# Modelling of extrusion process and application of Taguchi method and ANOVA analysis for optimization the parameters

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crossref http://dx.doi.org/10.5755/j01.mech.18.3.1881

## 1. Introduction

Among different shaping techniques, extrusion process is one of the most important techniques used in a wide variety of industrial application. This process is an attractive production method in industry for its ability to achieve energy and material saving, quality improvement and development of homogenous properties throughout the component [1]. Extrusion is one of the important forming processes where put a mass of metal in a container. This process is divided into two types depending on the direction of motion for metal, the first type is called direct extrusion which is used in this study as shown in Fig. 1 and the second type is called indirect [2]. Heat can be used in extrusion process, but in this study cold extrusion is used [3].

Over the last years, the field of metal forming is characterized by dynamic development. There are several reasons for this, but one of the most significant is the use of computers and powerful software, which radically changed the approach and the way of process design and planning [4]. In nearby time from now, upper bound theory has been used to calculate pressure of extrusion, it includes velocity field on inlet and outlet of the die and it results from reduction of the area and the change of metal flow and through which forming energies can be calculated [5]. This theory is so complex as compared with finite element method (FEM); whereas FEM gives accurate solutions through the using of engineering analysis [6]. There are many FE software, however, in this study ABAQUS finite element package is used [7].



Fig. 1 Overall configuration of direct extrusion process

As in the case of every shaping operation, the pressure needed in extrusion process is significantly affected by the process tuning parameters. There are several process parameters in this technique, among which die angle, coefficient of friction and the ratio of reduction of the area are of utmost importance and precisely controllable. Therefore, it is worth to study the effects of the process parameters on the process response characteristics. In recent years, determining an optimal set of process parameters values to achieve a certain output characteristics has been the prime interest by many researchers. Although there are few studies in modeling and optimization of process parameters in extrusion, most of them are limited to the particular circumstances and are computationally complex.

The present study attempts to make the use of FE data to relate important process parameters to process output variables, through developing empirical regression models for various target parameters. To validate FE model, the results gained by FE simulation have been compared with analytical method and good agreements were achieved.

In the next stage, the signal to noise method is used in order to identify a proper set of process parameters that can produce the minimum pressure needed for extrusion in the considered range. To validate the regression model, a sample extrusion process was solved using analytical methods, Ideal Work Approach, and the pressure needed in this process was determined. Then, the regression formula has been applied to the process parameters and pressure in extrusion as an output variable was found. Finally, a comparison between the results gained by Ideal Work Approach and Regression Model was made. According to the comparison, the regression model is in good agreement with analytical method and FE simulation.

## 2. Material properties

The material used for this study was aluminum alloy 2A12T4, with chemical composition listed in Table 1. Some of the mechanical properties of the material were obtained from tensile test in the rolling direction and stress-strain values are tabulated in Table 2.

Table 1 Chemical composition of aluminium alloy (2A12T4)

Chemical element	Al	Cu	Mg	Mn	Si	Fe	Ti
Percentage	93	4.29	1.34	0.46	0.14	0.31	0.02

For theorical computations [8] and FE simulations, it is often necessary to represent an experimentally determined stress-strain curve by an empirical equation of suitable form. When the material is rigid-plastic, it is frequently convenient to employ the Ludwick Power Law

$$\sigma = k \varepsilon^n \tag{1}$$

where k is constant stress and n is a strain-hardening exponent usually lying between zero and 0.5. According to experimental data presented in Table 2 and using the least squares method, the stress-strain equation was derived

$$\sigma = (570.558)\varepsilon^{0.1089} \tag{2}$$

Table 2

Experimental data from tensile test

Stress	Strain
0.01	346.15
0.02	370.19
0.03	394.23
0.04	402.91
0.05	413.59
0.06	417.47
0.07	427.18

#### 3. Analytic method (Ideal Work Approach)

Sometimes, the axially symmetric deformation in the extrusion or wire drawing of a circular rod or wire can be simulated by tension test [9].

The fact of necking that would occur in a tension test can be ignored. The ideal work is

$$w_i = \int_{0}^{\varepsilon} \sigma \, d\varepsilon \tag{3}$$

where  $\varepsilon = ln\left(\frac{A_0}{A_f}\right)$ . With power-law hardening

v

$$v_i = \frac{k \,\varepsilon^{n+1}}{n+1} \tag{4}$$

Fig. 2 illustrates direct or forward extrusion. A billet of diameter  $D_0$  is extruded through a die of diameter  $D_1$ . Except for the very first and last material to be extruded, this is a steady-state operation. The volume of metal exiting the die,  $A_1 \Delta l_1$ , must equal the material entering the die,  $A_0 \Delta l_0$ , so the total external work  $W_a$  is

$$W_a = F_e \,\Delta l \tag{5}$$

where  $F_e$  is the extrusion force. Substituting the work per volume as

$$w_a = \frac{W_a}{A_0 \,\Delta l} = \frac{F_e \,\Delta l}{A_0 \,\Delta l} = \frac{F_e}{A_0} \tag{6}$$

The extrusion pressure  $P_e$  must equal  $w_a$ . Although  $w_a = w_i$  for an ideal process, for an actual process  $w_a > w_i$ . Therefore

$$P_e > \int \sigma d\varepsilon \tag{7}$$

Now, by lumping the insufficient terms and defining deformation efficiency

$$\eta = \frac{W_a}{W_i} \tag{8}$$

In extrusion process, it is often between 0.60 and 0.75 depending on lubrication, reduction, and die angle.





