Simulation of the part-to-part vibratory alignment under impact mode displacement

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1. Introduction

The part-to-part alignment in assembly position predetermines the reliability of the automated assembly. Promising are vibratory methods of the alignment, based on various locating techniques of the parts and different ways of vibratory excitation. One of the being assembled parts may be excited by the low frequency vibrations, causing the part to vibrate like a solid body, or elastic vibrations of the part may be forced. The piezoelectric vibrators are used to excite the elastic vibrations [1, 2]. The electromagnetic or pneumatic vibrators may be used to cause the part to vibrate as a solid body.

Part-to-part alignment of the being assembled parts may occur as a result of impact mode motion of the movably based mating part, which was subjected to vibratory excitation. The impact motion of the part is such a motion, which occurs within a small time interval and causes rapid change in velocity of the part [3]. During the direct central impact, velocities of the colliding bodies are directed along the line of impact, i.e. along the common normal to the surfaces which are contacting during the impact. Considering the oblique impact, additionally it is necessary to analyze the emergent tangential component of the impact impulse, which depends on the characteristics of the impact and on friction between the contacting surfaces.

The analysis of the vibro-impact processes generally is based on the presumptions that magnitudes of the arising forces are relatively high, but the influence on the impacting parts is limited due to short duration of the impact. The colliding surfaces are smooth therefore during central impact the friction may be neglected.

The influence of the impact may be analyzed based on the Newton hypothesis, when the restitution coefficient is defined by relative normal velocities of the impacting bodies before and after the collision [4]. In the case of oblique impact, when colliding body is tilted relative to the supporting base, Newton’s hypothesis is used to calculate normal component of the velocity. Generally the presumption is made, that surfaces of the colliding bodies are smooth and both the direction and magnitude of tangential component of the velocity remain constant during the impact. To solve practical problems the hypothesis is used, that tangential component of the post-impact velocity is nondependent on the normal component, but is dependent on the constant $\lambda$ ($\lambda$ is instantaneous coefficient of friction during the impact), which is predetermined by the characteristics and state of colliding bodies [3]. Thus the tangential component of the impact impulse is proportional to the relative velocity of the colliding bodies. The other existing hypothesis states that impact collision should be analyzed applying dry friction law [5].

Commonly impact systems are analyzed applying the more simple theory, i.e. the stereomechanic theory of impact. This theory considers instant impact and determines the impact moment and post-impact state of the analyzed impact systems [6, 7], applying the velocity restitution coefficient. Then kinematical characteristics of the colliding parts are defined by the common theory of solid body mechanics, not going into details about the impact process as such.

Previous investigations of different researchers are mostly related to the dynamics of vibro-impact systems with direct central impact. As examples may be considered works of Babitsky [8] and Kobrinskij [9] which provide detailed analyses, assuming that the dynamics of vibro-impact systems can be reduced to only oscillatory motion. Nonlinear dynamics of impacting oscillators has received a considerable theoretical and experimental attention in scientific publications [10, 11].

The system, which represents the process of impact mode alignment, has a particular set of characteristics. The first peculiarity is that the point, where the part impacts the supporting base, changes during the alignment because the part not only displaces relative to the supporting base, but also performs rotational motion. Therefore, impact collision of the part and supporting base is presented by two-dimensional model of the impact. The second peculiarity is that elastic locating elements prevent the displacement and rotation of the part. Therefore, the part is able to displace from the static towards the dynamic equilibrium position. Impact mode alignment occurs only if the part bounces, losing contact with the supporting base. To cause this it is necessary the excitation force to be higher than total of the initial force of the part pressing to the supporting base and gravity.

The main objective of the presented paper is to analyze the impact mode motion of the movable assembly component and determine the parameters, which have significant influence on the character and duration of the component motion and provide conditions for reliable joining of the being assembled parts.

