temperature gradient for $\alpha = 2$ and $\gamma = 1$

In this figure, at early times when $x \to 0$, we use from power series solution as follow:

$$\theta(x,t) = \theta_0(x) + t\theta_1(x) + t^2\theta_2(x) + \cdots$$

(11)

$$\phi(x,t) = \phi_0(x) + t\phi_1(x) + t^2\phi_2(x) + \cdots$$

(12)

At $\theta(t)$ by derivation than time and place and then situation in the Eqs. (5) and (6), we obtain simplified equations as below:

$$\theta_1 = (\theta_0^* - \theta_0 + \phi_1) + t(\theta_1^* + \phi_1 - \theta_1)$$

$$\theta_0^* - \theta_0 + \phi_1 = \theta_1$$

$$\theta_1^* + \phi_1 - \theta_1 = 0$$

(13)

$$\alpha\phi_1 = (\phi_0^* + \gamma(\theta_0 - \phi_1)) + t(\phi_1^* + \gamma(\theta_1 - \phi_1))$$

$$\phi_0^* + \gamma(\theta_0 - \phi_1) = \alpha\phi_1$$

$$\phi_1^* + \gamma(\theta_1 - \phi_1) = 0$$

(14)

So boundary equations (10b) change according to the power series solution as below:

$$\theta(x = 0,t) = 1 \to \theta_0(0) + t\theta_1(0) = 1$$

$$\theta_0(0) = 1$$

$$\theta_1(0) = 0$$

(15)

$$\theta(x = \infty,t) = 0 \to \theta_0(\infty) + t\theta_1(\infty) = 0$$

$$\theta_0(\infty) = 0$$

$$\theta_1(\infty) = 0$$

(16)

We solve Eqs. (13) and (14) with boundary Eqs. (15) to (18) using from shooting method. Result of numerical solution of Eqs. (5) and (6) at the boundary $x = 0$ and early time $t = 0.001$ is equal result of analytical solution of Eqs. (13) and (14). This result for $\Delta x = 0.15$ is 1.1281.

We also repeat above progress for late time. In Fig. 2, at late times when $x \to \infty$, we use from other power series solution:

$$\theta(x,t) = \theta_0(x) + \frac{1}{t}\theta_1(x) + \frac{1}{t^2}\theta_2(x) + \cdots$$

(19)

$$\phi(x,t) = \phi_0(x) + \frac{1}{t}\phi_1(x) + \frac{1}{t^2}\phi_2(x) + \cdots$$

(20)

At $\theta(t)$ by derivation than time and place and then situation in the Eqs. (5) and (6), we obtain simplified equations as below:

$$-\theta_1 = \frac{1}{t^2}(\theta_0^* + \phi_0 - \theta_0) + t(\theta_1^* + \phi_1 - \theta_1)$$

$$\theta_0^* - \theta_0 + \phi_1 = 0$$

(21)

$$-\alpha\phi_1 = \frac{1}{t^2}(\phi_0^* + \gamma(\theta_0 - \phi_1)) + t(\phi_1^* + \gamma(\theta_1 - \phi_1))$$

$$\phi_0^* + \gamma(\theta_0 - \phi_1) = 0$$

(22)

We solve Eqs. (21) and (22) with previous boundary equations using from shooting method. The result of numerical solution of Eqs. (5) and (6) at the boundary $x = 0$ and late time $t = 1000$ is equal result of analytical solution of Eqs. (21) and (22). This result for $\Delta x = 0.15$ is 0.0333. Thereupon we can compare values of temperature gradients using results of Compact numerical method and perturbation method at early and late times. Compact numerical method is reliable and expandable to different cases.

5. Result and discussion

In this section we are going to present the complete description about behaviour of temperature gradients in the different conditions. Diffusivity ratio $\alpha$ and porosity-modified conductivity ratio $\gamma$ are two important parameters in this section. In the previous sections, we certified accuracy of results for temperature gradients using Compact numerical method.

