The structural synthesis of the parallel robots

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crossref http://dx.doi.org/10.5755/j01.mech.17.3.505

1. Introduction

Parallel kinematic chains were initially proposed in the context of tire-testing machine and flight simulators. In 1942, Willard L.V. Polland designed a first parallel, robot for painting [1]. In 1947, Eric Gough invented a new six degree-of-freedom (DOF) parallel robot. In 1965, Stewart published a paper describing a 6 DOF motion platform that was designed as an aircraft simulator. Stewart platforms are extensively analyzed as structural solutions or as conceptual solutions of the parallel elements [1]. In the early ’80s, Reymond Clavel used a parallelogram mechanism to construct a parallel robot – Delta- with 6 DOF (3 translations and 3 rotations).

Parallel manipulator is a complex closed loop structure. Parallel robots have inherent advantages over conventional serial mechanisms: high rigidity, high load capacity, high velocity and high precision [2].

The parallel robot is a mechatronic system and requires the design in the spirit of the mechatronic philosophy [3 - 6]. Reduced workload involves not talking about a universal parallel robot. The use of a parallel robot in a given application implies a careful analysis of the structure and of the essential parameters. Structural synthesis of mechanisms for parallel robot is addressed in detail in [2], is found in an approach based on the Lie Group of displacements in [7] or on topological synthesis of translational parallel manipulators [8]. Structural analysis and synthesis viewpoints for the TRIGLIDE robot, with relevance in medicine [9, 10], are encountered in [3, 11, 12]. Synthesis aspects of 6-DOF complex structures are addressed in [13, 14].

This paper proposes a method for structural synthesis of the parallel robotic mechanism using two concepts: the mechatronic philosophy and the kinematic connections. The mechatronic philosophy, as a support in the design of parallel robot structure, makes the object of Section 2 as well as the structural analysis of the mechanisms related to the concept of kinematic connections. The design of the robot structure is addressed in Section 3. Section 4 studies kinematic connection and structural analysis of the mechanisms. Section 5 deals with principles of structural synthesis based on the concept of the kinematic connection; models are reviewed in Section 6. Conclusions-Section 7 and References- Section 8 complete the paper.

2. “Mechatronic philosophy” & parallel robot design

Parallel robot belongs through concept, design and application to the mechatronic product class designed using the principles and procedures of the “mechatronic philosophy”. The V design model - V design cycle - is currently recognized and accepted in the field of mechatronic design (Fig. 1).

The systemic approach and the methods for developing new ideas play a key role in this growth. Taking a decision in the design process can be greatly simplified by the system decomposition according the system function. Patterns development is achieved by evolving from abstract to concrete, from simple to detail. Fig. 2 shows a case of system decomposition for a parallel robot, in correspondence with the previously facts.

3. Conceptual design of the parallel robot mobility

Definitions and explanations on the notations and names in the field of parallel robots are widely presented in [3]. A parallel robot is defined as the "... robot in which end effector is connected in parallel to reference link by \( k \geq 2 \) kinematic chains called limbs or legs" and a fully parallel robot as "...a parallel robot in which the number of limbs is equal to the robot mobility and each limb inte-
grates just one actuator” [3]. Under this concept, one can consider that a parallel mechanism is typically made of two rigid bodies, one movable and one fixed, connected by at least kinematic chains. A fixed platform (PF) is used as support/ frame for the assembly and positioning of the created mechatronic system - the parallel robot. The end effector (EF) for the given application is attached to a mobile platform (MP) of a specific geometrical shape and the characteristic point P is associated to the effector.

The mobile platform will be assembled to the fixed platform in a mechanical structure, through a parallel mechanism. Relative positioning in space of the two platforms offers various design alternatives to achieve the desired application, depending upon the specific employment requirements (Fig. 3).

![Fig. 3 Relative position of the two platforms within the structure of the parallel robot](image)

Structural synthesis generates the structure of parallel mechanisms that integrate the two platforms. The complexity of the application to be achieved has a decisive role in determining this structure and the number of the DOF of the mobile platform.

4. About the structural analysis of the parallel mechanisms and the kinematic connection

For a given mechanism, the desmodromy is obtained as a result of a number of constraints imposed in the relative motion of the elements and of the ruled and governed kinematic pairs. The totality of the means that achieve constrains in the relative motion of the elements in a mechanism are named connections [1]. The connections (K) and their definitions are presented in Table 1.

