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A new algorithm for cylindrical worm gears dimensioning based on the hydrodynamic lubrication conditions between the teeth flanks

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1. Introduction

A method for increasing the lifetime of the cylindrical worm gears is to ensure conditions of hydrodynamic lubrication between the flanks of the meshing teeth. In the paper the main parameters of the cylindrical worm gears are determined taking into account the load conditions and some parameters connected to the lubrication.

2. Cylindrical worm gear module calculation based on hydrodynamic lubrication

For cylindrical worm gears with hydrodynamic lubrication used in the mechanical transmission field, based on the expressions from [1, 2] the module can be determined by the following relation

$$m_x \geq \frac{2}{q+z_2+2x} \sqrt[1.39]{\frac{T_2^{0.13}\lambda(R_{a1}+R_{a2})}{21h^* C_\alpha^{0.6}\eta_{OM}^{0.7} n_1^{0.7} E_{red}^{0.03}}} \quad (1)$$

where the lubrication thickens $h^* = 0.018 + \frac{q}{7.86(q+z_2)} + \frac{1}{z_2} + \frac{x}{110} - \frac{\frac{z_2}{z_1}}{36300} + \frac{2\left(0.5+\sqrt{q+1}\right)}{370.4} - \frac{\sqrt{2q-1}}{213.9}$, q the diameter factor, z_1 the number of the threads (starts) of the worm, z_2 the number of the teeth of the worm wheel, x the specific addendum modification of the wheel, T_2 the moment of torsion on the axle of the worm wheel in Nm, λ the safety coefficient (λ = 1, ..., 2), R_{a1} and R_{a2} the arithmetic average roughness on the teeth of the worm and on the worm wheel measured in µm (R_{a1} = 0.4 µm at corrected worm and R_{a2} = 1.6 µm for the milled worm wheel), $C_\alpha = 1.7\times10^{-8}$ m^2/N is the viscosity pressure variation exponent in the case of mineral oils, η_{OM} the oil viscosity at ambient pressure and at temperature of the meshing area in Ns/m^2, n_1 the angular speed of the worm in min^{-1}, E_{red} = 140144 MPa is the elastic modulus of the teeth (for bronze worm wheel CuSn12 and steel worm).

The m_x module and the q diameter factor determined from Eq. (1) must satisfy the strength conditions of the transmission and the efficiency should be as high as possible.

3. Lubricated worm gears efficiency

If the lubrication conditions are accomplished between the teeth flanks during the meshing the efficiency can be calculated with the formula from [3] as

$$\eta = \frac{1}{1 + \frac{\mu}{cos(\alpha_n)} \frac{V_{12}}{V_1 cos(\beta_1)}} \quad (2)$$

where μ is the friction coefficient between the flanks of the teeth in contact, α_n the worm tooth profile angle at normal section (α_n can be considered 20°), V_{12} relative velocity between the teeth flanks on the rolling cylinder of the worm, V_1 peripheral speed of the worm on the rolling cylinder of the worm, β_1 worm tooth declination angle on the rolling cylinder.

For worm gears with the wheel made of bronze CuSn12 and the worm from hardened steel HRC > 45. The friction coefficient can be calculated using the relation in [3]

$$\mu = \frac{0.04}{\sqrt[4]{V_{12}}} = \frac{0.04}{\sqrt[4]{\frac{\pi m_x n_1}{60\times1000}\sqrt{z_1^2+q^2}}} \quad (3)$$

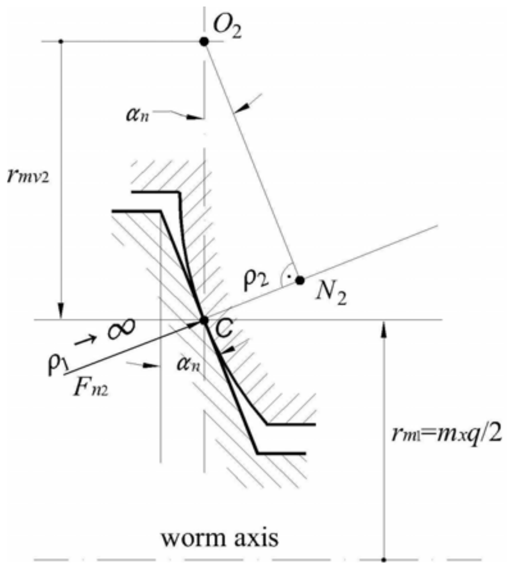

Fig. 1 Teeth contact between the worm and the worm wheel in the normal section

Considering the contact point C (Fig. 1) where the common perpendicular between the axes stings the division cylinder of the worm, using relations (2) and (3), the following relation can be established for the efficiency

$$\eta = \frac{z_1 q cos(\alpha_n)}{z_1 q cos(\alpha_n) + \frac{0.04\left(z_1^2+q^2\right)}{\sqrt[4]{\frac{\pi m_x n_1}{60\times1000}\sqrt{z_1^2+q^2}}}} \quad (4)$$

