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Modification of Chen model of plasticity for early ages applications

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1. Introduction

At present, the emphasis is often put on the speed of construction process. The time schedule of a construction is often rather tight and so it is sought to expedite partial activities. Technological pause necessary for concrete to gain sufficient strength before it can be loaded is one of the decisive factors.

The accelerated removal of formwork and entrance of workers, or even vehicles, on the concrete at the age of few hours are more and more frequent. This often results in overloading of yet hardening concrete. At this stage, the concrete microstructure is not fully developed. The effect of overloading, of course, depends on specific application, as overloading can also cause proper consolidation of concrete, which is actually a favorable result. On the other hand, the formwork is not infinitely stiff. Therefore, the engineers try to reduce the effect of hydrostatic pressure by reducing the height of poured lifts, where the decisions on the height of lifts are based on the estimation of the instant, from which concrete can, or at least partially, support itself. A more rigorous analysis of the processes inside the formwork is therefore desirable as it can yield more reliable estimation which would result in safer construction, or risk control.

In the case of concrete it is necessary to use a model with different loading functions in tension and compression, which takes into account the different strengths in compression and tension. All of this is satisfied by the Chen model of plasticity, which is described in detail in [1]. The Chen model of plasticity was originally derived for solid materials, especially hardened concrete, but with sufficient modification it is possible to extend the range of application of this model to yet hardening concrete. In [2], there are the basics describing the evolution of strength for the concrete at early ages. The paper [3] presents concise data on properties of concrete at the ages ranging from about the initial setting time until years, such as the compressive strength, with stating clear the dependence on temperature and water-cement ratio. Recently, the composition of concrete has also progressed and it is focused on the high-strength concrete (HSC) with using rapid hardening cement. Uniaxial experiments and resulting models focused on HSC are described in [4] and [5]. A function describing the evolution of concrete microstructure, which can be easily obtained from simple experiments, is also presented in [5]. For multidimensional analysis of mechanical behavior of concrete loaded at early ages, one of the essential input parameters is the evolution of Poisson's ratio, as it also depends on the evolution of concrete microstructure. A relation between Poisson's ratio and load level for hardening concrete can be found in [6].

In this paper, combination of the above mentioned Chen model of plasticity and the evolutionary function of concrete is proposed. The Chen model of plasticity is very suitable for modeling of hardened concrete due to different strengths considered in compression and tension. The evolutionary function of microstructure describes the evolution of mechanical parameters of concrete at early ages. Therefore, all material parameters considered in the Chen model of plasticity are dependent on progressing hydration, described by the evolutionary function. The modification of the original model allows the description of concrete behavior at early ages. To illustrate the applicability of the proposed approach, a real structure was considered. The example described the problems related to the construction of the new Border bridge on the newly constructed highway D8 connecting Prague and Dresden, when fresh concrete needs to be transported to the location of placement across already concreted sections. Therefore, the deformation of prematurely loaded concrete deck by truck tires was investigated. All parameters necessary for the analysis are given in the paper. Also, an approach to express the uncertainty contained in the material description, when only few experimental data are available, is suggested.

2. Evolutionary function of concrete

During the first few hours after water is mixed with cement, concrete undergoes a quite rapid change when it transforms from a liquid to a solid state. Mechanical behavior of solidifying and further hardening concrete is influenced by progressing hydration and the microstructure of concrete is developing very rapidly.