Table 1

<table>
<thead>
<tr>
<th>Connection</th>
<th>Symbol and DOF</th>
<th>Association with the theory of mechanisms</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_d$</td>
<td>$K_d(-L_k)$</td>
<td>Kinematic pair</td>
</tr>
<tr>
<td>$K_g$</td>
<td>$K_g(-L_k)$</td>
<td>1 element + 2 kinematic pairs</td>
</tr>
<tr>
<td>$K_c$</td>
<td>$K_c(-L_k)$</td>
<td>Kinematic chain</td>
</tr>
</tbody>
</table>

The DOF number of the connection is defined by

$$L_k = 6n_m - \sum_{i=1}^{d}c_i - \sum L_{pk} - \sum L_{slk}$$  \hspace{1cm} (1)

Where: $n_m$ is the number of the elements, $c_i$ is the number of the $i$ class kinematic pairs enclosed in the connection; $L_{pk}$ and $L_{slk}$ are the DOF number of the passive links, respectively the number of the superfluous DOF introduced in the mechanism by means of the connection.

Table 1 presents the mode of symbolizing the degrees of freedom $L_k$ of each connection. Using the symbols and notations from the theory of mechanisms and machines, one can present the equivalences of the structures for the previous described kinematic pairs. Table 2 summarizes the variations of the $K_d$ connections with the number of DOF in the structure of planar and spatial mechanisms. In Table 3, variants of the connections $K_g$ are presented, while in Table 4 a number of possibilities to achieve a connection $K_c$ are outlined.

Structural synthesis of a mechanism envisages determining the mechanism’s structural scheme, meaning the number of the elements, the class of kinematic pairs and the way they are assembled in an ultimate totality with a well-determined motion of the elements.

Let $n_m$ be the number of the driving elements – inclusively those representing driving kinematic links – and $n_s$ the number of the driven elements. The mechanism desmodromy is ensured if the sum of DOF for the connections that realize the link between the mentioned elements ($\sum L_k$), is given by the relationship (2) for a spatial mechanism, respectively (3) for a planar one

$$\sum L_k = -(5n_m + 6n_s)$$  \hspace{1cm} (2)

$$\sum L_k = -(2n_m + 3n_s)$$  \hspace{1cm} (3)

In a broad approach, structural synthesis of the mechanism pursues the following path:

- step 1: the sum of DOF is determined for the connections that have to be introduced between the fixed motor elements and the driven elements;
- step 2: nature and position of the introduced connections are established.

The previous methodology, typically applied in synthesizing planar or spatial mechanisms, is as well proper for the structural synthesis of the parallel robots.

Table 2

<table>
<thead>
<tr>
<th>Variants for the $K_d$ connection</th>
</tr>
</thead>
<tbody>
<tr>
<td>Connection</td>
</tr>
<tr>
<td>------------</td>
</tr>
<tr>
<td>$K_d$ Type $R$</td>
</tr>
<tr>
<td>$K_d$ Type $P$</td>
</tr>
<tr>
<td>$K_d$ Type $S$</td>
</tr>
<tr>
<td>$K_d$ Type $H$</td>
</tr>
</tbody>
</table>
Table 3

Variants for the $K_B$ connection

<table>
<thead>
<tr>
<th>$K_B$ Type</th>
<th>Planar / Spatial mechanism DOF: $L_a = -1 / L_b = -4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R - R$</td>
<td>$K_B(2)$</td>
</tr>
<tr>
<td></td>
<td>$K_B(1)$</td>
</tr>
<tr>
<td></td>
<td>$K_B(3)$</td>
</tr>
<tr>
<td>$P - R$</td>
<td>$K_B(2)$</td>
</tr>
<tr>
<td></td>
<td>$K_B(1)$</td>
</tr>
<tr>
<td></td>
<td>$K_B(3)$</td>
</tr>
<tr>
<td>$S - S$</td>
<td>$K_B(2)$</td>
</tr>
<tr>
<td></td>
<td>$K_B(1)$</td>
</tr>
<tr>
<td></td>
<td>$K_B(3)$</td>
</tr>
</tbody>
</table>

Table 3 (continuation)