4. Contact pressure check for the module determined by the hydrodynamic lubrication conditions

The contact pressure between the teeth flanks can be determined by the Hertz formula given in [4, 5]

$$\sigma_H = \sqrt{\frac{F_n}{L_k}\frac{\rho_1+\rho_2}{\rho_1\rho_2}\frac{1}{\pi\left(\frac{1-\nu_1^2}{E_1}+\frac{1-\nu_2^2}{E_2}\right)}} \le \sigma_{Ha} \tag{5}$$

where F_n is the normal force on the tooth flank in the normal section of the worm tooth (Fig. 1), L_k the length of the contact line between the teeth flanks in contact ($L_k \approx 0.55 m_x q$), ρ_1 and ρ_2 the radii of curvature of the profiles in contact in normal section, ν_1 and ν_2 the Poisson coefficients for the material of the worm and of the wheel ($\nu_1 = 0.30$ for the steel worm and $\nu_2 = 0.35$ for the bronze worm), E_1 and E_2 the elastic modules of the worm and the wheel ($E_1 = 2.1\times10^5$ MPa for steel worm and $E_2 = 0.883\times10^5$ MPa for the bronze CuSn12 wheel), σ_{Ha} the admissible contact pressure of the wheel from the worm gear ($\sigma_{Ha} = 425$ MPa).

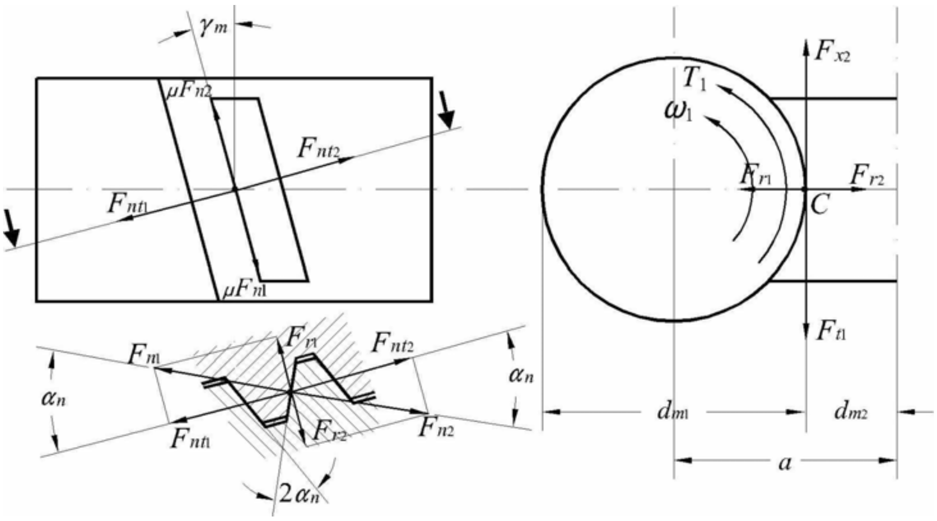

Fig. 2 Forces acting on the worm and the worm wheel in the contact point

The F_{n2} normal force on the tooth worm wheel can be determined from Fig. 2 using the relation

$$F_{n2} = \frac{F_{t2}}{cos(\alpha_n)\left(cos(\gamma_m) - tan(\varphi_1)sin(\gamma_m)\right)} = \frac{\frac{2T_2}{d_{m2}}}{cos(\alpha_n)\left(cos(\gamma_m) - tan(\varphi_1)sin(\gamma_m)\right)} \tag{6}$$

where F_{t2} is the tangential force on the worm wheel, $d_{m2} = m_x(z_2+2x)$ the rolling cylinder diameter of the worm wheel in [mm], $\gamma_m = atan(z_1/q)$ the lifting angle on the reference cylinder of the worm, $\varphi_1 = atan(\mu/\cos(\alpha_n))$ the reduced friction angle.