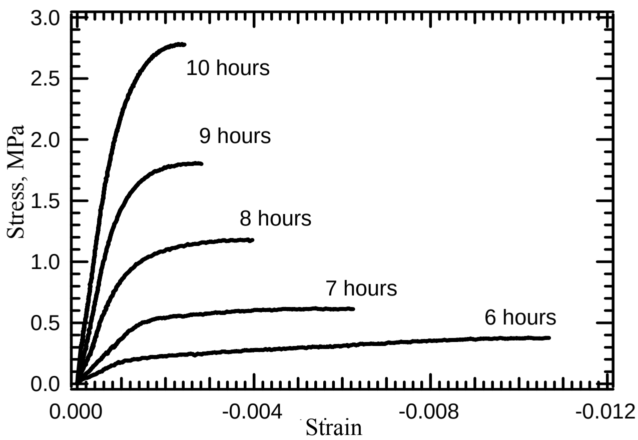

Fig. 1 Stress-strain diagrams of concrete at various ages

In Fig. 1, the experimental stress-strain diagrams for concrete of the ages 6 to 10 hours are shown. With respect to this well known behavior of the rapidly developing strengths, the material parameters of concrete in our model have to be formulated as a function of hydration. The progress in hydration can be expressed in various ways. In our case, the most favorable way of using the degree of hydration was assumed. The degree of hydration is described by a function, which was derived for experimental investigation of microstructure evolution. The evolutionary function, as its name implies, is introduced in order to describe the evolutionary changes in the microstructure of solidifying and hardening concrete and therefore to control mechanical behavior of concrete in the modeling. The function of microstructure evolution, which expresses the effect of aging, was identified from experiments on the evolution of penetration resistance, pullout resistance and compressive strength of concrete at the ages up to the final setting time, as shown in the paper [4], and is given by

$$h(t_n) = a_5 \left(\frac{a_3 t_n^{a_2}}{a_1 + a_3 t_n^{a_2}} \right)^{a_4} \tag{1}$$

where t_n is a normalized time with respect to the final setting time; W/C is the water/cement ratio (in decimal) and a_1, a_2, a_3, a_4 and a_5 are empirical parameters: $a_1 = 10,\ a_2 = 9.164 - 7.2\frac{W}{C},\ a_3 = 0.7,\ a_4 = 1,\ a_5 = 15$.

The evolutionary function was derived for high-strength concrete using rapid hardening Portland cement. All experimental data used for the derivation of this function was obtained under constant temperature of 20°C for two types of mixes for concrete at the ages up to the final setting time.

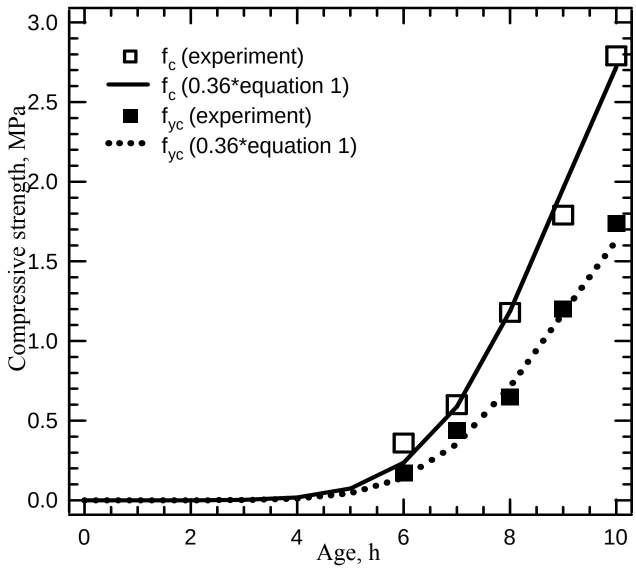

Fig. 2 Compressive strength evolution

Fig. 2 shows the comparison between the compressive strength evolution expressed by the evolutionary function (1) (scaling parameters are given in Fig. 2) and experimental data obtained from [5].

3. Chen model of plasticity

The Chen model of plasticity is a three-parameter model for concrete displaying isotropic hardening, [1]. This model expresses elastoplastic behavior of hardened concrete.

The typical behavior of concrete shows varying stress-strain characteristics under tension and compression. Therefore, the loading function of Chen model is composed of two parts. The part of loading function f^c is valid in the compressive area and it is parabola. The part of loading function f^t describes tension-tension and tension-compression area and it is hyperbola. Since the parts of loading function are different it is important to determine the correct stress-state zone. The zoning of the biaxial stress states is obvious from (σ_1, σ_2) space, see the Fig. 3.

