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Biparametric shakedown design of steel frame structures

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1. Introduction

The mathematical model of minimal volume design of steel structure, which implicates plastic strains and is subjected to the varying repeated load, is created based on energy principles of the mechanics of deformable body [1–3], shakedown of the structures [4–7] and mathematical programming theories [8, 9]. Varying repeated load (VRL) is a system of loads, which can vary independently [10, 11]. In all cases, the load is treated as quasi-static, and VRL is described by the lower and upper bounds of varying loads rather than a particular loading history (i.e. by the law dependent on time *t*).

In this work, only the deterministic formulation of structures' continuous optimization problems is analyzed. Usually only one design parameter of element's cross-section is optimized (e.g. an area or resistance moment of element's cross-section), the other parameters were expressed by mean curves [12]. However, this approach is very unaccurate especially in the case of discrete optimization problems [13], when the evaluation of dispersion of geometrical characteristics of profile assortment sets is still important [14]. A new principle of biparametric optimization for optimal shakedown design is investigated in this paper, where already two design parameters of separate element's cross-section are selected for optimization [15–17]. This treatment expands the applicability of optimization problem in optimal design of steel frame structures by performing the optimization of biparameters in admissible fields of geometrical characteristics of profile assortment sets, and allows us to estimate the dispersion adequately.

The structure adapted to VRL is safe in terms of plastic failure, but not always satisfies serviceability requirements, for example it may have unallowable deflections [18, 19]. Therefore, not only strength (shakedown), but also stiffness conditions [20, 21] are included in mathematical models of plastic systems' optimization problem in this paper. The stiffness conditions are usually realized by restraining deflections and nodes' displacements of the structure [22, 23]. The evaluation of stability is also relevant to trusses [24]. The analysis and evaluation of design of structures with plastic strains' allows more effective use of its load-bearing capacities and design more economical projects [25, 26].

The structures are modeled by the bar equilibrium finite elements with interpolation functions of internal forces [27–29]. The elements are designed using HE, IPE, RHS (Rectangular Hollow Sections) steel profile assortments and considering dispersion of geometrical characterristics of profile assortment sets [17]. The biparametric optimization principle of the main (leading) design geometry Π_0 of cross-sections and the control-corrective (driven) geometry Π_1 is used to realize the design of elements [15, 17]. Idealization of distribution (expressed by approximation of mean curves) of geometrical characteristics of steel profiles determines the application of the objective function, which approximately expresses the volume of the structure [16, 17]. Similarly, the approximate minimization function of volume is well suited for practical application of the structures' optimization principles.

In general, the design task of volume minimization of steel structures, which is described above, is associated with the problem of nonlinear and non-convex mathematical programming (NLP) [8]. The convergence of the problem is being achieved by an iterative method, solving a number of nonlinear optimization problems. To simplify the solution of the NLP, the complementary slackness conditions can be moved into the objective function [25].

Actually, the optimization problem of steel structures is implemented in the modeling, analysis and optimal design system JWM SAOSYS Toolbox v0.47 developed in MATLAB environment by the authors [15, 17, 30]. A special solution module SAOSYS/EPSOptim-SD is designed for designing elastic-plastic structures subjected to VRL. The previous version of the architecture SAOSYS v0.42 [17] was based on the database principle. Now, after a number of improvements made it has become a structurized hierarchical model of fully object-oriented components. The reconstruction of SAOSYS v0.47 enabled us to use new integration and development facilities applied in modern information technologies. Besides the structural part of SAOSYS system, the graphical user interface (Fig. 1) is created, which may be used in structural modeling, the control of problem solution and graphical interpretation of the results of analysis.