Due to the same initial conditions in both phases, graphs in the Fig. 3 have the same start points. In the presence of heat generation, all of the graphs attain the steady state in the negative value of temperature gradients. In the
through time domain, initial condition of phases is constant in the first node and therefore temperature of second node can get to higher temperature values than the first node. Finally, temperature gradients of fluid and solid phases are stabilized in the negative values. Temperature gradient difference is created from the difference between the solid and fluid thermal diffusivities. In the Fig. 3, a, with progress in time, this difference becomes greater. So, the rate of difference quicken by the existence of generation term in the solid phase. As we know, $\alpha$ is the ratio of fluid diffusivity to solid diffusivity and when $\alpha < 1$:

\[
\frac{\alpha_f}{\alpha_s} < 1 \rightarrow \frac{k_f}{(\rho c)_f} < \frac{k_s}{(\rho c)_s} \rightarrow (\rho c)_s < (\rho c)_f
\]  

Unequal Eq. (23) states that specific heat of solid phase is less than fluid phase. This means that solid phase will be heated and cooled too early rather than fluid phase. So when $\alpha < 1$, the solid curve is placed lower than the fluid curve. According to the presence of heat generation term in this case, the decrease rate of temperature gradient of solid phase rather than fluid phase becomes greater and there is no contact point between solid and fluid graphs.

According to the unequal Eq. (23), in the Fig. 3, b when $\alpha > 1$, specific heat capacity of solid phase is more than fluid phase and solid phase will be heated and cooled too late and the solid curve is placed higher than the fluid curve, but warming late value of solid phase decrease. In the Fig. 3, b, the difference between temperature gradients is less than Fig. 3, a. For the influence of continuous heat generation, this difference dwindles and finally curves obtain a coincidence point. At very late times, graph slope decreases to zero and the temperature gradient in both phases moves towards the infinite with a constant value. Similar slopes zero at the late times represent that heat generation effect is counteracted. When the graphs slope is zero, heat transfer happens between phases yet, but it should be noted that due to the lack of heat generation effect, the amount of heat transfer always remains constant.

As it was stated previously, solid specific heat for $\alpha > 1$ and $\alpha < 1$, is larger and smaller than the fluid phase specific heat capacity respectively. The important point in Fig. 4 is that as the amount of $\alpha$ increase, time to counteract heat generation effect increases. Fluid phase graphs are stabilized in the negative value of temperature gradient with the constant slope zero and different diffusivity ratios $\alpha$ don’t affect on this value. Solid phase has the similar status too.

![Fig. 3 Temperature gradient: a - for $\alpha = 0.2$ and $\gamma = 1$; b - for $\alpha = 2$ and $\gamma = 1$](image)

![Fig. 4 Temperature gradient curves: a - for $\gamma = 1$ and different values of $\alpha$; b - zooming of a](image)

We find out that different diffusivity ratios $\alpha$ do not affect on final values of temperature gradient in the phases, but according to the Fig. 5, a and b, different conductivity ratios change final values of temperature gradient in the phases and final distance between two phases. According to the relation (9), $(1-\varepsilon) k_s$ becomes greater than $\varepsilon k_f$ with decreasing $\gamma$ or porosity decreases. With decreasing void volumes in the porous media, solid phase contribution in the heat generating increases more than before. This result is reverse with increasing $\gamma$ in the heat
transfer process.

Fig. 6 depicts temperature gradients for $\alpha = 4$ and different values of $\gamma$. Heat generation effect is counteracted in earlier times when $\gamma$ increases.

The differences of temperature gradient between solid and fluid phases are depicted in Fig. 7 for $\gamma = 1$ and different values of $\alpha$. The graphs state that when $\gamma$ increases, the value of the difference decreases and temperature gradient of the phases are more coinciding. Contact point of the phases tend to earlier time when $\gamma$ increases.