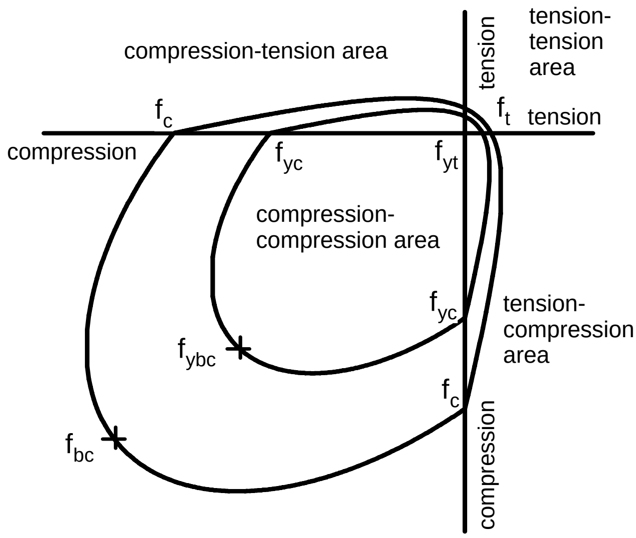

Fig. 3 Biaxial stress space

The curves in Fig. 3 represent two extreme situations. The inner curve represents initial yield surface defined with initial yield stresses; f_{yc} in compression, f_{yt} in tension and f_{ybc} in biaxial compression. The outer curve represents failure surface defined with ultimate stresses; f_c in compression, f_t in tension and f_{bc} in biaxial compression. An expansion of initial yield surface leads to subsequent loading surfaces.

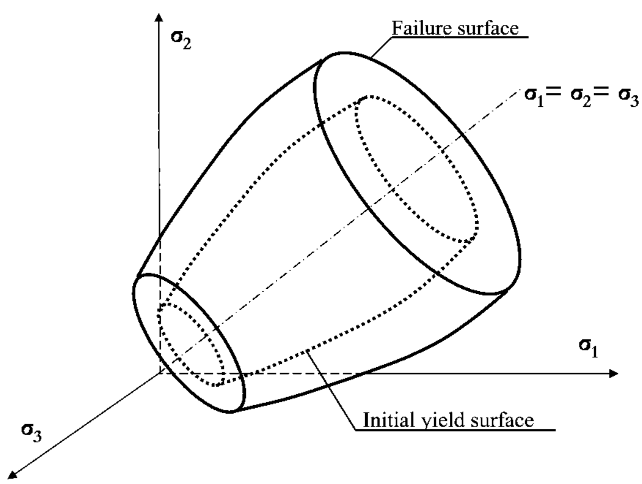

Fig. 4 Triaxial stress space

Fig. 4 shows the loading surfaces for general stress states in the $(\sigma_1,\sigma_2,\sigma_3)$ space. Also here the two extreme situations, initial yield surface and failure surface, again can be seen. For the triaxial loading of concrete it is not easy to select the correct stress-state zone, as in the case of biaxial loading. The appropriate region for general stress-state is determined according to the first invariant of stress tensor I_1 and the second invariant of stress deviator tensor J_2. The $\left(I_1,\sqrt{J_2}\right)$ space is separate by the simple linear functions

$$\sqrt{J_2}+\frac{I_1}{3}=0 \quad \text{and} \quad \sqrt{J_2}-\frac{I_1}{3}=0 \tag{2}$$

And the particular stress-state zones are established with the conditions:

- compression-compression zone

$$I_1<0 \quad \sqrt{J_2}+\frac{I_1}{3}<0 \tag{3}$$

- compression-tension zone

$$I_1<0 \quad \sqrt{J_2}+\frac{I_1}{3}>0 \tag{4}$$

- tension-tension zone

$$I_1>0 \quad \sqrt{J_2}-\frac{I_1}{3}<0 \tag{5}$$

- tension-compression zone

$$I_1>0 \quad \sqrt{J_2}-\frac{I_1}{3}>0 \tag{6}$$

For illustration, the equations for the compression-compression region introduced are given in the paper; it means the stress state corresponding to the Eq. (3). The equations for the tension-tension and tension-compression region are similar.