The curvature radius of the wheel profile in normal section can be determined using the relation

$$\rho_2 = \frac{1}{r_{mv2}sin(\alpha_n)} = \frac{2cos^2(\gamma_m)}{d_{m2}sin(\alpha_n)} \tag{7}$$

Replacing Eqs. (6) and (7) in Eq. (5) the following expression for the computation of the m_x module can be found [6]

$$m_x \ge \sqrt[3]{\frac{8}{0.55sin(\alpha_n)}\frac{T_2 q}{(z_2+2x)^2(q-\mu_1 z_1)}\frac{1}{\sqrt{z_1^2+q^2}}\frac{1}{\pi\left(\frac{1-\nu_1^2}{E_1}+\frac{1-\nu_2^2}{E_2}\right)}\frac{1}{\sigma_{Ha}^2}} \tag{8}$$

Using Eq. (8) it is possible to check if the module obtained on the conditions of hydrodynamic lubrication also satisfies the contact pressure conditions for the meshing teeth.

5. Worm shaft bending check

As shown in [6, 7] bending of the worm shaft can be determined using

$$f = \frac{l^3}{48E_1 I_{m1}}\sqrt{F_{t1}^2+F_{r1}^2} \le f_a \tag{9}$$

where l is the distance between the supports of the worm ($l = \psi_a a$, generally $\psi_a \approx 1.5, ..., 2$), $a = m_x(q+z_2+2x)/2$ the distance between the axes of the worm and wheel in [mm], $I_{m1} = \pi d^4_{m1}/64$ geometric moment of inertia in mm^4, $d_{m1} = m_x q$ reference diameter of the worm shaft, F_{t1} tangent force on the denture of the worm, F_{r1} radial force on the denture of the worm, f_a admissible bending deformation.

From Fig. 2 the F_{t1} and F_{r1} forces can be computed using the following relations

$$F_{t1} = \frac{2T_1}{d_{m1}} = \frac{2T_1}{m_x q} \tag{10}$$

$$F_{r1} = F_{n1} sin(\alpha_n) = \frac{2T_1\sqrt{z_1^2+q^2}\,tan(\alpha_n)}{m_x q(z_1+\mu_1 q)} \qquad (11)$$

where T_1 is the torque on the worm shaft.

Taking into account the presented relations the bending of the worm can be computed from Eq. (9) thus

$$f = \frac{l^3}{48E_1 I_{m1}}\sqrt{\left(\frac{2T_1}{m_x q}\right)^2 + \left(\frac{2T_1\sqrt{z_1^2+q^2}\,tan(\alpha_n)}{m_x q(z_1+\mu_1 q)}\right)^2} \qquad (12)$$

or finally

$$f = \frac{\psi_a^3}{3\pi E_1}\frac{(q+z_2+2x)^3}{m_x^2 q^5}\frac{z_1}{z_2}\frac{T_2}{\eta}\sqrt{1+\frac{z_1^2+q^2}{(z_1+\mu_1 q)^2}tan^2(\alpha_n)} \le f_a \qquad (13)$$

where $T_2 = \eta T_1 z_2/z_1$ is the torque on the worm wheel, f_a the admissible bending deformation [4] ($f_a = 0.004m_x$ at hardened worm and $f_a = 0.01m_x$ at improved worm).

6. Algorithm implementation and numerical results

Matlab was used to find the main parameters of the cylindrical worm gears that satisfy the required hydrodynamic lubrication conditions, bearing capacity and stiffness at curving using the following algorithm:

```
k=1;
A=[];
for q=7:1:17
 for x=-1:0.1:1
  eta <- relation (4)
  f   <- relation (13)
  mx  <- relation (1)
  fmx <- right part of relation (8)
  %% f<=0.01*mx - improved worm
  %% f<=0.004*mx - hardened worm
  if (eta>=etamin) &&
     (f<=0.01*mx) && (mx>=fmx)
   A = [A;[q x eta f 0.01*mx mx fmx]];
   k=k+1;
  end;
 end;
end;
try
 B=sortrows(A, 3);
 for i=1:k-1
  fprintf('%f … %f\n', B(i,1), …,
B(i,7));
 end;
catch exception
 disp(exception.message);
end;
```