<table>
<thead>
<tr>
<th>$K_B$ Type</th>
<th>Planar / Spatial mechanism DOF: $L_a = -1 / L_b = -4$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P - H$</td>
<td>$K_B(2)$</td>
</tr>
<tr>
<td></td>
<td>$K_B(1)$</td>
</tr>
<tr>
<td></td>
<td>$K_B(3)$</td>
</tr>
<tr>
<td>$R - H$</td>
<td>$K_B(2)$</td>
</tr>
<tr>
<td></td>
<td>$K_B(1)$</td>
</tr>
<tr>
<td></td>
<td>$K_B(3)$</td>
</tr>
<tr>
<td>$U$</td>
<td>$K_B(2)$</td>
</tr>
<tr>
<td></td>
<td>$K_B(1)$</td>
</tr>
<tr>
<td></td>
<td>$K_B(3)$</td>
</tr>
</tbody>
</table>

Table 4

Possibilities of achieving $K_C$ connections

<table>
<thead>
<tr>
<th>Connection</th>
<th>Connection’s DOF</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$L_k = 12 - \sum_{j=1}^n l_j - L_{pk} - L_{dr}$</td>
</tr>
<tr>
<td>$K_C$</td>
<td>$L_k = -2$</td>
</tr>
</tbody>
</table>

5. Structural synthesis of parallel robots’ mechanisms

Practical solutions of mechanisms for parallel robots, which are applied also in medicine, consider 2 DOF for planar variants and DOF $\geq 3$ for spatial variants.

Within a spatial general approach, $n$ motor elements $n_1 = n$, identified by $E_i, i = 1, \ldots, n$ together with the fixed element “0” will constitute $n$ kinematic connections corresponding to the actuators of the parallel robot. The connections belong to the $K_{4(-2)}$ category, type $R, P$ or $H$ (Table 2), so that each connection limits a number of $L_{10} = -2$ DOF. The mobile mechanical structure has a single driven element $n_r = 1$, identifiable by the mobile platform (MP) (see Fig. 4). A particular case of the mechanism structures for the parallel robot is given when the mobile platform is reduced to a point ($n_r = 0$).

Fig. 4 Planar structure synthesis for a parallel robot

Accordingly to the relationship (3), the DOF that must be introduced using kinematic connections will be:

$$L_k = -(2n + 3)$$ (4)

Taking into account that the $n$ motor elements belong to $n$ connections $K_{4(-2)}$, the inserted DOF are:

$$\sum L_k = -2 n$$ (5)

From the previous relationships (4) and (5), it follows that other kinematic connections should be inserted; their DOF sum should equate the value $\Delta L_k$:

$$\Delta L_k = L_k - \sum L_k = -(2n + 3) - (-2n) = -3$$ (6)

The number of the DOF distributed on each parallel chain must be an integer $L_k \in Z$. Table 5 shows the analysis of a potential distribution of the DOF.

As a result of the earlier analysis, one concludes that symmetrical planar parallel robot with $n_1 = 1$ can be obtained for $n = 3$.  

Table 5

Potential DOF’s distributions on parallel chains

<table>
<thead>
<tr>
<th>Case</th>
<th>$n=2$</th>
<th>$L_k = \frac{\Delta L_k}{n} = \frac{-3}{n} \notin Z$</th>
<th>Asymmetric structure: 3 $K_{4(-2)}$ connections used</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$n=3$</td>
<td>$L_k = \frac{-3}{n} = -1$</td>
<td>Symmetric structure of type $3 \times L_{1k} = 3 \cdot (-1) = -3$; 3 $K_{4(-1)}$ connections used</td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td>Redundant asymmetric solution: 3 $K_{4(-1)}$ and $K_{0(0)}$ connections used</td>
</tr>
</tbody>
</table>
In the specific case of a mechanism structure for the parallel robot \((n_a = 0)\), the number of the DOF that must be introduced by means of kinematic connections is

\[
L_a = -(2n + 3 \cdot 0) = -2n
\]  

(7)

After the constitution of the motor kinematic connections, one can easily determine, based on relationship (5), that other connections must be introduced, which have to accumulate the number of DOF equal to

\[
\Delta L_k = L_k - \sum L_k = -(2n) - (-2n) = 0
\]  

(8)

The construction of the parallel robot is achieved by attaching \(K_{c(0)}\) connections to the motor elements.

Within a general spatial approach, \(n\) motor elements \(n_m = n\), nominated by \(E_i\), \(i = 1, \ldots, n\) will compose, together with the fixed element “0”, \(n\) kinematic connections that are equivalent to the actuators of the parallel robot. If considering these connections from within the \(K_{kl} = 5\) category of \(R, P, \) or \(H\) type (Table 1), each of them limit \(L_{k0} = -5\) DOF. The mobile mechanical structure has a single driven element \(n_c = 1\), identifiable by the mobile platform (MP) (Fig. 5).