The failure surface is assumed in the compression-compression region

$$f_u^c(\sigma,h)=J_2+\frac{A_u(h)}{3}I_1-\tau_u^2(h)=0 \tag{7}$$

The initial yield surface in the compression-compression region is given by

$$f_0^c(\sigma,h)=J_2+\frac{A_0(h)}{3}I_1-\tau_0^2(h)=0 \tag{8}$$

where $A_0(h)$, $\tau_0(h)$, $A_u(h)$ and $\tau_u(h)$ are material constants which can be determined from simple tests. They are determined as functions of ultimate stresses under uniaxial compression $f_c(h)$, and under equal biaxial compression $f_{bc}(h)$, and of the initial yield stresses under similar conditions $f_{yc}(h)$ and $f_{ybc}(h)$.

$$\left.\begin{aligned} A_0(h)&=\frac{f_{ybc}^2(h)-f_{yc}^2(h)}{2f_{ybc}(h)-f_{yc}(h)}\\ \tau_0^2(h)&=\frac{f_{yc}(h)\,f_{ybc}(h)\left(2f_{yc}(h)-f_{ybc}(h)\right)}{3\left(2f_{ybc}(h)-f_{yc}(h)\right)}\end{aligned}\right\} \tag{9}$$

$$\left.\begin{aligned} A_u(h)&=\frac{f_{bc}^2(h)-f_c^2(h)}{2f_{bc}(h)-f_c(h)}\\ \tau_u^2(h)&=\frac{f_c(h)\,f_{bc}(h)\left(2f_c(h)-f_{bc}(h)\right)}{3\left(2f_{bc}(h)-f_c(h)\right)}\end{aligned}\right\} \tag{10}$$

All material parameters used in the previous equations are dependent on the function of the microstructure evolution, h. Therefore, it is obvious that the loading surfaces are changing with increasing time. With increasing strength of concrete the loading surfaces are expanding.

This is evidenced in Fig. 5, where the dotted lines denote the initial yield surface and the failure surface at the age of 7 hours and the solid lines denote the initial yield surface and the failure surface at the age of 9 hours.

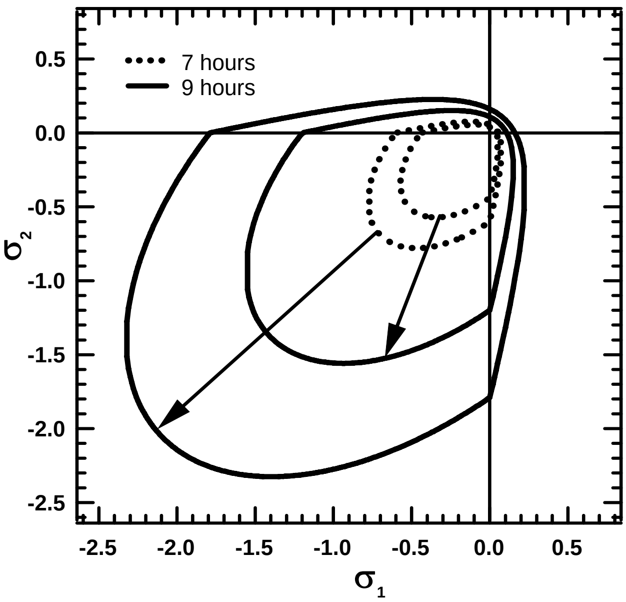

Fig. 5 Evolution of initial yield and failure surface

The modified Chen model of plasticity was implemented to the existing open program SIFEL, see [7]. The functionality of the programmed model was verified on several simple examples, one of them is shown bellow.