6. Conclusion

In this note, we have considered the local thermal nonequilibrium due to the temperature sudden change and great heat generation in porous media. Energy equations presented by Neild and Bejan [6] and Kaviany [16] were used as governing equations. After nondimensionalising, new parameters such as diffusivity ratio, scaled interfacial heat transfer coefficient and porosity-modified conductivity ratio were defined. Governing equations are solved numerically using Compact method and results valid using perturbation method. The effect of defined parameters on the behaviour of solid and fluid phases in porous media is investigated and results are presented in the form of various graphs. The results showed that after the passage of
time, temperature gradients in both phases reached to a negative fixed amount and remain constant. The diffusivity ratio affected the behaviour and positioning of temperature gradient in the both phases. Also, the effect of porosity-modified conductivity ratio was discussed to reach equilibrium conditions. In two final figures, the difference amount of temperature gradient for all states between both phases was presented.

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References

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STUDY OF LOCAL THERMAL NONEQUILIBRIUM IN POROUS MEDIA DUE TO TEMPERATURE SUDDEN CHANGE AND HEAT GENERATION

Summary

In this paper we examine the effects of temperature sudden change in boundary $x = 0$ and great heat generating on the creation of local thermal nonequilibrium (LTNE) in the semiinfinite stagnant porous media. Two energy equations in the transient state and in the presence of heat generating are used as the governing equations in each phase. These partial governing equations solve numerically. For partial derivatives, we use from Compact finite difference method that is the continuous method for calculation of derivatives and for progress in time, we use from RK4. So we test values of heat transfer rate for early and late times using perturbation and shooting methods. Rate of heat transfer between phases depict in the figures. Results show that effects of heat generating and LTNE are restrained at very late times and graphs slope decrease to zero. In this time, the temperature gradient moves towards the infinite with a constant value. Also, effect of different nondimension parameters on behavior of temperature gradients is verified. When diffusivity ratio $\alpha$ increases, time to counteract heat generation effect increases. Fluid and solid graphs are stabilized in the negative value of temperature gradient with the constant slop zero and different diffusivity ratios $\alpha$ don’t affect on this value. So for $\alpha > 1$, the difference between solid and fluid phases will increase and vice versa, for $\alpha < 1$ it will decrease.

Heat generation effect is counteracted in earlier times when conductivity ratio $\gamma$ increases. So The graphs state that when $\gamma$ increases, the value of the difference decreases and temperature gradient of the phases are more coinciding and Contact point of the phases tend to earlier time. Other characterizations are explained in detailed.

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ИССЛЕДОВАНИЕ МЕСТНОЙ ТЕПЛОВОЙ НЕУРАВНОВЕШЕННОСТИ В ПОРИСТОЙ СРЕДЕ В ЗАВИСИМОСТИ ОТ ВНЕЗАПНЫХ ИЗМЕНЕНИЙТЕМПЕРАТУРЫ И ГЕНЕРАЦИИ ТЕПЛА

Резюме

В этой статье исследуется эффект внезапного изменения температуры в предельном слое $x = 0$ и быстрая генерация тепла, создавая локальную тепловую неуравновешенность в полубесконечной застойной пористой среде. Два уравнения энергии в переходном состоянии при генерировании тепла использованы как основные в каждой фазе. Эти частные основные уравнения решены числовым методом. Для расчета частных производных использован COMPACT от RK4, который является известным сберегающим время расчета методом производных. Исследована скорость передачи тепла на ранней и поздней стадиях при помощи возмущения и приближения. Скорость возмущения тепла между фазами приведена на рисунках. Результаты показывают, что эффект генерации тепла и местная тепловая неуравновешенность снижается в конечной фазе и уклон кривых снижается до нуля, а градиент температуры изменяется до бесконечности с постоянной скоростью. Кроме того подробно объяснено влияние различных бездименсных параметров на изменение температурного градиента.

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