Determination of the digitally filed geodesics on a cross-section of an ellipsoidal mandrel

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1. Introduction
Filament winding is a method of forming composite structures using automated machines. A mandrel is pre-manufactured matching the geometry of the structure to be produced and mounted between the jaws of a coil comprising a control system rotation of the mandrel and a control system of the guide wire. The wire that wraps around the mandrel, generally along geodesic paths, goes across the guide wire or the eye’s guide. Timing of the eye guide’s movements and rotation of the mandrel permit to define the paths of the filament [1, 2]. The following steps are required for the winding:

- Step 1: mathematical analysis of the structure to be achieved;
- Step 2: programming of the equations obtained for the simulation of the winding on the computer screen and the preparation of the control parameters of the winder;
- Step 3: development of the control program of the winder by a digital control using the appropriate command parameters;
- Step 4: transfer of the control program to the CNC (Computer Numerical Control);
- Step 5: control of the winder by the CNC for the winding.

This work, which falls within Step 2 above, sets out to determine the control parameters of the winder and store them in a database to develop the control program of the winder for a defined CNC. The parameters to be determined numerically are rotation of the mandrel on its axis, the translation of the eye guide along an axis parallel to the mandrel and rotation of the eye guide around an axis perpendicular to the chuck, to prevent wrinkling of the ribbon.

Cross-section of the structure is elliptical relative to the axes (o, x). If b denotes semi-minor axis of the ellipse and c semi-major axis of the ellipse, then the coordinates of point M will be defined in the following Eq. (1):

\[ x = U \sin \beta_0, \]
\[ y = U \cos \beta_0, \]

where

\[ U = \frac{bc}{\sqrt{c^2 \sin^2 \beta_0 + b^2 \cos^2 \beta_0}}. \]

From the relation of Clairaut [6] which defines the equation of the geodesic on the surface of revolution of radius R, if \( \psi \) is the angle of removal and \( R \sin \psi = \text{constant} \), we obtain the following expression of the geodesic on the ellipsoidal surface:

\[ t \psi = \frac{b \sin \psi_{\text{MAX}}}{\sqrt{1 - b^2 \sin^2 \psi_{\text{MAX}}}}. \]

Furthermore, the relation defining the geodesic on a surface according to the ellipsoidal rotation of the chuck elementary \( d\beta_0 \) around its axes (o, x) and axial translation
Fig. 1 Setting the structure, where $\Psi$ is the angle of the geodesic, $\beta_0$ the angle of rotation of the mandrel (or amplitude of rotation of M around the $x$ axis), $\theta$ the angle between $\vec{A}_{\beta_0}$ and $\vec{A}_v$, $\vec{A}_v$ is the unit vector tangent to the geodesic in M, $\vec{A}_{\beta_0}$ is the unit vector tangent to the meridian, $\vec{A}_x$ the unit vector tangent to the parallel M, $\vec{A}_x$ the unit vector normal to M, $dx$ the displacement along $x$ to an angular increment of $d\beta_0$, $P_{\beta_0}$ the plan files (containing the point M and the axes [o, x]), $M_1$ is the point of removing the tape on the mandrel, and E the position of the eye guide.

$dx$ elementary point of removal is given in Fig. 1:

$$\frac{d\beta_0}{d\Psi} = \frac{1}{UH} \left( \frac{\cos^2 \beta_0 + \sin^2 \beta_0}{c^4 + b^4} \right)^{1/2}.$$  \hspace{1cm} (3)

When we eliminate the current angle $\Psi$ by combining Eqs. (2) and (3), we get the final expression of the geodesic on a reference ellipsoid:

$$x_2 - x_1 = \frac{(\beta_2 - \beta_1) b^6 c^{i+16}}{2 \sin \Psi_{\text{max}}} \sum_{i=1}^{i=16} D_i \sqrt{\frac{c^2 - \sin^2 \Psi_{\text{max}} (b^2 \cos^2 \beta_0 + c^2 \sin^2 \beta_0)}{c^4 \sin^2 \beta_{i0} + b^4 \cos^2 \beta_{i0}}} ,$$  \hspace{1cm} (5)

where $\beta_{i0} = \frac{\beta_2 + \beta_1}{2} + \frac{\beta_2 - \beta_1}{2} l_i$, $l_i$ is the roots of the Legendre polynomial of degree 16, $D_i$ is the coefficients of the quadrature formula, $\beta_2$ is the variable and $\beta_1 = \beta_2 - d\beta_0$, $d\beta_0$ is to be chosen, values adopted $\pi/n$, $n = 12, 90$ or 180.

