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Surface area determination of ragged metallic bodies

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1. Introduction

The amazing development in the thermal engineering literature during the last three decades has opened up totally new and challenging research areas. One of them is the determination of surface area of metallic bodies. Modern advancements in surface area determination (SAD) have enabled the calculation of areas of several complex shapes. More recently, problems in ragged metallic bodies (RMB) have evolved.

Research into SAD primarily aims at calculating the cost of coating or electroplating processes based on material cost and electroplating time [1, 2]. SAD research is related to the engine block of a vehicle since engines with large surface areas have high tendency that the heat generated by the engine would be conducted away easily as the heat will tend to spread all over the large area of the engine block.

Despite the immense benefits and opportunities embedded in ragged metallic SAD research, no compelling documentation on how to determine the surface area of a RMB exists. This work is therefore an answer to the urgent call of providing a solution to this long-neglected problem. Two natural questions that emerge in trying to determine a suitable method to quantify the surface area are (a) what is the best method to estimate the surface area of the RMB? and (b) how do we evaluate how good a method is? These questions are addressed in this work.

The concept of SAD may properly be ascribed to the classical investigations of Meeh (see [3]) who recognized the paucity of actual measurements of the surface area of living things. Many years later, it became established that SAD could be extended to non-living objects.

The vast amount of studies on living things could be visualized from two main angles. The first is the body of research on animals that focuses on cattle and swine, among others. The entire studies in this domain have scientifically reported satisfactory methods for the comparison of heat production in animals of different sizes. The study by Hogan and Skouby [3] is an example. In human medicine, a large amount of research aims at preventing medication errors in human therapy. Applications are in the treatment of cancer [4, 5].

For non-living things, extensive studies have been carried out in soil science and some aspects of manufacturing. The soil science literature has a compendium of studies in activated slag samples, slice hull, fine-grained soil and sea oil sand, among others.

The research carried out by Baechler and Jolie [6] investigated into activated slag samples by cold neutron tomography. The study on wetted surface area of a slice hull was carried out from a lock headship drawing of PI-100-01 in 1994. The SAD of fine-grained soils was the main focus of Cerato and Lutenegger [7]. Their work described an experiment on soil using the Ethylene Glycol Monoethyl Ether (EGME). The test involves saturating a soil sample with EGME and then removing the excess EGME, in a vacuum desiccator until the EGME forms a monomolecular layer on the soil surface. Test results were presented demonstrating the effect of various test parameters on the results.

Omotoso et al. [8] determined the surface area of inter-stratified phyllosilicates in athaba sea oil sands from sychrotion X-ray scattering domain size. The ability to quantify the surface area contribution from individual phyllosilicates in the oil stand matrix is important for developing models to predict tailings behaviour and understand water chemistry.

2. Surface area computational method

The heat transfer method is discussed as follows. Consider two objects with irregular and regular shapes respectively having the same mass, M, (Fig. 1). The surface area of object *2* can easily be determined while that of object *1* may pose some problems. By intuition, let us assume that the surface area of object *1* is greater than that of *2* because of its irregular shape. Imagine a particular flow through the surfaces of the two objects at the same rate b. At the end of time t, more of b is expected in object (1) having an area A_1 than in object *2* with an area A_2. If this argument is valid in all situations, then the rate of flow is proportional to the surface area. Mathematically, $b \alpha A$; and $b = kA$ where the sign of proportionality is to be substituted with " = k" (see Fig. 1).

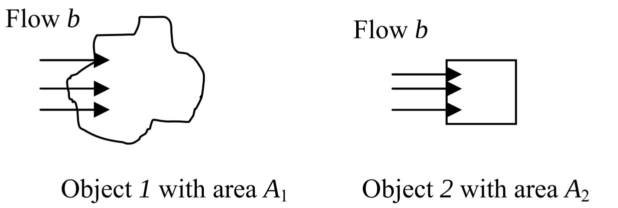

Fig. 1 Object *1* and object *2*

If we apply this to the relationship between rate and the areas of the two objects, then $A_1 = A_2 b_1/b_2$. Thus, we could calculate the surface area, A_1, of the irregular shaped object. The value of b could be determined using the principles of Newton's law of cooling.