The results from Table 1 and Table 2 were obtained for the following input data: $z_1 = 1$, $z_2 = 41$, `eta-min` $= 0.85$, $\eta_{OM} = 0.08$ Ns/m^2, $n_1 = 1000$ min^{-1}, $T_2 = 587$ Nm, $\lambda = 1$, $\psi_a = 1.5$. Two `for` cycles are used to control the variation of q and x main parameters. The q is changing between 7 and 17 with step 1 and x between -1 and 1 with step 0.1. The values of m_x are computed using the (1) expression and stored in the `mx` variable. Only the results that have the efficiency (`eta` variable) computed using Eq. (4) higher than the given `etamin` value and that are satisfying Eqs. (8) and (13) are stored in the A matrix. These rows are sorted ascending by the efficiency column (the third column in the A matrix), stored in B matrix and then printed. The bending from Eq. (13) is given in the f (left part of the inequality) and f_a (right part of the inequality) columns, while the module from Eq. (8) is given in the `fmx` (is the right part of inequality). Exceptions must be handled using `try … catch` as the restrictions can leave the A matrix without any rows, in which case the sorting would cause error.

7. Conclusions

Based on the numerical results from Table 1 and Table 2 in the case of the cylindrical worm gears of ZI, ZA, ZN and ZK type, with hydrodynamic lubrication between the teeth flanks, the axial module m_x must be high, while the q diameter factor must be low. From Table 1, for hardened worm, the x specific addendum modification can be chosen from the [-1, -0.2] domain. From Table 2, where the worm is improved, the values for the specific addendum modification cam be choose form the [-1, -0.2] domain. Numerical results confirm the hypothesis of the literature [5] as to have hydrodynamic lubrication between the flanks of the teeth, at worm gears, the specific addendum modification is good to be negative. With the help of these values the geometrical dimensions of the gear can be determined while also having knowledge of the efficiency.

Table 1

Main parameters for hardened worm

q	x	η	f<	f_a	m_x >	fmx
7	-0.2	0.850	0.0008	0.096	23.91	5.35
7	-0.3	0.851	0.0008	0.097	24.28	5.36
7	-0.4	0.851	0.0007	0.099	24.66	5.38
7	-0.5	0.852	0.0007	0.100	25.05	5.40
7	-0.6	0.852	0.0007	0.102	25.46	5.42
7	-0.7	0.853	0.0007	0.103	25.87	5.44
7	-0.9	0.853	0.0006	0.105	26.30	5.45
7	-0.9	0.854	0.0006	0.107	26.74	5.47
7	-1.0	0.854	0.0006	0.109	27.19	5.49

Table 2

Main parameters for improved worm

q	x	η	f<	f_a	m_x >	fmx
7	-0.2	0.850	0.0008	0.239	23.91	5.35
7	-0.3	0.851	0.0008	0.243	24.28	5.36
7	-0.4	0.851	0.0007	0.247	24.66	5.38
7	-0.5	0.852	0.0007	0.251	25.05	5.40
7	-0.6	0.852	0.0007	0.255	25.46	5.42
7	-0.7	0.853	0.0007	0.259	25.87	5.44
7	-0.8	0.853	0.0006	0.263	26.30	5.45
7	-0.9	0.854	0.0006	0.277	26.74	5.47
7	-1.0	0.854	0.0006	0.272	27.19	5.49

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NAUJAS CILINDRINĖS SLIEKINĖS PAVAROS MATMENŲ NUSTATYMO ALGORITMAS, ĮVERTINANTIS HIDRODINAMINDELFI.LT ES KRUMPLIŲ ŠONŲ TEPIMO SĄLYGAS

R e z i u m ė

Straipsnyje aprašytas naujas būdas cilindrinės sliekinės pavaros geometriniams parametrams nustatyti, įvertinant tepalo plėvelės susidarymą ant krumplių šonų. Atsižvelgiant į tai, nustatytas ašinis modulis m_x ir kiti parametrai, užtikrinantys gerą sliekinės pavaros efektyvumą bei patenkinantys guolių apkrovimo ir slieko lenkimo sąlygas. Pagrindinių parametrų skaitiniams rezultatams nustatyti naudota programa Matlab.

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A NEW ALGORITHM FOR CYLINDRICAL WORM GEARS DIMENSIONING BASED ON THE HYDRODYNAMIC LUBRICATION CONDITIONS BETWEEN THE TEETH FLANKS

S u m m a r y

The paper gives a new method for obtaining the geometrical dimensions for cylindrical worm gears based on the conditions of forming a lubrication film between the teeth flanks. Based on this requirement the axial module m_x is determined together with other parameters so that the worm gear will have good efficiency while the bearing strength and worm shaft bending conditions are satisfied. Numerical results were obtained for the main parameters using the Matlab programming environment.

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