2. Model of the impact collision of being aligned components

The impact models along the normal and tangential directions present the impact collision of the movably based part and supporting base and consist of elastic and damping elements, representing deformation of the base during the impact. It is assumed, that impacted surface is massless.

The movable $M$ mass part is able to displace
along the $X$ and $Y$ directions and turn relative to the locating base (Fig. 1). The position of the movable part is characterized by the coordinates $X_c, Y_c$ of the mass centre of the part and by turn angle $\phi$. As a result of the provided vibratory excitation along the assembly direction ($Y$), the movable part displaces over the locating base. Impact collision of the movable part and supporting base is classified as an oblique and causes reactions $N_{11}$ and $N_{12}$, sliding friction forces along the contacting surfaces and deformation forces of the base prevent displacement of the movable part.

Based on the made presumptions, differential equations of motion of the body, contacting the locating base, are expressed as follows

$$
\begin{align*}
MX_c'' + H_x X_c' + K_x (X_c - X_u) &= F_x, \\
MY_c'' + H_y Y_c' + K_y (Y_c - Y_u) &= F_y, \\
I\ddot{\phi} + H_{\phi}\dot{\phi} + K_{\phi} (\phi - \phi_u) &= M_{\phi},
\end{align*}
$$

(1)

here $F_x, F_y$ are projections of the reaction forces onto the $X, Y$ axes; $F_{11} = K_{xy} Y \sin \alpha$ is reduced excitation force; $M_{\phi}$ is the reaction moment; $K_x, K_y, K_{\phi}$ are the corresponding constraint rigidities, illustrating the movably based part’s ability to displace/turn along the corresponding direction; $K_{xy} = K_{x1} + K_{x2}, K_{y} = K_{y1} + K_{y2}; Y_1$ is the deformation of the supporting base. The values $F_x, F_y, M_{\phi}$ are calculated by the expressions

$$
\begin{align*}
F_x &= N_1 (\sin \phi - f \cos \phi \sin \nu), \\
F_y &= N_1 (\cos \phi + f \sin \phi \sin \nu), \\
M_{\phi} &= N_1 (Q + fG \sin \nu),
\end{align*}
$$

(2)

here $N_1 = N_{11} \cos \phi + N_{12} \sin \phi$ is the reaction force of the locating base, acting onto the body; $N_{11} = H_1 Y_1' + C_1 Y_1; N_{12} = H_2 X_1' + C_2 X_2; N_1$ is normal reaction force at the contact point $A_1; C_1, C_2, C_{\phi}$ and $H_1, H_2$ are the corresponding rigidity and damping coefficients of the locating base.

The relative velocity of the body in respect of the base is $V = X_c' \cos \phi + Y_c' \sin \phi + G \phi'$. When the body is not contacting the locating base, its motion is expressed by the equations

$$
\begin{align*}
MX_c'' + H_x X_c' + K_x X_c &= 0, \\
MY_c'' + H_y Y_c' + K_y Y_c &= F_y, \\
I\ddot{\phi} + H_{\phi}\dot{\phi} + K_{\phi} \phi &= 0.
\end{align*}
$$

(3)

To express the dimensionless equations of the movably based part motion the following notations have been used:

$$
\begin{align*}
\tau &= pt; ^* = d/d\tau; p^2 = K_1/\mu; v = \omega/p; x_e = K_1 Y_1/F_1; \\
y_e = K_2 Y_2/F_1; \ l = K_1 L/F_1; \ g = K_2 G/F_1; \\
I_{11} &= \left(\left[I \left[ K_1 / F_1 \right] \right] \right); h_1 = H_1 / \sqrt{K_1 M}; h_2 = H_2 / \sqrt{K_2 M}; \\
h_\phi = H_\phi \left( p^2 M / F_1^2 \right); \ k_x = K_x / K_1; k_y = K_y / F_1; \\
q &= K_1 Q / F_1 = \frac{1-x_e}{\cos \phi} - \tan \theta; n_1 = N_1 / F_1; n_2 = N_2 / F_1; \\
f_x &= F_1 / F_1; f_y &= F_2 / F_1; m_\phi &= M_{\phi} / F_1; \nu_1 = C_1 / C_1; \\
\nu_2 = C_2 / C_2; \ h_1 = H_1 / \sqrt{K_1 M}; \ h_2 = H_2 / \sqrt{K_2 M}; \\
y_1 &= K_1 Y_1 / F_1; \ x_2 &= K_2 X_2 / F_1; \ k_z = C_z / C_z.
\end{align*}
$$