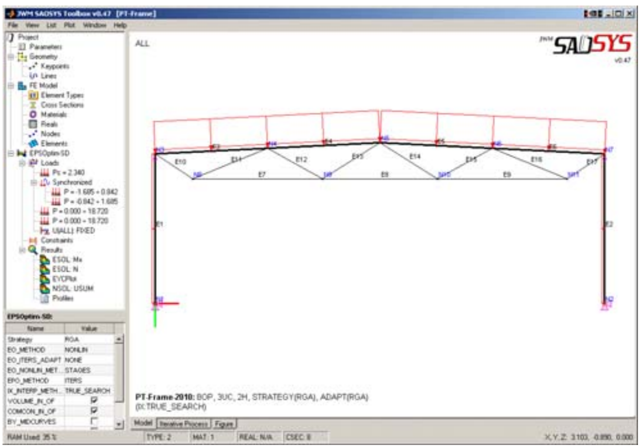

Fig. 1 Frame design model in SAOSYS system

The possibilities of the system SAOSYS are demonstrated by a numerical example of industrial building frame design with strength, stability and stiffness constraints. The assumption of small displacements is adopted in calculations.

2. A general mathematical model of the structure optimization problem

The elastic-plastic bar structures of known topology subjected to varying repeated and dead loads are considered. In the case of monotonic load $\boldsymbol{F}(\eta) = \eta\cdot\{F_i \mid i = 1, 2, ..., m \Leftrightarrow i \in DOF\}$, all loads F_i vary proportionally to one general multiplier η. Here, m is the number of degrees of freedom of the structure (DOF). The real structures are usually subjected to different, unrelated effects and their combinations $\boldsymbol{F}(t) = \{F_i(t) \mid i \in DOF\}$, which vary in time t [7, 10–12]. Eliminating the loading history, the VRL vector $\boldsymbol{F}(t)$ may be described only by the lower and upper bounds $\boldsymbol{F}_{inf}$, $\boldsymbol{F}_{sup}$, which are not related to the time t: $\boldsymbol{F}_{inf} \le \boldsymbol{F}(t) \le \boldsymbol{F}_{sup}$.

The stress-strain state of the structure at shakedown depends on the loading history $\boldsymbol{F}(t)$. For example, the displacements of the elastic-plastic structure may be described by the sum of elastic and residual components – $\boldsymbol{u}(t) = \boldsymbol{u}_e(t) + \boldsymbol{u}_r(t)$. When loading is given only by the vectors $\boldsymbol{F}_{inf}$ and $\boldsymbol{F}_{sup}$, it is possible to calculate only the lower and upper bounds of displacements range. Therefore, in the case of VRL, only the lower and upper values of node displacements $\boldsymbol{u}_{inf}$, $\boldsymbol{u}_{sup}$ are used, such that $\boldsymbol{u}_{inf} \le \boldsymbol{u}(t) \le \boldsymbol{u}_{sup}$. The elastic displacements here can be calculated by the formula $\boldsymbol{u}_e(t) = [K]^{-1}\boldsymbol{F}(t)$ [27]. Meanwhile, the residual displacements $\boldsymbol{u}_r(t)$ of the structure at shakedown vary unevenly and various methods [12, 20, 22, 26] were created to define the limit values of residual displacements $\boldsymbol{u}_{r,inf}$, $\boldsymbol{u}_{r,sup}$. Slightly simplified stiffness conditions-constraints [25] are used in the mathematical model presented in this paper. Thus, the global displacement vectors of lower and upper bounds are calculated as follows

$$\boldsymbol{u}_{e,inf} = [\beta_{sup}]\boldsymbol{F}_{inf} + [\beta_{inf}]\boldsymbol{F}_{sup} \tag{1}$$

$$\boldsymbol{u}_{e,sup} = [\beta_{sup}]\boldsymbol{F}_{sup} + [\beta_{inf}]\boldsymbol{F}_{inf} \tag{2}$$

where

$$[\beta] = [\beta_{inf}] + [\beta_{sup}]; \; -\beta_{inf,i,j} \ge 0, \; \beta_{sup,i,j} \ge 0 \tag{3}$$

here, $[\beta] \equiv [K]^{-1}$ is the influence matrix of elastic displacements; $[K]$ is the stiffness matrix of the structure.

Assume that the structure is subjected to μ ($\mu \le m$) unrelated varying repeated loads and load sets. It is possible to create $p = 2^{\mu}$ different load combinations, which compose the following final VRL combination matrix of the structure

$$[F] \equiv \left[\boldsymbol{F}_j \middle| \, j = 1, 2, ..., p \Leftrightarrow j \in J\right] \tag{4}$$

The matrix of elastic internal forces of the structure is as follows

$$[S_e] \equiv \left[\boldsymbol{S}_{e,j} \middle| \, j \in J\right] = [\alpha][F] \tag{5}$$

where $[\alpha]$ is the influence matrix of internal elastic forces.