The starting point of the analysis is Eq. (1). It obviously applies to a body (object) with infinite thermal conductivity, whose uniform temperature $T(t)$ changes with time if immersed into the fluid with infinite thermal capacity, so that the temperature of the surrounding fluid T_s remains constant. The initial temperature of the body is denoted by $T(0) = T_0$. The changing temperature of the body is given by Eq. (1) as follows

$$T(t) = T_s + (T_0 - T_s)e^{-bt} \qquad (1)$$

where b gives the temperature change constant.

This experiment shows the combined effects of conduction, convention and radiation. It requires a stop-watch, two thermometers, a hot cup of tea and a lid. A thermometer is required to record the ambient temperature while the other monitor the coffee's slow slide to thermal equilibrium.

2.1. Description of the experiments

Apparatus for the experiments (a) two samples of the metallic object (mild steel), (b) two thermometers (similar) with readings ranging between –10 and 110 °C, (c) two similar plastic containers, (d) a stop watch, and (e) a good source of water.

Aim of the experiments – To determine the surface area of two or more samples from a known surface area of one sample. We would then check the deviation from the real value of the surface area of one sample relative to the other and determine the percentage error and the reliability of the method.

Experiment 1 – Two bars of length 128 mm, and diameter 42 mm, having the same mass and volume were labelled as samples A and B. Sample B was cut into two equal parts to make the surface area of the 2-piece object greater than that of the single piece of sample A (Fig. 2).

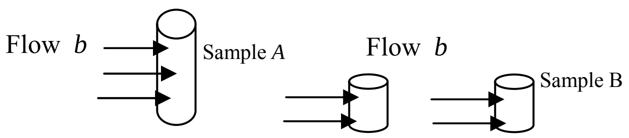

Fig. 2 Samples A and B

The same volume of water is poured into each container and the thermometer introduced noting the ambient temperature of the surroundings. The water is heated to its boiling point and the temperature noted. Samples A and B were introduced into each container and the stopwatch started for readings at 5 minutes interval over 40 minutes.

Experiment 2 – Three different shapes of mild steel materials with the same mass are considered. The first sample, tagged C, is a rectangular bar with dimensions 50 × 40 × 100 mm. The second sample, tagged D, is a rod with a circular cross section of diameter 50 mm and length 102 mm. The third sample, tagged E is a rectangular bar of dimension 50 × 50 × 80 mm (Fig. 3).

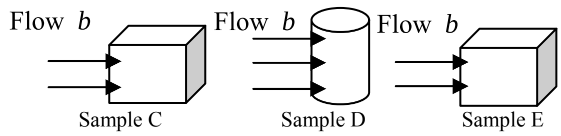

Fig. 3 Samples: C - 50 × 40 × 100 mm; D - diameter = =50 mm, length = 102 mm; E - 50 × 50 × 80 mm

The third sample is made ragged by deforming it to an irregular shape. Experiments were performed with samples C and D at the same time while the two objects were suspended in the water. They were not allowed to rest on the base of the container or even touch any part of it to ensure proper heat transfer and minimization error.

Experiment 3 was performed with samples D and E, as described above.

2.2. Experimental results and observations

The following are the results of the experiments (Table).

Description of Table – Table shows the ambient and maximum boiling temperatures of water for samples A,B,C, and D.

The experiments were performed 10 times for each of the samples and at an interval of 5 minutes.

Experiment 1 – The high percentage error could be associated with the following factors (a) The low thermal conductivity of mild steel. (b) During the experiment some parts of the metals that were touching the containerat the base of the container. (c) The experiments were performed one after the other. Having noticed these shortcomings, we then performed the experiments with cautions and in a more accurate manner to guide against errors.