From the relationship (2), the number of the DOF that have to be introduced by kinematic connections is:

\[
L_a = -(5n + 6)
\]  

(9)

Because the \(n\) motor elements belong to \(n\) connections \(K_{kl} = 5\) a certain number of DOF are added:

\[
\sum L_k = -5n
\]  

(10)

Based on the relationships (9) and (10), other new kinematic connections must be inserted, with DOF

\[
\Delta L_k = L_k - \sum L_k = -(5n + 6) - (-5n) = -6
\]  

(11)

Fig. 5 Generalized case for the synthesis of the spatial structure incorporated into the parallel robot

The number of DOF allocated on each parallel chain must be an integer \(L_k \in \mathbb{Z}\). An analysis of a potential distribution of the DOF is shown in Table 6.

A similar analysis can be performed too in the case were the robot actuator is equivalent to a \(K_{b(-1)}\) connection (linear actuator – Table 3).

6. Examples of structural syntheses for parallel robots’ mechanisms

Case 1

The synthesis of planar parallel mechanisms can readily offer multiple structural variants. Two 5th class motor kinematic pairs – in any combination (see Table 1) - constitute two \(K_{kl} = 5\) connections that count

\[
\sum L_k = -4\) DOF (Fig. 6); the number of the motor elements is \(n_m = 2\). If admitting for the parallel robot the existence of one and the only one driven element, \(n_c = 1\), one can determine the number of the required DOF that must be added by means of connections, that is \(L_a = -(2 \cdot n_m + 3 \cdot n_c) = -(2 \cdot 2 + 3 \cdot 1) = -7\).

Given the sum of the introduced DOF, the balance of the DOF further requires the introduction of a number of connections to totalize the value of \(\Delta L_k = L_k - \sum L_k = -7 - (-4) = -3\) DOF. Three DOF can be distributed through three \(K_{b(-1)}\) connections, which, together with the mobile platform, would build the parallel kinematic chains (Fig. 7). Thus, a planar parallel robot is achieved. Fig. 8 shows the robot structure.
The insertion mode of the connections allows any functional combinations; one can obtain new variants for the parallel robot. Conforming to the connections’ property No. 2, two $K_{B(-1)}$ connections from the total of three connections provided to be introduced, allow obtaining a parallelogram kinematic chain ($K_{C(-2)}$ connection) without the modification of the DOF sum (Fig. 9).

Additional interleaving of two $K_{C(0)}$ connections, besides the $K_{B(-1)}$ connections, without modifying the sum of the DOF:

\[
\text{step 1: } K_{C(0)} + K_{C(-2)}; \\
\text{step 2: } K_{C(0)} + K_{B(-1)}.
\]

The resulted variant of the parallel robot structure is shown in Fig. 11.

Case 2
One considers the case in Fig. 12. The fixed platform (FP) is equipped with three motor elements $n_m = 3$ arranged in the same plane and making the motor kinematic pairs A, B and C. As shown before, the three motor kinematic pairs are assimilated each of them by a $K_{A(-2)}$ connection. The mobile platform (MP) is equivalent to the driven element $n_i = 1$.

Synthesis of the parallel mechanisms that must be interleaved between motor elements and driven element keeps the following steps:

- step 1 – computation of DOF sum for the connections that must be interleaved (from the relationship (3)): $L_k = (2 \cdot 3 + 3 \cdot 1) = -9$;
- step 2 – distribution of the DOF over the connections that have to be interleaved. Because three $K_{A(-2)}$ connections were formerly interleaved, they
already accumulate \( \sum L_k = 3 \cdot (-2) = -6 \) DOF.

The balance of the DOF involves that a number of
\[ \Delta L_k = L_k - \sum L_k = -9 - (-6) = -3 \] DOF still have to be
introduced. The three DOF will be distributed over three
\( K_{B(-1)} \) connections. The mechanism in Fig. 13 is got.

\[ \Delta L_k = L_k - \sum L_k = -4 - (-4) = 0 \]

This balance requires the introduction in the second
step of a connection \( K_{C(0)} \). The parallel structure in
Fig. 15 is obtained. The end effector position in the synthesized
structure is shown in Fig. 16.

\[ \Delta L_k = L_k - \sum L_k = -4 - (-4) = 0 \]

Planar motion of a point on a given path, based on
a parallel structure, can be achieved using a mechanism with two DOF.