This equation explains axial position of point M files based on the incremental rotation of the mandrel. We deduce the axial position of the eye E guidance, given the spatial configuration of the whole (Fig. 2):

$$X_E = -\frac{1}{\tan \Psi} \left( \frac{U \sin \theta_0 - y_{E0}}{\cos \theta_0 + U_0 \sin \theta_0} \right) H + x ,$$  \hspace{1cm} (6)

with $U_0 = \sin \beta_0 \cos \beta_0 \left( \frac{1}{c^2} - \frac{1}{b^2} \right)$.

Rotation of the eye $\Phi_E$ guide E around its axis is calculated by the following expression:

$$\Phi_E = \tan \phi = \frac{H}{\tan(\Psi) \left( \frac{\sin \theta_0}{U^2} + U_0 \cos \theta_0 \right)} .$$ \hspace{1cm} (7)

2.2. Incremental procedure

The winding operation is characterized by an incremental rotation of $d\beta_0$ the point of removal, and the resulting absolute angle is the angle of rotation of the mandrel $\beta_0$. While the fibre is wound along a sine wave around
the mandrel, the abscise $x$ of the filed point varies in an incremental step of $\Delta x$ defined by geodesic Eq. (5). At the same time the guide of the eye performs a translational motion $X_E$ so that the trajectory is parallel to the axes $(o, x)$, the elementary displacement $\Delta X_E$ is proportional to the guidance of the eye that controls movement, and the coordinates $Y_E$ and $Z_E$ remain unchanged during the movement, including $Z_E = 0$. The eye also performs a rotational movement $\Phi_E$ around the parallel axes $(o, y)$ to avoid fibre bandage creasing.

The control of the CNC coil is reduced to one of these elements; they therefore need to be known precisely for a better coil. Their knowledge is possible through the following expressions:

- $\Delta X_E$, the change in the eye guide position is calculated by the program as: $\Delta X_E = X_E^{\text{(new)}} - X_E^{\text{(previous)}}$;
- $\Phi_E$ is calculated from Eq. (7);
- $X_E$ is calculated by the program from Eq. (6);
- $x$ is calculated by the system, so that $x_{\text{(new)}} = x_{\text{(previous)}} + \Delta x$;
- $d\beta_0$ is to be chosen: we chose the values $\pi/n$, $n = 12$, 90 or 180;
- $\beta_0$ that results, will be calculated so that $\beta_0^{\text{(new)}} = \beta_0^{\text{(previous)}} + d\beta_0$;
- $\Delta x$ is calculated from Eq. (5) defined above.

These data will be stored in a matrix file format with $n$ rows and seven columns as shown in the Table below.

<table>
<thead>
<tr>
<th>$j$</th>
<th>$\beta_i$</th>
<th>$\Delta x$</th>
<th>$x_i$</th>
<th>$X_E$</th>
<th>$\Delta X_E$</th>
<th>$\Phi_E$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$\beta_1 = 0 + \frac{\pi}{n}$</td>
<td>$\Delta x_1$</td>
<td>$x_1 = x_0 + \Delta x_1$</td>
<td>$X_{E1}$</td>
<td>$\Delta X_{E1}$</td>
<td>$\tan \Phi$</td>
</tr>
<tr>
<td>2</td>
<td>$\beta_2 = \frac{2\pi}{n}$</td>
<td>$\Delta x_2$</td>
<td>$x_2 = x_1 + \Delta x_2$</td>
<td>$X_{E2}$</td>
<td>$\Delta X_{E2}$</td>
<td>$\tan \Phi$</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>$i$</td>
<td>$\beta_i = \frac{i\pi}{n}$</td>
<td>$\Delta x_i$</td>
<td>$x_i = x_{i-1} + \Delta x_i$</td>
<td>$X_{Ei}$</td>
<td>$\Delta X_{Ei}$</td>
<td>$\tan \Phi$</td>
</tr>
<tr>
<td>$n$</td>
<td>$\beta_n = \pi$</td>
<td>$\Delta x_n$</td>
<td>$x_n = x_{n-1} + \Delta x_n = L$</td>
<td>$X_{En}$</td>
<td>$\Delta X_{E_{n-1}}$</td>
<td>$\tan \Phi$</td>
</tr>
</tbody>
</table>

3. Application example

A cross-section of the winding chuck constant in the shape of an ellipse was formed with a semi-minor axis 40 mm, semi-major axis 60 mm, length 500 mm and angular increment of rotation of the mandrel $\pi/180$. The following shows the steps of the program and the results obtained.