Table

Ambient and maximum boiling temperatures of water for samples A, B, C, and D

	t, min		$T(t)$, °C		$T(t) - T_0$, °C				T_0 T_s, °C		$T(t) - T_0$, °C	
S/N	A,B	C,D	A	B	A	B	C	D	A,B	C,D	C	D
1	0	0	96	96	66	66	98	98	66	67	67	67
2	5	5	76	75	46	45	82	82.5	66	67	51	51.5
3	10	10	73	71	43	41	77.5	78	66	67	46.5	47.0
4	15	15	67	66	37	36	73	73.5	66	67	42	42.5
5	20	20	63	62	33	32	69	69.5	66	67	38	38.5
6	25	25	60	59	30	29	66.6	66.5	66	67	35	35.5
7	30	30	57	56	27	26	63	63.5	66	67	32	32.5
8	35	35	54	54	24	24	60.5	61	66	67	29.5	30
9	40	40	52	51	22	21	58.5	59	66	67	27.5	28
10		45					56.5	57	66	67	25.5	26

Experiments 2 & 3 – As a result of improvement on the first experiment, the percentage error is reduced to a reasonable value. Also, if the material had been changed from mild steel to a metallic material like aluminium or copper the deviation is expected to drop to about 4 to 5% which makes the heat transfer method of estimating surface area more reliable, applicable, and authentic. This method is then used to estimate the surface area of a sample E but with ragged body.

Taking logarithm of both sides of Eq. (1), we have

$$ln\,(T(t) - T_s) = ln\,(T_o - T_s) - bt \qquad (2)$$

If Eq. (2) is represented as $y = mx + c$, the slope of the curve is b where $y = ln\ (T(t) - T_s)$, $mx = -bt$, while $c = ln\ (T_0 - T_s)$. The solution to the problem gives $b_A = 0.024956$. The area for A is therefore 19660.09 mm^2. For sample B, $b_B = 0.025312666$ where the experimental area for B is 19941.08683 mm^2. The actual area of B is given by 22430.9715466 mm^2. The percentage error in experimental area to the actual area could be calculated as Error × 100 %/Actual area = 11%. For sample C, b_C = 0.019625454, while for sample D, $b_D = 0.019291878$. The experimental area for C is therefore 20294.05361 mm^2.

The actual area for sample C = 22000 mm^2, and the error is 7.8%. For sample E, $b_E = 0.019813818$. The experimental area for E is therefore 20628.16346 mm^2. This value is the experimental area with an error of about 8%. To get the actual value of the surface area of the RMB, we will only convert it from its present value of 92% to 100% that is required of it. Thus, we have the surface area of the RMB to be 22421.99168 mm^2. Hence, this is the actual surface area of sample E. A ragged body tends to have a higher surface area than the regular shape with the same mass. The surface area of the metal body with dimension 50 mm by 50 mm by 80 mm is 21000 mm^2. This surface area increased when the body was deformed, implying that they have the same mass since their volumes are the same.

3. Determination of the accuracy of the formula

In the determination of the accuracy of the model proposed in this work, we use the least square method. We start with taking the approximation function in the form

$$y(t) = a + ct^2 \tag{3}$$

The difference between the approximating function and the exact function is

$$D(t) = y(t) - [T_s + (T_0 - T_s)e^{-bt}] \tag{4}$$

This difference should be small. Now, substituting for $y(t) = a + ct^2$ in the preceding expression, we have

$$\begin{aligned} D^2(t) = {} & a^2 + 2act^2 + c^2t^4 + [T_s^2 + 2T_s(T_0 - T_s)e^{-bt} + \\ & +[(T_0^2e^{-2bt} - 2T_0T_se^{-2bt} + T_s^2e^{-2bt}) - 2a[T_s + (T_0 - T_s)e^{-bt}] + \\ & + 2ct^2\,[T_s + (T_0 - T_s)e^{-bt}] \end{aligned} \tag{5}$$

From the above expression, we could introduce the integral function to obtain

$$\begin{aligned} D^2 = {} & \frac{1}{p}\int_0^p \Big\{ a^2 + c^2t^4 + 2c\left(a - T_s\right)t^2 + \Big[\left(T_0 - T_s\right)^2 e^{-bt} + \\ & + 2T_s\left(T_0 - T_s\right) - 2a\left(T_0 - T_s\right) - 2ct^2\left(T_0 - T_s\right)e^{-bt} \Big\} dt \end{aligned} \tag{6}$$

In an effort to progress on the modelling aspect of the work, we embark on defining elementary integrals A, B, C, N and M as follows.