(4)

Dimensionless equations of motion under body-supporting base contact are expressed as follows:

$$
\begin{align*}
\ddot{x}_c + h_1 \dot{x}_c + k_x (x_c - x_u) &= f_x, \\
\dot{y}_c + h_2 \dot{y}_c + (y_c - y_u) &= \sin \tau - f_y, \\
\mu_\phi \dot{\phi} + h_\phi \dot{\phi} + k_\phi (\phi - \phi_u) &= m_\phi, \\
n_2 + f_\phi &= 0,
\end{align*}
$$

(5)

where $f_x = n_1 (\sin \phi - f \cos \phi \sin \nu)$; $f_y = n_1 (\cos \phi - f \sin \phi \sin \nu)$; $m_\phi = n_1 (q + fG \sin \nu)$; $n_1 = n_{11} \cos \phi + n_{12} \sin \phi; n_1 = h_1 y_1 + k_1 y_1; n_2 = h_2 x_2 + k_2 y_2; \nu = \dot{x}_c \cos \phi + \dot{y}_c \sin \phi + b \phi; x_u, y_u, \nu_u$ are the coordinates of static equilibrium position.

Equations of the body motion without contact with the supporting-base

$$
\begin{align*}
\ddot{x}_c + h_1 \dot{x}_c + k_x x_c &= 0, \\
\dot{y}_c + \dot{y}_c + y_c &= \sin \tau, \\
\mu_\phi \dot{\phi} + h_\phi \dot{\phi} + k_\phi \phi &= 0.
\end{align*}
$$

(6)

Collision of the body with the point $A_2$ may be expressed by the analogy.
3. Simulation of vibratory displacement of the part under impact mode

To analyze the impact collision of the part with contact points of the supporting base the dynamical model of the impact was made. Generally, with no contact with the supporting base, normal reaction is equal to 0. Normal reaction at the points of contact was calculated replacing the supporting base by elastic and damping elements. During the oblique collision of the movable part and supporting base because of the elastic and damping elements, the supporting base gets deformed along the $Y_1$ axis direction by the $X_1$ and along the $X$ axis direction by the value $X$ correspondingly (Fig. 1).

The equations of the movably part motion Eqs. (4), (5) have been solved numerically in MATLAB.

Simulation of the vibratory alignment of the parts by impact mode was carried out applying the particular excitation amplitude and part-to-part pressing force aiming to cause occurrence of the impact. Initially, during the simulation, the distance between the part and locating base was calculated. The analyzed distance between the part and the supporting base provides possibility to identify contact or noncontact state between the part and the supporting base. If mentioned distance is less or equal to zero, then the contact between the movable part and the locating base is identified, along with the normal reaction force, which is perpendicular to the surface of the part. The calculated normal reaction $n_i$ depends on deformation of the supporting base. Having calculated the normal reaction, it is possible to solve the equations of the body motion and define coordinates $x_C$, $y_C$ of the mass center and turn angle $\phi$ of the part. These results provide the possibility to estimate the character of the parts alignment process.

Due to positioning errors, initially coordinates of the points $B_1$ and $B_2$ of the movably based part are displaced relative to the points $A_1$ and $A_2$ of the base. Varying the coordinates of the mentioned points, it is possible to analyze the positioning errors influence on the efficiency of the parts’ alignment process, when chosen criterion is alignment duration. The influence of joint clearance on the alignment process characteristics was simulated varying the distance between the origin of the coordinate frame and $A_1$ and $A_2$ points.