#### 4. Modelling development

As earlier mentioned, the important controlling process parameters in extrusion include die angle, coefficient of friction and the ratio of reduction of area. In this study, extrusion pressure has been chosen as the main process response characteristics to investigate the influence of the above parameters. We first develop a FE model to simulate extrusion process and validate the FE results using analytical method, then present a mathematical model to relate the process control parameters to the process response characteristics. The empirical model for the prediction of extrusion pressure in terms of the controlling parameters will be established by means of piecewise linear regression analysis.

## 4.1. FE simulation

In recent years, the FEM has been an effective tool to evaluate the metal forming processes [10]. It is possible to drastically reduce the lead-time of new parts and products by proper implementation of a simulation technique into development and research. Much research based on the finite element analysis has been done to analyze the metal forming processes. The accuracy and efficiency of FEA determine whether the simulation results are successful and reliable or not [11]. One question being frequently asked is whether a model is valid or invalid. This leads to research on model verification and model validation in this study.

There are two types of FEM codes, which can be used for metal forming simulation, i.e. static codes and dynamic codes. Generally, without going into detail, both types of codes are based on equations of motion. In this research, the static approach was used and represented by ABAQUS/Standard [7] to simulate the extrusion process of a bar made from aluminium. All the simulations performed in the current study were run on a Pro Mc 700 Laptop Computer and the FE results were validated by the analytical method. Furthermore. the results of the ABAQUS/Standard simulation are in close agreement with those obtained with ABAQUS/Explicit.

Fig. 3 shows the half of the cross-section of finite element model.



Fig. 3 Layout of finite element model

Table 3 shows the pressure needed for extrusion operation as well as the process specifications. By the comparison of FE and analytical results presented in Section 5, good agreements were achieved for extrusion pressure.

Table 3 Process specifications as well as FE and analytical results for extrusion pressure

α	μ	R %	FE result for extrusion pres- sure, MPa	Analytic result for extrusion pressure, MPa
15	0.04	36	315	300.376

4.2. Design of experiments

The FE results were obtained using design of experiment (DOE) technique. Table 4 shows input parameters in different levels and Table 5 shows some of the FE settings obtained by Taguchi DOE matrix.

	Level	Level	Level	Level	Level
	one	two	three	four	five
Die Angle ( $\alpha$ )	5°	7.5°	10°	12.5°	15°
Coefficient of friction $(\mu)$	0.0	0.02	0.04	0.06	0.08
$\frac{\text{Reduction ratio}}{\left(\frac{A_0 - A_f}{A_0}\right) \times 100}$	7.8	15.36	22.56	29.44	36

Input parameters in different levels

Table 5

Table 4

FE settings obtained by Taguchi DOE matrix

Row	Die angle $\alpha$	Coefficient of friction $\mu$	$\frac{\text{Reduction ratio}}{\left(\frac{A_0 - A_f}{A_0}\right) \times 100}$
1	1	1	1
2	1	2	2
3	1	3	3
		•••	
16	4	1	4
17	4	2	5
18	4	3	1
25	5	1	5

As shown a total of 25 FE tests were performed to gather the required data. Table 6 shows the results gained from FE simulation according to Taguchi DOE matrix.

In Table 6, the first three columns show the pro-

Result gained from FE simulation

Row	Die angle α	Coefficient of friction $\mu$	Reduction ratio $\left(\frac{A_0 - A_f}{A_0}\right) \times 100$	Extrusion pressure, MPa
1	5	0.0	7.80	54.8
2	5	0.02	15.36	196
3	5	0.04	22.56	214
			•••	
16	12.5	0.0	29.44	259
17	12.5	0.02	36.00	276
18	12.5	0.04	7.84	100
			•••	
25	15	0.0	36.00	289

cess parameters settings given by Taguchi DOE matrix. The last column is the measured process output resulted from different FE tests. The general form of a regression mathematical model is as follows

$$P = 25.7 + (44113)\mu^{2} + (8.91)R - (0.356)\alpha^{2}$$
(9)

Different regression functions (linear, curvilinear, logarithmic, etc.) are fitted to the above data and the coefficients values  $(a_i)$  are calculated using regression analysis. The best model is the most fitted function to the experimental data. Such a model can accurately represent the actual extrusion process. Therefore, in this research, the adequacies of various functions have been evaluated using analysis of variance (ANOVA) technique.

The model adequacy checking includes the test for significance of the regression model and the test for significance on model coefficients [12]. ANOVA results recommend that the quadratic model is statistically the best fit in this case. Statistical analysis shows that the associated P-value for the model is lower than 0.02; i.e. 98.00% confidence. This illustrates that the model is statistically significant. Based on ANOVA, the values of and adjusted  $R^2$  are over 89% for output parameter. This means that regression model provides an excellent explanation of the relationship between the independent variables and response.

For illustrative purposes, the distributions of real data around regression lines and residual analysis of regression model are illustrated in Fig. 4.

This figure demonstrates a good conformability of the developed models to the real process and hence is used to represent the actual process.

The best levels for process parameters in extrusion using Signal to Noise method were obtained. Fig. 5 shows the best levels in order to minimize the extrusion pressure.

## 5. Sample extrusion problem

In this sample process, a bar of the material is reduced from 9 to 7.2 diameter ( $\eta = 0.7$ ). The strain hardening behavior of this material is approximated by  $\sigma = (570.558)\varepsilon^{0.1089}$ . Then, the work per volume in extru sion process is calculated using ideal work approach



Residual plots for P







$$w_i = (570.55) \times \frac{(0.4462)^{1+0.1089}}{1.1089} = 210.263 \,\mathrm{MPa}$$
 (10)

$$P_e = w_a = \frac{239.946}{0.7} = 300.376 \,\mathrm{MPa} \tag{11}$$

This sample problem is also resolved by regression formula according to the following process parameters:

>  $\eta = 0.7 \Rightarrow \mu = 0.04$ > Reduction of Diameter = 9 to 7.2, i.e. R = 36%.

Then we have the following 
$$(2 - 2)^2 (2 - 2)^2 (2 - 2)^2$$

$$P_e = 25.7 + (44113) \times (0.04) + (8.91) \times (36) - -(0.356) \times (15)^2 = 336.94 \,\text{MPa}$$
(12)

As it can be seen here, the regression model is in good agreement with analytical method. Hence, the adequacy of the regression formula can be also achieved by this comparison.

 $\geq \alpha = 15^{\circ}$ 

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# 6. Conclusions

Extrusion pressure is one of the most important specifications in extrusion process; therefore, the study of this parameter is of utmost importance. This study addresses modeling and optimization of the process parameters for extrusion operation. To model the process, a set of FE data has been used to evaluate the effects of various parameter settings in extruding 2A12TA aluminum alloy. Taguchi method has been employed in order to gather FE data. Then a mathematical model was developed in order to relate the process control parameters to the process response characteristics. The empirical model for the prediction of extrusion pressure in terms of the controlling parameters was established by means of piecewise linear regression analysis.

In the next, analysis of variance (ANOVA) was used to determine optimal values of input parameters to achieve minimum extrusion pressure in the same conditions. As it can be seen above, the optimal process parameters for a certain reduction of area are  $\alpha = 15^{\circ}$  and  $\mu = 0.02$ .

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## EKSTRUZIJOS PROCESO MODELIAVIMAS IR TAGUCHI METODO BEI ANOVA ANALIZĖS PRITAIKYMAS PARAMETRAMS OPTIMIZUOTI

### Reziumė

Šiame tyrime modeliuoti ir optimizuoti ekstruzijos proceso parametrai. Duomenų bazė imituojant aliumio lydinio ekstruziją sudaryta įvairiems parametrams. Tam vietoje eksperimento naudota BEM programa ABAQUS. Šiame procese naudojami kintamieji yra puansono kampas, trinties koeficientas ir ploto sumažėjimo laipsnis. Viena svarbiausių gamybos charakteristikų yra presavimo slėgis, nustatytas naudojant įvairius parametrų rinkinius. Taguchi metodas ir regresinis modeliavimas panaudoti ryšiams tarp jėjimo ir išėjimo parametrų nustatyti. Modelio adekvatumas įvertintas dispersijos (ANOVA) analize. Proceso parametrų rinkinio įtaka proceso rezultatams parodyta grafiškai. Darbo tikslas yra nustatyti tinkamus proceso parametrų dydžius ir surasti geriausią presavimo parametrų reikšmes, sutaupyti laiko bei lėšų naudojant regresijos modelį vietoj BE imitavimo.

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## MODELING OF EXTRUSION PROCESS AND APPLICATION OF TAGUCHI METHOD AND ANOVA ANALYSIS FOR OPTIMIZATION THE PARAMETERS

## Summary

This research addresses modeling and optimization of the process parameters for extrusion operation. To model the process, a set of data has been used to evaluate the effects of various parameter settings in extruding 2A12TA aluminum alloy. The set of data used in this paper was achieved by ABAQUS finite element package instead of carrying out the experiments. The process variables considered here include die angle, coefficient of friction and the ratio of reduction of area. Extrusion pressure, as one of the most important output characteristics, has been evaluated based on different parameter settings. The Taguchi method and regression modeling are used in order to establish the relationships between input and output parameters. The adequacy of the model is evaluated using analysis of variance (ANOVA) technique. The pairwise effects of process parameters settings on process response outputs are also shown graphically. The objective is to determine proper levels of process parameters in order to obtain a certain level of extrusion pressure and save time as well as cost, using regression model instead of running FE simulations.

Keywords: extrusion, Taguchi method, ANOVA analysis.

Received June 20, 2011 Accepted June 13, 2012