By introducing notations for A and B, we note that

$$A = \frac{1}{c^2 p}\int_0^p c^2 t\,dt;\ B = \frac{1}{p}\int_0^p 2c\frac{\left(a - T_s\right)}{2c\left(a - T_s\right)}t^2\,dt \tag{7}$$

From the expression for A, the symbol c^2 could be cancelled out. Thus, we have new expressions in simplified forms as equations (8) and (9).

$$A = \frac{1}{p}\int_0^p t\ \ dt \tag{8}$$

$$B = \frac{1}{p}\int_0^p t^2 dt \tag{9}$$

Following the same procedure, we obtain values for C, N, and M

$$C = \frac{1}{p}\int_0^p e^{-bt}\ dt \tag{10}$$

$$N = \frac{1}{p}\int_o^p t^2 e^{-bt}\ dt \tag{11}$$

$$M = \frac{1}{p}\int_0^p p\ dt \tag{12}$$

Note that

$$\begin{aligned} \overline{D}^2 = {} & a^2M + c^2A + 2c\left(a - T_s\right)B + \left[\left(T_0 - T_s\right)^2 + 2\left(T_0 - T_s\right) - \right. \\ & \left. - 2a\left(T_0 - T_s\right)\right] M - 2c\left(T_0 - T_s\right) N \end{aligned}$$

When we differentiate $\overline{D}^2$ with respect to a, we get

$$\frac{d\overline{D}^2}{da} = 2aM + 2cB - 2\left(T_0 - T_s\right)M = 0 \tag{13}$$

When we differentiate $\overline{D}^2$ with respect to c, we have

$$\frac{d\overline{D}^2}{dc} = 2cA + 2\left(a - T_s\right)B - 2\left(T_0 - T_s\right)N = 0 \tag{14}$$

From Eq. (13)

$$2a + 2c\,\frac{p^2}{3} - 2(T_0 - T_s) = 0 \tag{15}$$

From Eq. (14)

$$cp + \frac{2}{3}\left(a - T_s\right)p^2 + 2\left(T_0 - T_s\right)\left(\frac{pb + 2 + p}{b^2}\right)e^{-bp} = 0$$

This implies that

$$a = (T_0 - T_s) - \frac{cp^2}{3} \tag{16}$$

From above

$$A = \frac{10^3}{5} = 200,\quad B = \frac{10^2}{3} = 33.33$$

$$C = \frac{-1}{10\ x\ 1.5}\ e^{-10\ x\ 1.5} = -2.039\ x\ 10^{-3},$$

4. Cost computation on estimated surface area

The surface area of a RMB is estimated in order to calculate the cost of electroplating the metallic body which is determined by (a) Cost of the coating material, (b) Labour cost, (c) Equipment cost. For the cost of coating material, we adapt and improve on the model suggested by Lenau and Mazilli [9,10], calculated as

$$C_m = p \times q_m \times S \times t(1 + i)^{-x} \tag{17}$$

where C_m is cost of material; p is price of the coating material, DKK/part; q_m is density of the material, g/dm^2·µm; S is surface area, dm^2/part; t is coating thickness, µm; i is interest rate, x is period of time (DKK – currency value of Denmark).

The prices used in this analysis are calculated from their average quotations. A corrective multiplicative coefficient (1.7 for common metals and 1.4 for non-common metals) is used to take into consideration the difference between the quoted value and the real price of the metal in the market.

The labour cost depends on the parts throughput time, and the hourly cost of labour. The throughput time includes plating time and set-up time. The set-up time includes the time spent to prepare the part, to change the baths or electrodes and for possible post processing. The hourly cost of labour depends on an average cost of the labour, which includes workers wage, overheads and profit. With the parameters set, the labour cost can be calculated by the formula below:

$$C_1 = (w^1 \times n \times T_b + w^2 \times t_a \times q)/(q \times 60) \tag{18}$$

The number of baths (n) required for plating a batch of (q) parts is: $n = (S \times q)/b$, where S is estimated surface area; b is the bath content capacity in dm^2, empirically estimated to be one tenth of the bath size in litres (b = 0.1 × bath size). Finally, the set-up time t_a, min/part depends on the part's size, the type of production (local, industrial) and the size of production batch. The equipment cost can be calculated using the following formula (C_e denoting the equipment cost).

$$C_e = [W_e \times (n \times T_b + q \times t_a)]/(q \times 60) \tag{19}$$

The calculation of all other parameters such as T_b, t_a, n, has been shown in the previous discussion and still remain the same. Total Cost: The total cost (C_t), DKK/part is the sum of the three previously analysed costs, which are the cost of material (C_m), labour cost (C_1) and equipment cost (C_e):

$$C_t = C_m + C_1 + C_e. \tag{20}$$

This is the overall cost of electroplating a metallic object.