The value of the body coordinate $x_C$, which represents the part-to-part misalignment error in assembly position, initially was set to be positive. During the alignment, the body moves towards the hole’s axis and so the misalignment error is compensated (Fig. 2, a). The tilt angle $\phi$ changes during motion of the body (Fig. 2, e). Under the existing joint clearance between the body and the supports, the body, during its slip down into the hole ($\tau \approx 3.1$), may be slightly tilted.

The vibratory displacement and turn of the body by impact mode is accompanied by complex dynamic phenomena. Depending on parameters of the dynamic system and excitation and on their matching, displacement of the movably based body may be of more intensive or rather slow character. Within particular ranges of the parameters values, displacement of the part is characterized by single impacts, which generally are accompanied with repeated small impacts. Improperly chosen parameters of the dynamic system and excitation may result the transition from the impact mode to the nonimpact mode displacement of the body. If tilt angle of the body decreases during the alignment, while the initial pressing force remains unchanged, vertical component of the normal reaction increases, whereas the horizontal component – diminishes. In order the body would be able to bounce off the supporting base, it is necessary to provide higher amplitude of the excitation. Therefore, nonimpact mode displacement may occur. Smaller horizontal component of the normal reaction, results the increase in alignment duration. Such phenomena were noticed under relatively small magnitudes of the excitation amplitude and frequency. The impact mode displacement of different character is characterized by different normal reaction forces.

![Fig. 2](image-url) Time dependences of generalized coordinates of the body, velocities and normal forces during the impact mode displacement: a – $x_C(r)$; b – $y_C(r)$; c – $\dot{x}_C(r)$; d – $\dot{y}_C(r)$; e – $\phi(r)$; f – $n_1(r)$, $n_2(r)$; as $f_0 = 0.2$; $\delta = 0.09$; $x_{cl-o} = 0.2$; $y_{cl-o} = 0.895$; $\nu = 20$; $\phi_{l-o} = 0.025$

Time dependences of normal reactions (Fig. 2, f) characterize collision of the body and the supporting base during the impact mode displacement. Prior to the slip down into the slot, the body collides with the opposite edge of the supporting base, the normal reaction force $n_2$ emerges and shortly the part starts the slip down into the slot.

It was defined from the graphs of the simulation that impact collision of the body and supporting base, and also the impact mode displacement, may be of different character. Under higher magnitudes of angular rigidity coefficient $k_{\phi}$, the collision of the body and supporting base at $A_1$ point of contact in most cases occurs prior the slip down into the slot.

Under smaller angular rigidity $k_{\phi}$, the body is able to rotate by a larger angle and the normal reaction $n_2$ may emerge at the point $A_2$ far prior to the slip down into the slot. Under particular magnitudes of angular rigidity, excitation and initial pressing forces, several impacts of the body occur at the point $A_1$ on the supporting base, but later the impacts disappear.
During the impact mode displacement of the movable part, the increase in pressing force causes the increase in tangential component of the oblique impact impulse and, consequently, results more intensive and relatively shorter alignment duration. As initial pressing force increases, the turn of the movably based part is more intensive, but changes in alignment duration not always are positive. The dependences of alignment duration versus initial pressing force $f_0$, under different axial misalignment of the mating components, are close to linear (Fig. 4). When axial misalignment of the components increases, alignment duration also increases. Under more significant initial misalignment error, i.e. as $x_{c|p-o} = 0.3, x_{c|p-o} = 0.4$, impact mode prevails within a more narrow range of the initial pressing force. In this case, the axial misalignment is relatively large, whereas pressing force is insufficient, mode does not occur.