4. Example of uniaxial compressive loading

The modification of Chen model of plasticity was used for interpreting the uniaxial compressive strength test with standard cylindrical specimens with diameter of 10 cm and height of 20 cm. Material characteristics of the concrete in early ages are obtained from experiments. The experimental data used for this example are presented in literature [5], see Table.

Table
Yield stresses and ultimate stresses under uniaxial compression

Age of concrete	f_{yc}, MPa	f_c, MPa
6 hours	0.18	0.36
7 hours	0.44	0.60
8 hours	0.65	1.18
9 hours	1.20	1.79
10 hours	1.74	2.79

Table summarizes the yield stresses and the ultimate stresses from experimental data for the concrete of the ages of 6 to 10 hours. Chen in his paper [8] introduced a method of acquiring material parameters for his model from the uniaxial compressive strength. Even if ultimate stress in compression is known only, it is possible to calculate the remaining values from the simple formulas

$$\left.\begin{aligned} f_t &= 0.09 f_c \\ f_{bc} &= 1.16 f_c \\ f_{yc} &= 0.6 f_c \\ f_{yt} &= 0.054 f_c = 0.09 f_{yc} \\ f_{ybc} &= 0.6 f_{bc} = 1.16 f_{yc} \end{aligned}\right\} \tag{11}$$

From Table 1 it is obvious that the yield stress is approximately equal to 60% of the ultimate stress as well as Chen [8] shows in his equations. Therefore, the application of the relations (11) is justified.

In the example presented, the compression-compression area only is dealt with, therefore, it is sufficient to calculate biaxial yield and ultimate strength in addition to the experimentally obtained uniaxial compressive strength.

Fig. 6 shows a comparison between the computed results and experimental data at the ages from 6 to 10 hours. The solid lines denote modeling results and the dotted lines denote the experimental results presented in [5]. You can see the very good agreement of the results.

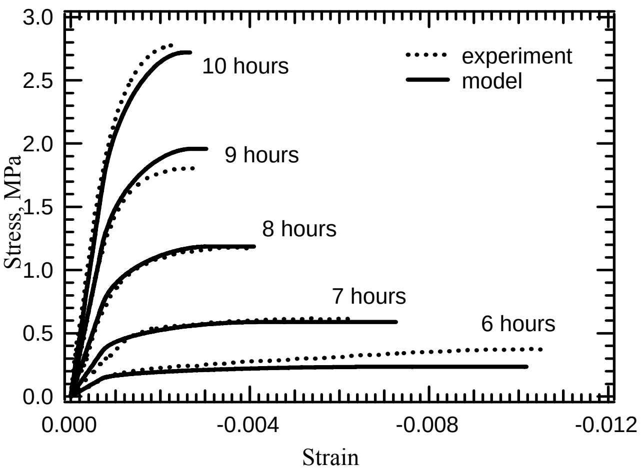

Fig. 6 Comparison between experimental and modeling

5. Example of application

To illustrate the applicability of the presented approach, a real structure was considered. The Border bridge is a part of the newly constructed D8 highway connecting Prague (Czech Republic) and Dresden (Germany). This composite bridge is about 500 meters long and overpasses a deep valley. The intermediate columns are about 50 meters tall, what prohibits concrete pumping directly from the bottom of the valley to the bridge deck, which is designed as reinforced concrete slab. Therefore, the concrete needs to be transported to the location of placement across the already finished reinforced concrete deck. As falling behind the schedule is a very possible threat, especially in this case, when the construction site is located in a mountainous area, where it is a subject to unfavorable weather conditions, a tool for the estimation of the earliest possible entrance to the newly concreted section of the deck is desirable, moreover, when the tool also provides some information on the possible damage caused by premature loading.