In correspondence with the expected goal, one establishes
the number of motor elements to \( n_m = 2 \); the
number of driven elements is \( n_c = 0 \). DOF of the connections
to be interleaved, with the aim of ensuring the desmodromy, is given by the equation (12) and balance of the
DOF is outlined by the equation (13)

\[ L_k = -(2 \cdot n_m + 3 \cdot n_c) = -(2 \cdot 2 + 3 \cdot 0) = -4 \] (12)

\[ \Delta L_k = L_k - \sum L_k = -4 - (-4) = 0 \] (13)

One refers to the previous problem, which is the
planar parallel robot that might locate point C on a given
curve. One admits the possibility to achieve two motor kinematic pairs by using two linear actuators. Equivalency
with the theory of the kinematic connections corresponds
to two \( K_{B(-1)} \) connections with their sum of the DOF given
by the following relationship:

\[ \sum L_k = (-1) + (-1) = -2 \] (14)

Balance of the remaining DOF equals \( \Delta L_k = -4 - (-2) = -2 \). To ensure the desmodromy,
the interleaving of a connection with \( L_k = -2 \) is required
(Fig. 17). Insertion of a connection \( K_{C(0)} \) allows achieving
a new variant of a parallel robot with two DOF
(Fig. 17) [13]. If a kinematic chain, composed of two connections - \( K_{B(-2)} \) and \( K_{C(0)} \) - is interleaved, balance of the
DOF is complied, but another variant of the parallel robot
is achieved, solving the expressed problem.
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Case 5

A \( K_{C(2)} \) connection was presented in Table 4 and is based on a universal kinematic pair. Another \( K_{C(2)} \) connection is achieved by the combination of two \( K_{B(-1)} \) connections built of spherical kinematic pairs (S) (Fig. 18).

![Fig. 17 Planar structure of a parallel robot with 2 DOF](image)

For a parallel robot with 3 DOF (Fig. 19), the \( K_{C(2)} \) connection is inserted between the motor elements and the mobile platform (MP). The structural variants derived from the Delta robot are achieved [2], Triglide [10].

![Fig. 18 \( K_{C(2)} \) connection – namely a parallelogram (Pa) achieved by means of two \( K_{B(-1)} \) connections](image)

![Fig. 19 The \( K_{C(2)} \) connection – namely a parallelogram (Pa), within the structure of a parallel robot](image)

7. Conclusion

1. Numerous applications of parallel robots in medical field bring this field closer to mechatronics. As robots are, by their own nature, mechatronic systems, designing a parallel robot according to mechatronic design philosophy is a logical decision that allows general systemic approach.

2. Analysis and structural synthesis are together the mandatory first stage in designing a parallel robot.

3. The connections method is a procedure for analysis and structural synthesis of mechanisms with broad applicability in the field.

4. The use of the connections method is simple and fast to implement.

5. The connections method offers many options in the inventics field because it can be combined with inventics procedures for developing parallel structures.

6. Numerous and diverse variants of planar and spatial parallel robots were defined for many practical applications based on the same design approach.

7. Ensuring desmodromy for mechanisms can involve dynamic approach too; in this case, the item of “dynamic connection” might be used as a continuation of the “kinematic connection”.

8. In order to achieve final design solution, one has to follow all the stages in the design of a mechatronic product, namely concept, detailed design, modelling and simulation & control.

References


high rigidity, high load capacity, high velocity and high precision. Parallel robot is a mechatronic system and requires to be designed in the spirit of mechatronic philosophy. This paper proposes a method of structural synthesis of parallel robotic mechanism according to the concept of mechatronic philosophy and of the kinematic connections.

The V design model - V design cycle - is adopted. The methodology considers an iterative process corresponding to the stages of product design development. During the design process, one cannot consider a universal parallel robot due to the reduced workload. The use of a parallel robot in a given application requires careful analysis of the structure and of the indispensable parameters. Structural synthesis of mechanisms for parallel robots is addressed in detail.

The kinematic connections are defined and classified; the conceptual mode of operation and the structural analysis of the mechanisms are noted. Principles of structural synthesis using the concept of the kinematic connections are discussed; edifying examples of planar and spatial parallel robot structures for this synthesis are summarized.

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THE STRUCTURAL SYNTHESIS OF THE PARALLEL ROBOTS

Summary

Parallel robots have received special interest due to their advantages over conventional serial mechanisms: