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Analysis of parametric excited vibrations of drive shafts caused by induction machines

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1. Introduction

Torsional vibrations in drive shafts are imperceptible by human senses. There is no noise, no vibrations on the machine bed human beeings can hear or feel. But measurements of the shaft torque show up to 25 times of the nominal machine torque [1] during the change over from one stable operation point to another like speed reversal or switch on situations. Especially in low-damped drive systems – as servo drives or direct driven machines – resonance excitation of torsional vibrations is critical to the mechanical strength of the shaft [2].

Resonance excitation of a vibrational system may be caused by different occurences:

1. impact;
2. excitation of the system with resonance frequency;
3. excitation by a sweep function.

An impact always happens at switch on, switch off operations. Excitation with resonance frequency is unusual but happens when the torsional resonance frequency is the net frequency or nearby. A sweep function excitation is the most problematically situation, because it runs continously through a wide range of frequencies. A sweep function is characterized by the following equation (1) and looks like Fig. 1.

$$f(t)=sin(2\pi F(t)t) \tag{1}$$

with $F(t)=k_I(t)$.

The research of torsional vibrations in various behaviors is also dedicated in papers [3-7]. This paper explains a sweep function excitation as the result of nonlinear parametric excitation.

2. Mathematical description of the induction machine for transient phenomena

Transient phenomena in an induction machine are difficult to describe, because quasistationary approaches cannot be employed, as no stable operation point during transients exists. Linearization is likewise inappropriate, as the nonlinear effects of interest in this instance are eliminated by definition. The Kovacs space vector theory [8] is suitable for describing induction machine transients. The space vector theory leads to a set of nonlinear differential equations which cannot be solved in a general manner. Numerical methods are necessary to solve this set of non–linear differential equations. So it is possible to describe the transient behaviour of induction machines in the time scale.

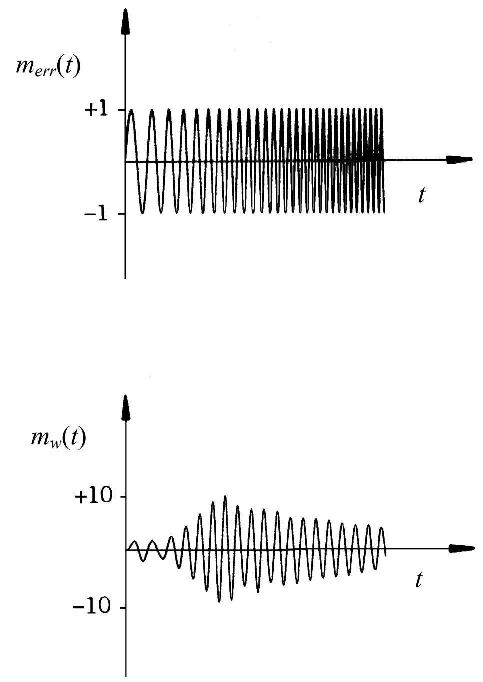


Fig. 1 Sweep function and excitation result

The description of the induction machine by the space vector theory in complex numbers looks as follows

$$\underline{u}_S(t)=r_S\underline{i}_S(t)+\frac{d}{dt}\underline{\Psi}_S(t)+jf_S\underline{\Psi}_S(t) \tag{2}$$

$$\underline{u}_R(t)=r_R\underline{i}_R(t)+\frac{d}{dt}\underline{\Psi}_R(t)+jf_R(t)\underline{\Psi}_R(t) \tag{3}$$

$$\underline{\Psi}_S(t)=x_S\underline{i}_S(t)+x_h\underline{i}_R(t) \tag{4}$$

$$\underline{\Psi}_R(t)=x_R\underline{i}_R(t)+x_h\underline{i}_S(t) \tag{5}$$

$\underline{u}$ is voltage space vector, $\underline{i}$ is current space vector, $\underline{\Psi}$ is flux space vector, f is frequency, r is resistor, x is inductive reactance, index S is stator, index R is rotor.

Variables are all space vectors and additionally the rotor frequency $f_R(t)$.

The equation for the electrical torque m_e is

$$m_e(t)=\frac{-Im\left\{\underline{\Psi}_R^*(t)\underline{i}_R(t)\right\}}{t_{m1}} \tag{6}$$

where t_{m1} is a normative time constant, which represents the time the nominal torque of the induction machine needs to speed up the rotor mass to nominal rpm.

These equations show three things:

1. the system has a product of variables $\frac{-Im\left\{\underline{\Psi}_R^*(t)\underline{i}_R(t)\right\}}{t_{m1}}$, which shows the nonlinearity of the set of equations;
2. the rotor frequency f_R is a time dependent parameter in equation (3) in combination with the rotor flux $f_R(t)\underline{\Psi}_R(t)$;
3. the system has only terms in the first derivation, what means, that the system is not able to swing.

Looking to the mechanical system, we have a free two–mass torsion oscillator. Θ_1 is representative for the rotor, where the eletrical torque M_e is the input to the torsional oscillator, i.e. drive system. Via a shaft, a flywheel with the mass Θ_2 is coupled to the rotor mass Θ_1. The shaft can be considered as a spring with the stiffness c and a damper with the damping constant k (Fig. 2).

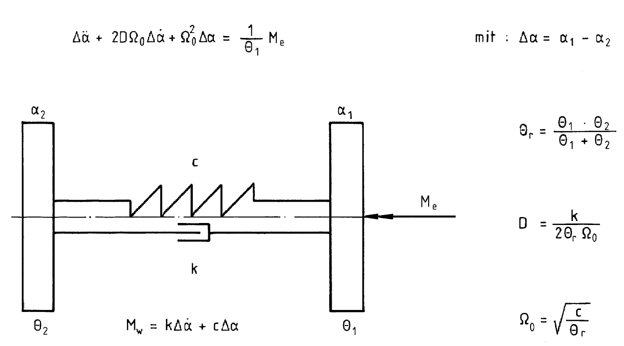


Fig. 2 Single degree–of–freedom torsional system

The differential equations for the torsional system are as follows [1]

$$\Theta_1 \frac{d^2}{dt^2}\alpha_1(t) + k\frac{d}{dt}\left(\alpha_1(t)-\alpha_2(t)\right) + \\ + c\left(\alpha_1(t)-\alpha_2(t)\right) = M_e(t) \tag{7}$$

$$\Theta_2 \frac{d^2}{dt^2}\alpha_2(t) - k\frac{d}{dt}\left(\alpha_1(t)-\alpha_2(t)\right) - \\ - c\left(\alpha_1(t)-\alpha_2(t)\right) = 0 \tag{8}$$

According to the second derivation of the variables $\alpha_1(t)$ and $\alpha_2(t)$ the mechanical system is able to swing. The torsional oscillator with low damping constant is a very good indicator for vibrations caused by the electrical torque $M_e(t)$.

Angular speed $\frac{d}{dt}\alpha_1(t)$ of Θ_1 is linked to the rotor freqency $f_R(t)$ as follows

$$\frac{d}{dt}\alpha_1(t) = \frac{2\pi}{p}\left(f_S - f_R(t)\right) \tag{9}$$

The set of differential equations from Eq. (2) to Eq. (9) has to be solved.

3. Numerical simulation of the run-up phase for an induction machine

A test constellation as in Fig. 2 is assumed to simulate the transient phenomena. To solve the described set of differential equation, a special numerical differential equation solver is used. To start the simulation, the standardized voltage space vector $\underline{u}_S$ Eq. (2) jumps from 0 to 1.

This jump function is causing electromagnetic compensation phenomena in the electrical torque M_e during start up (Fig. 3, a).

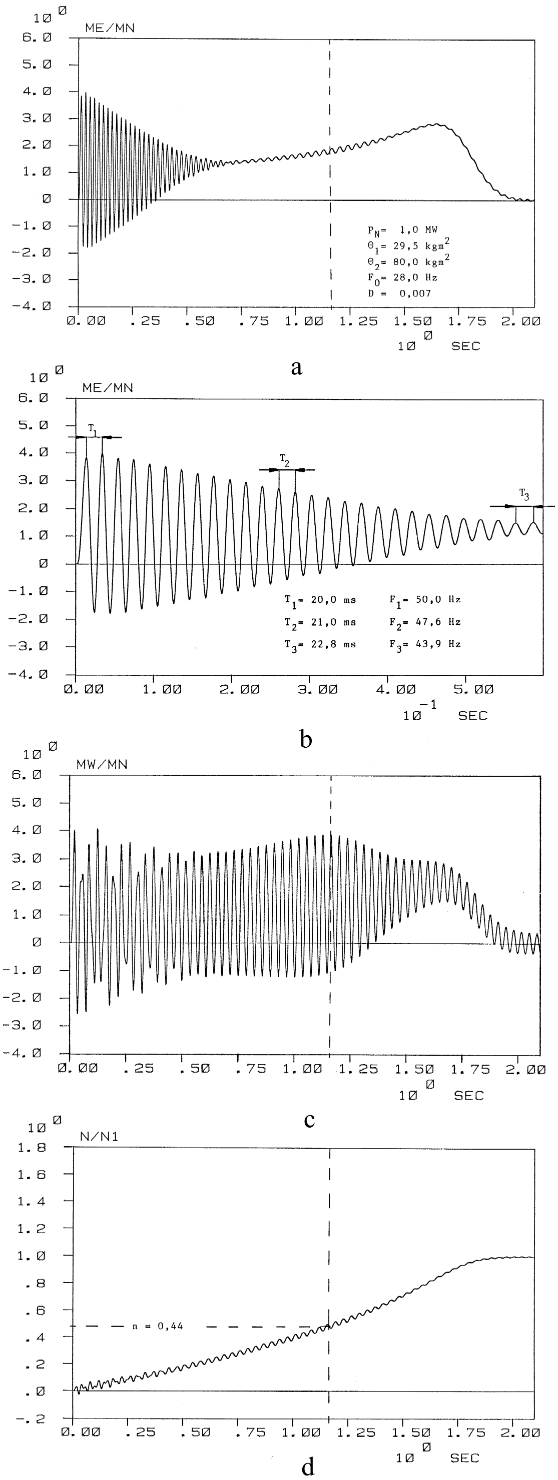


Fig. 3 a - simulation electrical torque (run-up); b - analysis of electrical torque after switch on; c - parametric excitation of the shaft torque (run-up); d - simulated speed (run-up)

The frequency starts with line frequency (50 Hz) and goes down to 43 Hz as analysed in Fig. 3, b.

The electrical torque shows the behavior of sweep function, as shown in Fig. 1. Responsible is the parametric excitation in Eq. (3), where the rotor frequency $f_R(t)$ is multiplied with the flux space vector $\underline{\Psi}_R(t)$. According to Eq. 6, $\underline{\Psi}_R(t)$ is one of the multiplier for the electrical torque M_e, which explains the sweep behavior.

The conclusion is, that the sweep frequency of the electrical torque runs from line frequency (50 Hz) at start up down to 0 Hz at nominal speed.

The shaft torque M_w shows in the first half second (Fig. 3, c) an overlay from the forced electrical torque (50 Hz - 43 Hz) and the resonance frequency of 28 Hz from the torsional oscillator.

Further on at about 1.2 seconds the amplitude of the shaft torque increases again with only the resonance frequency of 28 Hz of the torsional vibration system.

Looking at the speed (Fig. 3, d) at that moment, the relative speed is $n = 0.44$.

Eq. 9 can also be written in a normative way as follows

$$\frac{N}{N_1} = 1 - \frac{f_R}{f_S} \tag{10}$$

with $\frac{N}{N_1} = 0.44$, f_R is calculated to $f_R = (1-0.44)\cdot 50$ Hz = = 28 Hz.

28 Hz is the resonance frequency of the mechanical system. Thus parameter f_R leads to the excitation of torsional vibration system, that can be explained by Eq. (3). This is a typical parametric excitation phenomena.

Understanding the parametric excitation as a sweep function, the excitation condition can be determined very easily by Fig. 4.

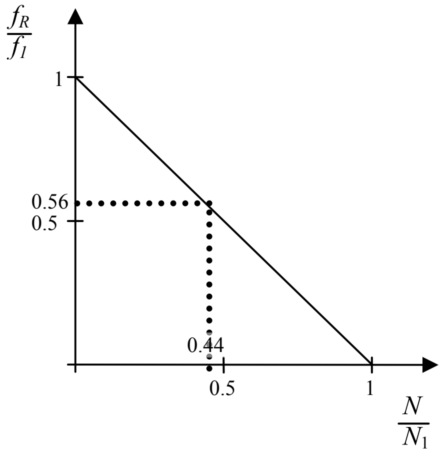


Fig. 4 Excitation condition diagram for run-up

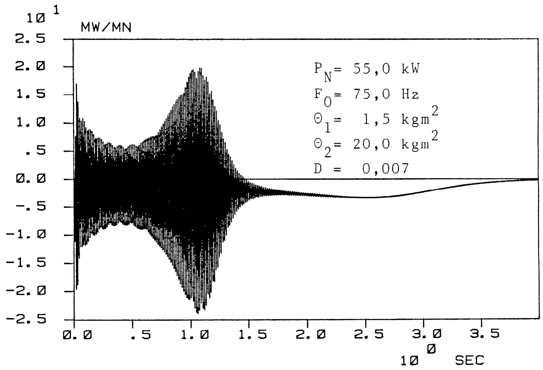
To prove this theory a simulation is made with resonance frequency of 75 Hz of the torsional vibration system. The motor is 55 kW induction machine with a squirrel cage rotor. As the maximum excitation frequency is 50 Hz, no resonance excitation is shown in the shaft torque.

To show the power of the space vector model it is to expect, that the resonance frequency should be excited during reversal, because the rotor frequency f_R runs from 100 Hz to 0 Hz during reversal. According to Eq. (10) the torsional system with the resonance frequency of 75 Hz should be excited at a speed of

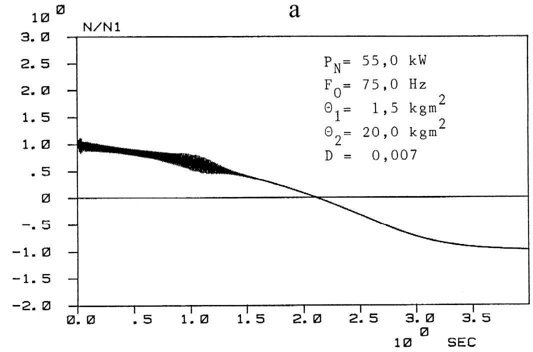
$$\frac{N}{N_1} = 1 - \frac{f_R}{f_S} = 1 - \frac{75\text{Hz}}{50\text{Hz}} = -0.5 \tag{11}$$

The simulation for reversal starts with the conditions at idle speed of 1500 rpm.

After the switch, the torsional oscillator is excited at its resonance frequency of 75 Hz (Fig. 5, a) by the impact of the electrical torque. After decline of the amplitude, the resonance frequency is again excited (Fig. 5, a) by the parameter f_R.



a



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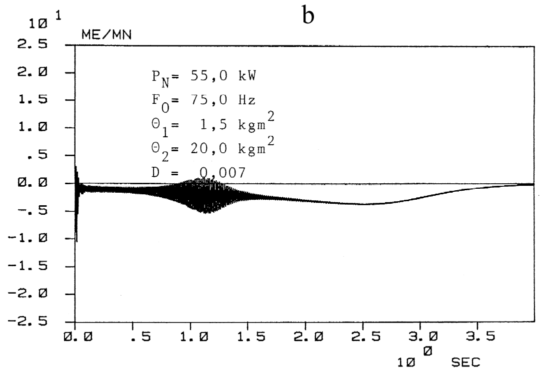


c

Fig. 5 a - parametric excitation of the shaft torque during reversal; b - simulation of the speed during reversal; c - simulation of electrical torque (reversal)

The parametric excitation takes place as calculated according Eq. (11) (Fig. 5, b).

High amplitudes of the shaft torque, which are up to about 25 times of the nominal torque of the electrical machine, shows the feedback of the mechanical system to the electrical system during the parametric excitation.

Fig. 6 shows the impact of electrical torque with about 20 times of the nominal torque. The shaft torque reacts with an amplitude of about 30 times of the nominal torque, due to the low damping of $D = 0.007$.

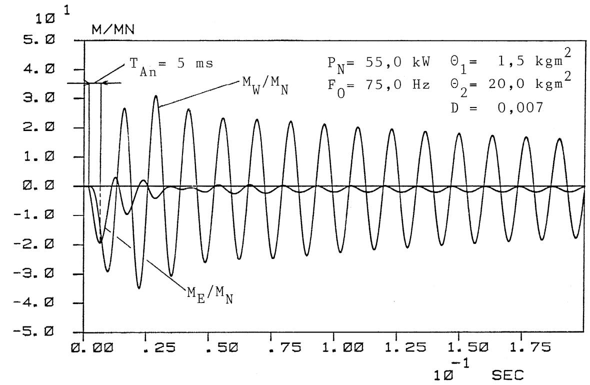


Fig. 6 Electrical impact and response of shaft torque at reversal switch

So the excitation diagram of Fig. 4 can be expanded as follows

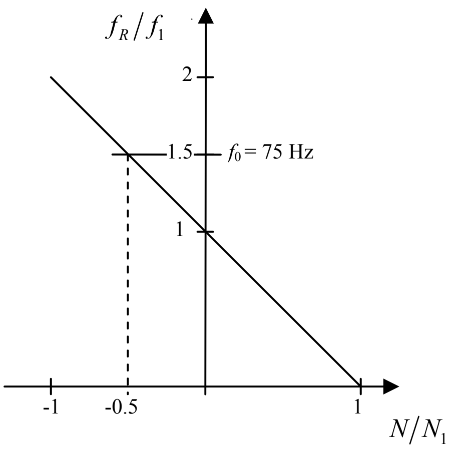


Fig. 7 Expanded excitation diagram

With the excitation diagram for an induction machine, it is very easy to predict parametric excitations of torsional vibrations during transients as speed up or reversal.

4. Experimental results

According to Fig. 2 a test rig was designed (Fig. 8).

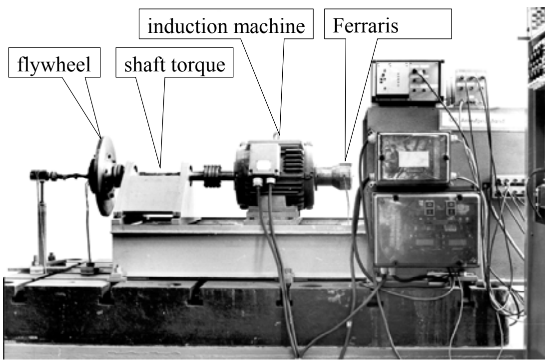
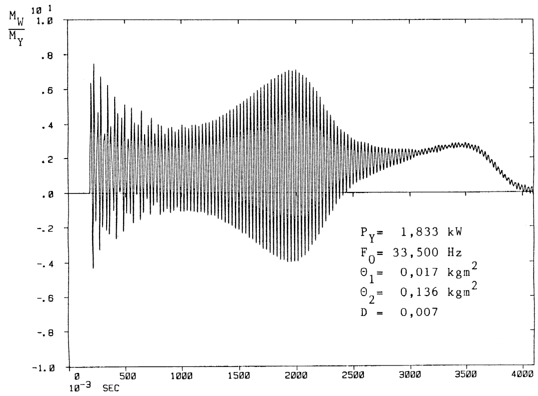
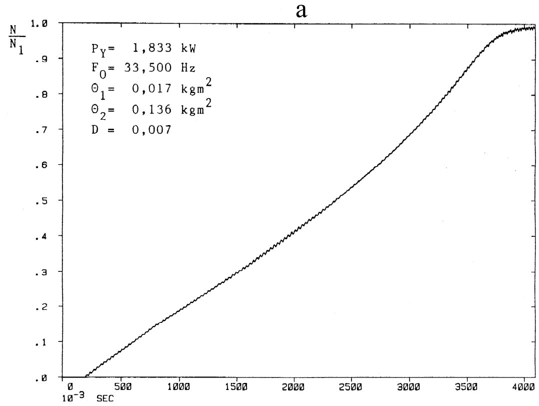


Fig. 8 Test rig to investigate torsional vibrations

The motor is a 1.8 kW induction machine with squirrel cage and synchronous speed N_1 of 3000 rpm. The resonance frequency of the mechanical system is $f_0 =$ = 33.5 Hz. No damping clutches are used so the damping is $D = 0.007$.



a



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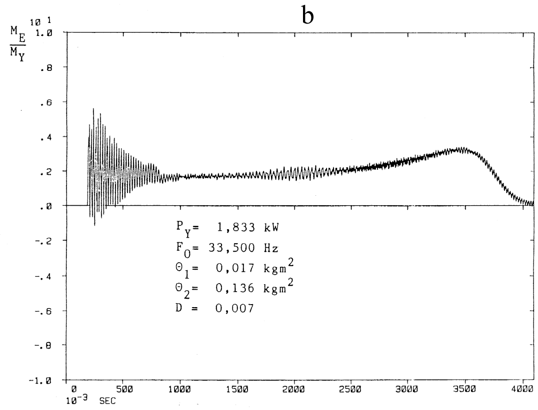
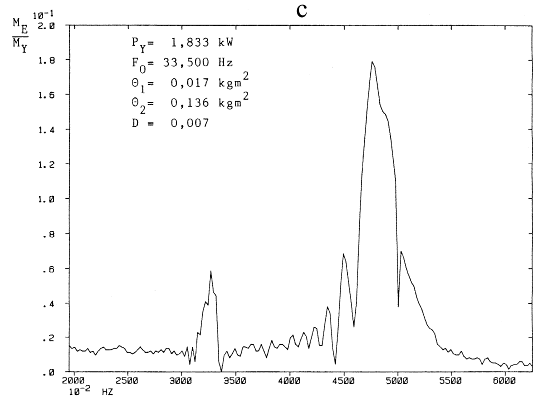


c



d

Fig. 9 a - measured parametric excitation of the shaft torque (run-up); b - measured speed (run-up); c - measured electrical torque (run-up); d - measured spectrum of the electrical torque (run-up)

To determine the electrical torque M_e, also angular acceleration $\ddot{\alpha}_1(t)$ of the rotor mass has to be measured according to Eq. (12).

$$M_e(t) = M_W(t) + \Theta_1\ddot{\alpha}_1(t) \tag{12}$$

The shaft torque has been measured with strain gauges, applied on the shaft. Angular acceleration $\ddot{\alpha}_1(t)$ has been measured with Ferraris sensor, which has been developed by the author [9]. The measuring chain was approved linear for the range 0 - 1000 Hz (–3 dB).

The shaft torque shows the same typical parametric excitation of the resonance frequency (33.5 Hz) as already simulated with the space vector theory (Fig. 3, c).

The measured speed shows, that the excitation happens, when the condition according to Eq. (10) is fulfilled

$$\frac{N}{N_1} = 1 - \frac{f_R}{f_S} = 1 - \frac{33.5\text{Hz}}{50\text{Hz}} = 0.33 \tag{13}$$

The electrical torque shows the typical transient phenomena with the sweep effect.

In the spectrum of the electrical torque (Fig. 9, c) sweep effect is also visible. It shows a wide range of frequencies below 50 Hz and lower amplitudes down to 0 Hz according the amplitudes in Fig. 9, c. The feed back of the mechanical resonance is also visible with a clear peak around 33 Hz.

The alternating torques after switch-on are clearly recognizable, with the values of roughly 6 times rated torque. Of special note is the fact, that the electrical torque frequently becomes negative, resulting for example in the much-feared “chattering teeth” effect on rigidly-coupled gears (geared motors) [10].

5. Conclusion

During transient phenomena the induction machine causes parametric all excited torsional vibrations. The excitation mechanism is a sweep that runs through all frequencies from 100 Hz to 0 Hz. In low damped systems this might cause an overload of torque in the mechanical system.

As amplitudes of the torque become also negative, in geared drives chattering of the teeth will reduce the lifetime of the gearbox.

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PARAMETRIŠKAI ŽADINAMO INDUKCINIŲ VARIKLIŲ VARANČIOJO VELENO VIRPESIŲ ANALIZĖ

R e z i u m ė

Tiriant pereinamuosius procesus, surijusius su parametriniu žadinimu ir silpnu kylančių virpesių slopinimu indukciniuose varikliuose, reikia spręsti netiesines diferencialines lygtis. Variklio elektrinių ir mechaninių kintamųjų dydžių derinys diferencialinėse lygtyse apsunkina vykstančių reiškinių fizikinę interpretaciją.

Šiame darbe parodyta, kaip varančiojo veleno nestacionarius virpesius, kylančius pereinamųjų procesų metu, galima paaiškinti parametriniu žadinimu. Žadinimo proceso analizė leido sudaryti diagramas, kuriomis naudojantis galima nustatyti virpesius žadinančių kritinių greičių diapazonus.

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ANALYSIS OF PARAMETRIC EXCITED VIBRATIONS OF DRIVE SHAFTS CAUSED BY INDUCTION MACHINES

S u m m a r y

A description of transient phenomena in an induction machine in connection with oscillation excitation in low–damped drive systems requires the solution of a system of nonlinear differential equations. The coupling of variables from the electrical and mechanical system in the differential equations complicates physical interpretability of the observed phenomena. The paper shows how nonstationary oscillations in the drive shafts, produced by transients in the induction machine, are explained in terms of parametric excitation. The knowledge of the excitation mechanism enables a simple excitation diagram to be constructed, indicating critical speed ranges for the excitation of oscillations.

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В. Турла, А. Якштас

АНАЛИЗ ПАРАМЕТРИЧЕСКИ ВОЗБУЖДАЕМЫХ КОЛЕБАНИЙ ВЕДУЩЕГО ВАЛА ИНДУКЦИОННЫХ ДВИГАТЕЛЕЙ

Р е з ю м е

Описание переходных процессов в индукционных двигателях, связанных с параметрическим возбуждением и недостаточным демпфированием возникающих колебаний требует решения систем нелинейных дифференциальных уравнений. Совокупность переменных величин электрической и механической частей двигателя в дифференциальных уравнениях усложняет физическую интерпретацию происходящих явлений. В данной работе показано, как нестационарные колебания ведущего вала, возникающие в переходных процессах, можно объяснить параметрическим возбуждением. Анализ процесса возбуждения позволил построить диаграммы возбуждения, с помощью которых можно определить критические диапазоны скоростей, возбуждающих колебания.

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