Material characteristics of the concrete in early ages are obtained from the experiments with simple techniques based on uniaxial compression and penetration tests. For example, the experimental data gathered by Byfors yielded, from today's point of view, very useful data on the evolution of modulus of elasticity and compressive strength of solidifying and further hardening concrete, [9]. This experimental data were used for the investigation of the reinforced concrete deck of the composite bridge, where a section of the concrete deck (see Fig. 7) under compression was modeled and analyzed at the ages from 6 to 24 hours. Fig. 7 shows the deformation of the hardening concrete under excessive compressive load.

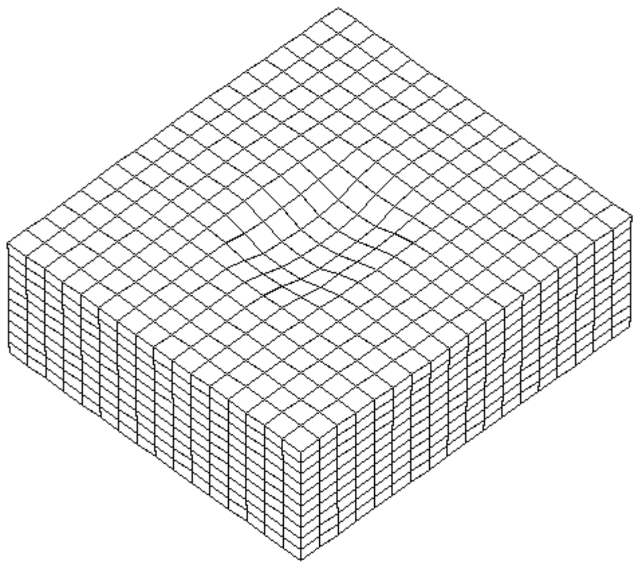

Fig. 7 Section of analyzed concrete deck - deformation

The main objective was to describe the behavior of the concrete deck under compressive load corresponding to a truck carrying fresh concrete. The data for loading force were obtained at a construction site and compared with Eurocode [10]. The force representing the truck tire is 90 kN for one wheel axis. Here the front axis was considered. The pressure on the slab was derived as a force of 45 kN on the contact area under the wheel, whose standard value is about 20 x 30 cm.

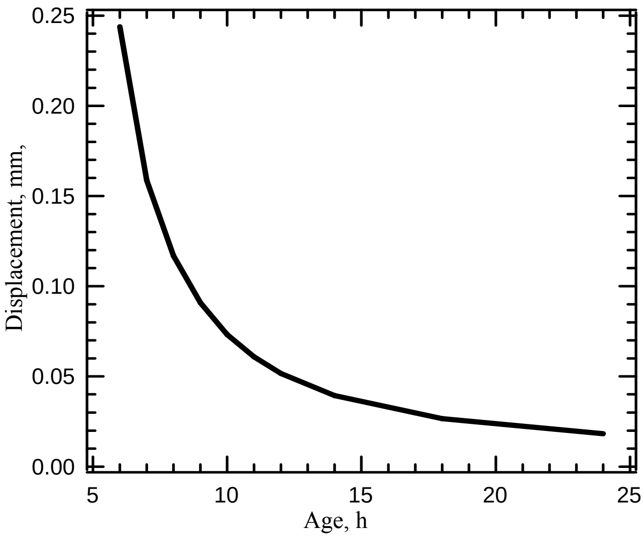

Fig. 8 Vertical displacement at various ages

Fig. 8 shows results of the analysis, where the decreasing displacement with increasing age of the concrete under constant loading can be seen. The effect of increasing strength at the ages from 6 to 12 hours is also obvious in this graph.