3.1. Data entry

Fig. 3 shows the data entry and start of the winding process which correspond to the winding simulation shown in Fig. 4.
3.2. Parameter file

The parameter file includes seven parameters in a table with seven columns where the number of rows equals the number of angular increments necessary to cycle the winding mandrel ellipsoidal:

- \( J \) is the serial number of the angular increment;
- \( \beta_0(j) \) is the angle swept by the point of removal after \( d-1 \) angular increment;
- \( \Delta x(j) \) is the increment of the axial point of removal to move from \( j \) to \( j-1 \);
- \( x(j) \) is the point position deposited on \( o, x \) to the increment \( j \);
- \( XE(j) \) is the eye position of the guide \( o, x \) to the increment \( j \);
- \( \Delta XE_j \) is the increment on the eye guide for the past \( j-1 \) to \( j \), and
- \( \Phi_E \) is the rotation of the eye guide to keep the tape flat during winding.

3.3. Rotation of the chuck

Fig. 5 shows the curve values of the rotation of the mandrel. One can notice that the curve of the angle \( \beta_0(i) \) is linear and increasing continually, that is to say that the core rotates in the same direction through the coil, which is what we want.

3.4. Variation of the abscissa of the point of removal

The curve in Fig. 6 presents a set of the values calculated for each incremental rotation of the mandrel. For better visibility, we present the values for four turns of the mandrel.

3.5. Abscissa of the point of removal

Fig. 7 shows the curve of the change in abscissa of the point of removal.

Determination of deposit geodesic parameters
Enter data \((L, b \text{ and } C)\) in milimeter

<table>
<thead>
<tr>
<th>Long. Mandrel</th>
<th>( L = 500 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nber divisions</td>
<td>( n = 180 )</td>
</tr>
<tr>
<td>Half small axis</td>
<td>( b = 40 )</td>
</tr>
<tr>
<td>Half big axis</td>
<td>( c = 60 )</td>
</tr>
</tbody>
</table>

Fig. 3 Data entry window and start of the execution

Fig. 4 Results of the simulation of the winding

Fig. 5 Curve representing the rotation of the mandrel

Fig. 6 Curve representing the variation of \( x \)

This parameter varies in the interval of 0 to 1.2. It decreases from 1.17060311 to 0, when the value of the angle of rotation of the mandrel is within the interval of \([0 + 2k\pi \text{ (1+2k)}]r\) and grows from 0 to 1.17060311 when this value is included in the interval of \([0 + 2k\pi \text{ (1+k)}]2\pi\) which shows that the cross-section of the form being wound is not circular. These values are calculated in absolute values by the system with the geodesic equation.
The x-coordinate of the point M representing the deposit varies in the interval of \([0 \ L]\), where \(L\) is the winding length of 500 mm in this case. This periodic variation is the movement back and forth and reflects the fact that the removal of the filament must be carried out back and forth several times until the full recovery of the surface. The undulating shape of this curve is the fact that its variation \(\Delta x'\) is not constant.

3.6. Horizontal axis of the eye guide

The horizontal axis of the eye guide varies in the same way as that of the point of removal. Its graph has the same shape as seen in Fig. 8.

![Fig. 8 Curve of the evolution of the x-axis of the eye guide](image)

The abscissa of point \(X_{E}\), where \(E\) represents the position of the eye guide along an axis parallel to the axis of rotation of the mandrel, also varies in the interval of \([0 \ L]\). This shift similar to the M point of deposit is due to the fact that the eye must guide the filament to compel the geodesic deposit. It must also make the round trip several times until the end of winding.

3.7. Variation of the abscissa of the eye guide

The curve in Fig. 9 shows the variation of the displacement of the eye corresponding to the guide from the point of deposit.