Fig. 3 Time-dependent graphs of generalized coordinates of the body, velocities and normal reaction forces during unsuccessful impact mode alignment: $a - x_c(\tau); b - y_c(\tau); c - \dot{x}_c(\tau); d - \dot{y}_c(\tau); e - \phi(\tau); f - \dot{\phi}(\tau); g - n_1(\tau)$, as $\nu = 20; k_p = 60; f_0 = 0.3; \nu_x = 0.15; \delta = 0.02; x_{c|p-o} = 0.2; y_{c|p-o} = 0.90; \phi_{p-o} = 0.003$

Under higher values of the joint clearance between the body and the supports, slip into the slot may occur without the contact with $A_2$ point on the supporting base and, therefore, in such cases, the normal reaction $n_2$ entirely does not occur.

Accordingly selected magnitudes of the excitation parameters and the initial pressing force, provide the possibility to reduce emerging impact forces and in such a way to adjust the character of the alignment process, thus influencing its reliability and efficiency. This is rather important factor, because the relatively large reaction forces, which emerge during the process of the parts alignment, may cause damage of the being assembled parts.

If both parameters of the system and vibratory excitation are not properly matched, because of provided vibratory excitation the impact mode displacement of the part is insufficient, to result a slip down into the slot (Fig. 3). The displacement of the body is accompanied by the impact collision with the single edge of the supporting base. This is seen from the $n_1(\tau)$ graph (Fig. 3, g).

Particular magnitudes of the parameters provide possibility to reduce impact forces and to influence the character of the impact and efficiency of the parts alignment. This is rather important, as relatively large reaction forces may cause damage to the parts and used devices.

Numerical simulation showed that the dependence of alignment duration on rigidity parameter $k_x$ is more noticeable under smaller excitation frequency $\nu$ (Fig. 5). To ensure efficient vibratory alignment of the parts by impact mode of displacement requires particular excitation frequency $\nu$.

Fig. 4 Alignment duration of the parts versus initial pressing force $f_0$ under different misalignment

Fig. 5 Alignment duration of the parts versus rigidity parameter $k_x$ under different excitation frequency $\nu$.

The dependences of the parts’ alignment duration on the excitation frequency $\nu$, under different axial misalignment of the components show, that alignment duration of the parts depends on excitation frequency and has relatively longer duration as frequency $2 < \nu < 4$ (Fig. 6). In frequency range $5 < \nu < 10$, the dependence of alignment duration on misalignment of the parts is most evident. Further increase in frequency ($\nu > 10$) showed relatively small influence on alignment duration.
From obtained graphical dependences of the parts’ alignment duration on excitation frequency (Fig. 6), the minimum duration of the alignment can be identified in the frequency range $5 < \nu < 10$, which nonsignificantly displaces under different axial misalignment of the components. Mentioned time dependences of the parts’ alignment duration on excitation frequency lead to conclusion, that it is possible to choose such a magnitude of excitation frequency, which ensures minimum duration of the alignment.

The character of the parts alignment by impact mode depends on the parameter of angular rigidity $k_\varphi$ (Fig. 7). Under smaller values of $k_\varphi$ ($k_\varphi < 20$), the alignment duration is relatively long because of the more intensive turn of the movably based part. Under relatively small force of the components pressing ($f_0 = 0.1$) and angular rigidity $k_\varphi = 10 – 60$, alignment duration diminishes as $k_\varphi$ increases. Further increase in angular rigidity results significant increase in alignment duration. Under initial pressing force $f_0 = 0.3$, the impact mode of displacement and turn of the part takes place within a more narrow range of the angular rigidity. Under relatively large pressing force ($f_0 = 0.4$), directional displacement and turn occurs within a rather narrow range of $k_\varphi$. Thus, to ensure alignment of the parts by impact mode, it is necessary to match the magnitudes of the $f_0$ and $k_\varphi$.