6. Description of uncertainties contained in material

Concrete is a composite material whose performance depends heavily of precise dosing of its main constituents, which are water, cement, sand and aggregate. Due attention needs to be paid to the amount of water, whose erroneous dosage has severe consequences for the ultimate performance of concrete. It should be noted that the amount of water dosed cannot be controlled perfectly, as the simple change in ambient humidity and ambient temperature results in condensation of water contained in the air on the surface of sand and aggregate particles. The actual amount of water in concrete can differ from the desired value by tens of liters, what means an error in the order of percents to tens of percents. Of course, there are mechanisms which try to maintain the just dosage. However, human factor, the variation of properties of the single constituents, weather conditions, all those factors influence the ultimate performance of the concrete structure, and not all factors can be described by a standard probabilistic parametric distribution. The work presented in [11,12] proposes a method for the assessment of ultimate performance of concrete structures using the concept of the fuzzy set theory. An approach to fuzzification of the Chen model of plasticity described above was presented in [11]. A similar fuzzification procedure was presented in [12], where the varying settlement of a tall reinforced concrete wall during construction was quantified with respect to the uncertainties contained in the material description. The advantage of the fuzzy set approach lies in the form of output values, which can immediately serve as input parameters for financial analysis of economical feasibility of the entire construction project. The difference between statistical and fuzzy concepts is described in, e.g. [13].

7. Conclusions

In this paper, a modification of Chen model was described, so that it is possible to use the model for solidifying and hardening concrete. The parameters of the model were defined as a function of microstructure evolution. The input data can be obtained experimentally with simple standard testing methods.

The results obtained from the modified Chen model of plasticity were compared with the results from experiments. Very good agreement of the results verified the possibility of using the Chen model of plasticity not only for hardened concrete but also for hardening concrete with yet evolving microstructure.

The applicability of the presented model was shown on a real structure, the Border bridge. The results showed the possible deformation of hardening concrete deck subjected to premature loading.

It was also shown that the proposed model could be further extended so that the effect of uncertainty contained in material description could be quantified.

Acknowledgement

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M. Frantova

ČENO PLASTINGUMO MODELIO MODIFIKAVIMAS ANKSTYVOSIOSE STADIJOSE

R e z i u m ė

Šiame straipsnyje aptartas Čeno plastingumo modelio pritaikymas ankstyvųjų kietėjimo stadijų betonui. Kadangi Čeno plastingumo modelis dėl skirtingo stiprumo gniuždant ir tempiant vertinimo labai tinka sukietintam betonui modeliuoti, stengiamasi pritaikyti jį kietinamo betono mechaninei būklei įvertinti. Modeliuojant vertinami medžiagos parametrai pasiekiami šiuolaikiškai hidratuojant. Hidratavimas išreiškiamas hidratavimo laipsniu. Taikant modifikuotą Čeno modelį gali tekti pailginti betono kietėjimą.

M. Frantová

MODIFICATION OF CHEN MODEL OF PLASTICITY FOR EARLY AGES APPLICATIONS

S u m m a r y

An extension of the Chen model of plasticity for the use with concrete in the very early ages is described in this paper. Since the Chen model of plasticity is very suitable for modeling of hardened concrete due to different strengths considered in compression and tension, it was sought to make use of this model also for the description of mechanical behavior of hardening concrete. The material parameters considered in the modeling have to be dependent on the progressing hydration. The hydration is quantified with help of the degree of hydration. Due to this modification the application range of the Chen model can be extended to hardening concrete.

М. Франтова

МОДИФИКАЦИЯ МОДЕЛИ ПЛАСТИЧНОСТИ ЧЕНА НА РАННИХ СТАДИЯХ

Р е з ю м е

В статье представлены возможности применения модели Чена на ранних стадиях отвердения бетона. Так как модель пластичности Чена применима для моделирования отвердевшего бетона благодаря разной оценки прочности при сжатии и растяжении созданная модель может быть использована для оценки механического состояния бетона. Параметры материала, оцениваемые при моделировании, достигается современным гидратированием. Благодаря оценке модификации модели Чена может увеличиться время отвердения бетона.

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