5. Future directions

In this work, we present a contribution from a logical theoretical perspective. However, the result can be applied only for the component when its heat transfer coefficient is known. This is a limitation of the study. Future expansion of the study could incorporate a modelling technique that would take care of the differences in the heat transfer coefficient of different ragged metallic components. Apart, the work could be extended to non-metallic ragged component of different material characteristics. An important component in this respect is the product made from fibre reinforced ragged plastic (FRRP) tubes for the application in oil and gas industry. Research indicates that they offer both engineering and economic advantages over conventional metallic ragged objects. Future investigations could establish a relationship between the existing model and other mechanical properties of metallic objects such as elasticity, plasticity, ductility, malleability, toughness, brittleness, strength, and hardness.

Investigations can also be made on fluids. This may be helpful, more importantly when a mathematical model that relates the parameters of the engine block to the fuel used to drive the engine is established. Another important relationship could be established between the sound made by different metals under conditions of test for sound. Some of these tests and insights proposed above are useful directions in the determination of surfaces in the chemical, biochemical branches.

The problems to be coped with for ragged – non-metallic materials are numerous. On the account that many of non-metallic materials are insulators, there is a problem with the heat transfer coefficient. The specific heat transfer coefficient may be very low and hence difficult to be monitored experimentally. Additionally, there may be unequal distribution of heat across the body of the non-metallic object due to poor conductive property of the object. This may therefore lead to bias and inaccurate measurement due to the heat transfer approach.

From the findings of this study, a top priority research is the development of SAD software that should be comprehensive, automated and useful as a management tool. It should centre on the SAD model structure. The development of the SAD computer software from both producer's and consumer's perspectives is essential. The relevant dimensions from producer's perspective are accuracy, capability, completeness, conformance, features, flexibility, simplicity and stability. However, by utilizing the SAD software, permanent records are created in a database that can be manipulated to provide specific product cost and reliability information.

Reports can be electronically transmitted to SAD database with the ability to access data entry screen, reports, and data submission modules though an Internet browser. SAD CD-ROM software could be put in place, and its functionality ported over to a server, enhanced, and made available at a website (i.e. www.SAD.model.com). It could be developed so that no special software is required and the operator can access the system from anywhere on the Internet.

A goal of the article is to sharpen our understanding of SAD research for a RMB. Clearly, we do not attempt to replace the existing literature by any means, but rather support it. The motivation for this research is that the SAD literature is at present fragmented. Many of the most influential articles on SAD appear in journals in fields as diverse as medicine, chemistry and soil science.

6. Conclusions

The determination of surface area of metallic bodies is a recent development. Thus, we have established a method for estimating the surface area of a RMB. The advantage of empirical approximation method over the heat transfer (HT) method is the fact that, one can use ordinary material to form the shape.

Given two metals of the same type, and mass, but with regular and irregular shapes, the surface area of the irregular (ragged) shaped body would be greater. We showed that to get a higher surface area, you could make the body ragged.

Hence, the higher the surface area of the engine block of an automobile, the more heat would be conducted away from the engine. It is therefore recommended that automobile companies should make the shape of the engine blocks more ragged to ensure high surface area and consequently, high rate of cooling, and consequently, improved engine performance efficiency.