It was determined, that impact mode alignment occurs within the range of significantly smaller values of the initial pressing force ($f_0 = 0.1 – 0.8$), if compared to nonimpact mode ($f_0 = 0.9 – 15$) [12]. The duration of the impact mode alignment is highly dependent on angular rigidity $k_\varphi$, and, as $k_\varphi = 50 – 60$, it is approximately 30% smaller, than that under non-impact mode [12]. The impact mode alignment is not recommended for the assembly of fragile parts and aiming to avoid surface damage of the mating parts.

Properly matched parameters of the system and excitation are prerequisites to prevent undesirable situations during the alignment and assembly.

4. Conclusions

1. The model of elastic interaction of the part and supporting base was made, considering the oblique impact. Interaction of the parts during the impact is taken into account by means of noninertial supporting base with the connected in parallel elastic and damping elements. Applying the model of elastic interaction of the part and the supporting base, the influence of the parameters of the dynamic system and excitation on impact mode process of the part-to-part alignment was defined.

2. The impact mode alignment occurs when excitation force is higher than the sum of the part-to-part pressing force and gravity of the movable part. During the impact mode, the movable part impacts obliquely. It was determined, that during the alignment it is able to impact colliding with the opposite edges of the immovably based part.

3. Highest influence on duration of the impact mode alignment have the excitation frequency and rigidity of the movable part. As the frequency increases up to $\nu = 6$, the duration of the alignment suddenly diminishes, but later it is marginally dependent on the excitation frequency. The alignment duration versus angular rigidity dependence has apparent minimum, which is situated within the $k_\varphi = 50 – 60$ range.

4. The area of the reliable alignment by the impact mode mainly is predetermined by the initial part-to-part pressing force and by angular rigidity of the movable part. When the pressing force is too high, the impact mode alignment occurs within a narrow range of angular rigidity values.
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Summary

Vibratory impact mode alignment of the being assembled parts is analysed in the presented paper. The dynamical model was made, which represents the interaction between movable body and supporting base during the oblique impact. Applying the model of the interaction, the influence of parameters of the dynamic system and vibratory excitation on the process of the impact mode independent alignment of the being assembled parts was analysed.

The movably based body has characteristic of different modes of impact displacement, which depend on the excitation frequency, part-to-part pressing force and on displacement and angular rigidity. The alignment duration dependences on system and excitation parameters were defined.

The parameters’ sets area, of the part-to-part pressing force versus angular rigidity of the body, was defined, as reliable alignment by impact mode is taking place.

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MODELIRUOVANIE SOVместного ВИБРАЦИОННОГО ЦЕНТРИРОВАНИЯ ДЕТАЛЕЙ ПРИ УДАРНОМ ПЕРЕМЕЩЕНИИ

Résumé

Dans la présente étude, le but est d’analyser le mouvement vibratoire de l’assemblage de deux pièces. Le modèle est constitué d’une pièce mobile et d’une pièce fixe. Le mouvement du système est déterminé par les paramètres de l’assemblage et la pression appliquée. La durée d’alignement est fonction des paramètres de l’assemblage et du processus d‘excitation.

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REZUMÉ

Strapsnyje nagrinėjamas automatiškai surenkamų detalių vibracinis tarpusavio centravimo smūginiai režimai. Sudarytas judančio kūno ir atramos sąveikos tampriusis modelis, esant ištržiam smūgiui. Naudojant sąveikos modelį, išitira dinaminės sistemos ir vibracijos žadinimo parametrų itaka smūginiam surenkamų detalių tarpusavio centravimo procesui.

Paslankiai bazuojamam kūnui būdingi skirtingi smūginiai poslinkio režimai, kurie priklauso nuo žadinimo dažnio, detalių prispaidumo jėgos, poslinkio ir posūkio standumo.

Nustatytos centravimo trukmės priklausomybės nuo žadinimo ir sistemos parametrų. Sudaryta smūginio režimo patikimai centrojamojų detalių prispaidumo jėgos ir kūno posūkio standumo derinių sritis.

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