![Fig. 9 Curve of the change in eye guide position](image)

Negative values \(\Delta X_{E}\) correspond to the values calculated during the return of the winding. This is justified by the fact that this variable is calculated by the formula: \(X_{E}(j) - X_{E}(j-1)\). In this phase \(X_{E}(j)\) is always less than \(X_{E}(j-1)\).

3.8. Rotation of the eye guide

The eye guide must occupy a position which favors the tape so that it does not wrinkle its ends. This position must be constantly adjusted to an angle depending on the position of the deposit or rotation of the mandrel. Fig. 10 shows the variation of this angle.

![Fig. 10 Curve representing the rotation of the eye guide](image)

The rotation of the eye guide is in the range of \([6.60369 \times 10^{-22} \ 5.7712 \times 10^{-6}]\), and the smallest deviation occurs when the spindle is rotated \(\frac{3\pi}{2} + 2k\pi\), and the largest in the proximity of \(\frac{\pi}{7} + 2k\pi\). The inflection point observed on the curve corresponds to a rotation \(\pi + 2k\pi\) of the chuck.

4. Conclusions

1. This work allowed us to simulate the winding structure matrix composites in thermoplastic or thermosetting resin reinforced with continuous fibers and prepare a data file to program a numerical control designed to control the winder.

2. A calculation error has to be acknowledged. For a structure of the size of the sample application above, the error recorded by a winding cycle is 0.407 mm on the movement of the eye guide. This error does not pose a problem in a cycle, given the non-zero coefficient of friction between the mandrel and the tape submitted. The programming of the CNC considers this error cancelled at the end of each winding cycle.

3. This work can be replicated for the coil structures’ truncated elliptical cross-sections, and all structures of revolution, for the coil itself.

References

1. Suong, V.H. 2009. Principle of the manufacturing of composite materials, Department of Metal and Industrial Engineering, Concordia University, Quebec, Canada, DEStech Publications, Inc., 205-231.


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**SKAITINIS GEOFEDZINIŲ LINIŲ ELIPSINIO ĮTVARO SKERSINIAME PĮŪVYJE NUSTATYMAS**

**R e z i u m ė**

Straipsnyje pateikti vejamo pluošto gijų elgsenos elipsinio profilio skersiniame pjūvuje tyrimų rezultatai. Atliktas tyrimas leido palaipsniui nustatyti skaitines besišukančio įtvaro ir vediklio kreipiančiųjų poslinkio reikšmes slenkant ir sukantis, darant prielaidą, jog šie laikė suderinti pokyčiai griebtuvu leidžia nuimti suvytą įtakos geodezinės linijų kryptimi.

Naujajame integruotame metodė gautos įtvaro sukimų aprašančios skaitinės reikšmės yra kaupiamos duomenų failė. Šis programiškai sudarytas failas toliau naudojamas kaip duomenų bazė, kurios pagrindu skaitinio valdymo sistema formuoja vijimo valdymo programą.

Skaitiniam paklaida dėl gijos susisukimo nedaro ženklesnės įtakos vediklio jūzdėjimui ir neįtakojos ciklo. Tai parodo, jog taikoma priemonė yra efektyvi ir dera su šiuo vijimo metodu.

**DETERMINATION OF THE DIGITALLY FILED GEODESICS ON A CROSS-SECTION OF AN ELLIPSOIDAL MANDREL**

F.E. Betene, P. Kah, J.A. Atangana, J. Martikainen, A. Ohandja, B. Mvola

**S u m m a r y**

This paper presents the results of a study on the weaving beams across a straight elliptical cross-section by filament winding. The study determines, block by block, numerical values of the displacements of the rotating mandrel and guides of the eye in translation and in rotation, provided that the timing of these changes provides a removal of the tape with the chuck according to geodesic lines.

The numerical values obtained by an incremental method applied to the rotation of the mandrel are then stored in a file. This file is generated by the program as a basis for data to then develop the program executable by the digital control in order to control the weaver.

The error of the calculation recorded by filament wound is insignificant on the movement of the eye guide and does not affect the cycle. This is a demonstration of the efficiency of the application and responds to the composite winding method.

**Keywords:** filament winding, ellipsoidal mandrel, numerical control, simulation.

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