References

1. http://preserve.nal.usda.gov:8300/jag/v25/v25i10/250419/a250419.htm.
2. www.ipt.dtu.dk/~ap/ingpro/surface/empirica.htm, Department of Manufacturing Engineering, DTU, Building 425, DK-2800 Lyngby.
3. **Hogan, A.G., Skouby, C.I.** Journal of Agricultural Research, *25*, 1923, p.419.
4. **Banerjee, S., Sen, R.** J Appl Physiol, 7, 1955, p.585.
5. **Gehan, E.A., George, S.L.** Cancer Chemother Rep, 54, 1970, p.225.
6. **Baechler, S., Jolie, J.** Determination of the Surface Area of Activated Slag Samples, 2002. http://lch.web.psi.ch/pdf/anrep01/E-03cement.pdf.
7. **Cerato, A.B., Lutergger, A.J.** Determination of Surface Area, 2002. journalsip.astm.org/Journals/geotech/pages/396.htm.
8. **Omotoso, O.E., Mikula, R.J., Stephen, P.W.** Surface Area Determination of Interstratified Phyllosilicates in Athaba sea oil sands from Synchrotron X-Ray Scattering Domain Size. www.dxcicdd.com/01/pdf/d-067.pdf.
9. **Lenau, T., Mazilli, A.** Empirical Calculation of the Surface Area of an Object, 1996.
10. **Lenau, T., Mazilli, A.** Electroplating Cost Estimation. Institut for Produktion og Ledelse, 1998. www.ipt.dtu.dk/~ap/ingpro/surface/elecomk.htm, Tekniske Universitet.

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METALINIŲ DETALIŲ ŠIURKŠČIŲ PAVIRŠIŲ PLOTO NUSTATYMAS

R e z i u m ė

Detalių paviršiaus plotas vaidina svarbų vaidmenį, ypač kai jas veikia didelės apkrovos. Didelę reikšmę turi tokių detalių paviršiaus ploto ir jų tūrio tarpusavio santykis. Šiame straipsnyje nagrinėjamas šiurkščių metalinių detalių paviršiaus plotas, ypač atkreipiant dėmesį į detales, turinčias tą patį šilumos perdavimo koeficientą. Dvi iš tos pačios medžiagos pagamintos vienodos temperatūros detalės įdėtos į indus su tuo pačiu kiekiu karšto vandens. Straipsnio pagrindinė idėja paremta vandens temperatūrų pasikeitimo abiejuose induose palyginimu. Didesnį paviršiaus plotą turinčios detalės temperatūra keičiasi greičiau. Temperatūrų skirtumas įgalina apskaičiuoti vienos detalės paviršiaus plotą, jeigu yra žinomas kitos detalės paviršiaus plotas. Bandymų rezultatams aprašyti ir analizuoti pateikta keletas elementarių matematinių priklausomybių. Šie rezultatai lyginami mažiausių kvadratų metodu (labiausiai tinkamu šiuo atveju).

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SURFACE AREA DETERMINATION OF RAGGED METALLIC BODIES

S u m m a r y

Surface area plays an important role for all components, especially those under extreme loading conditions. For such a component, the relation between the surface area and the subject volume is very important. The contribution of this paper is in the determination of the surface area of a ragged metallic component, in particular, components having the same transfer heat coefficient. The basic idea of the paper is concerned with immersing two compared bodies with identical temperatures and materials into separate containers filled with the same amount of hot water and measuring temperature change of the water in both containers. The body with greater surface experiences greater temperature rate. The difference enables us to calculate the surface of one body if the surface of the other body is known. Some mathematical calculations are introduced with a description of the experiments and analysis of the experimental results based on the elemental formula.

Т.А. Салау, О.А. Аедоку, С.А. Оке

ОЦЕНКА ПЛОЩАДИ ШЕРОХОВАТОЙ ПОВЕРХНОСТИ МЕТАЛЛИЧЕСКИХ ДЕТАЛЕЙ

Р е з ю м е

Площадь поверхности деталей играет важную роль, особенно при больших нагрузках. Большое значение имеет связь между площадью поверхности и объемом детали. В статье исследуется площадь поверхности шероховатых металлических деталей, имеющих одинаковый коэффициент передачи тепла. Идея статьи основана на сравнении изменения температуры двух деталей, изготовленных из того же материала и с идентичной температурой, погруженных в сосуды с одинаковым количеством горячей воды. Деталь с большей площадью имеет большую скорость изменения температуры. Разница температур дает возможность определить площадь поверхности одной детали, если известна площадь поверхности другой детали. Для описания и анализа результатов эксперимента представлено несколько элементарных математических зависимостей. Для сравнения результатов применен метод наименьших квадратов.

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