In addition to VRL, the structures are also subjected to dead load $\boldsymbol{F}_c$. The vectors of elastic displacements and internal forces of the structure subjected to dead load are as follows: $\boldsymbol{u}_{e,c} = [\beta]\boldsymbol{F}_c$; $\boldsymbol{S}_{e,c} = [\alpha]\boldsymbol{F}_c$.

The structure is modeled by the equilibrium finite elements with interpolation functions of internal forces [27–29]. The finite elements of truss (LINK11) and frame (BEAM31) types are used for discretization [16, 17]. Yield and stability (truss elements) conditions are controlled at the nodes of the elements.

The main characteristics $\boldsymbol{\Pi}_0 \equiv \{\Pi_{0k} \mid k \in K\}$ of the elements' cross-sections, such as, cross-section areas $A_{0k} \Leftrightarrow \Pi_{0k}$, $k \in K_{11}$ for the truss elements and plastic resistance moments $W_{pl,y0k} \Leftrightarrow \Pi_{0k}$, $k \in K_{31}$ for the frame elements are optimized (Fig. 2), where $K = K_{11} \cup K_{31}$. In addition, based on the biparameters as main (leading) $\boldsymbol{\Pi}_0$ and the control-corrective (driven) $\boldsymbol{\Pi}_1 \equiv \{\Pi_{1k} \mid k \in K\}$ ($I_{min,k} \Leftrightarrow \Pi_{1k}$, $k \in K_{11}$; $A_k \Leftrightarrow \Pi_{1k}$, $k \in K_{31}$) principles of cross-section geometry, the elements are designed using HE, IPE, RHS steel profile assortments and considering dispersion of geometrical characteristics of profile assortment sets [15–17].

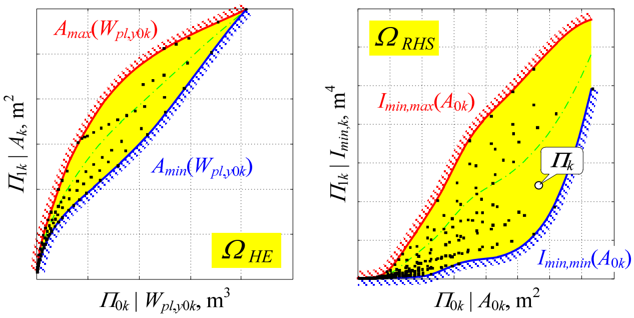

Fig. 2 Admissible fields Ω of the discrete characteristics of HE and RHS profiles

The project of minimal volume V of the structure should be sought, when the optimality criterion is secured and strength, stiffness and stability (truss part) requirements are evaluated. A general mathematical model for the problem of minimal volume of the structure is as follows

find
$$min \; V = \boldsymbol{L}^T \boldsymbol{A}(\boldsymbol{\Pi}_0) + \sum_j \boldsymbol{\lambda}_j^T \boldsymbol{\varphi}_j \tag{6}$$

subject to:
$$\boldsymbol{\varphi}_j = [Z]\boldsymbol{\lambda} - [B]\boldsymbol{\Pi}_0 \le -\boldsymbol{Y}_j, \quad \forall j \in J \tag{7}$$

$$\boldsymbol{\lambda} = \sum_j \boldsymbol{\lambda}_j \tag{8}$$

$$\boldsymbol{\lambda}_j \ge \boldsymbol{0}, \quad \forall j \in J \tag{9}$$

$$[H]\boldsymbol{\lambda} \ge \boldsymbol{u}_{min} - \boldsymbol{u}_{e,c} - \boldsymbol{u}_{e,inf} \tag{10}$$

$$[H]\boldsymbol{\lambda} \le \boldsymbol{u}_{max} - \boldsymbol{u}_{e,c} - \boldsymbol{u}_{e,sup} \tag{11}$$

$$\boldsymbol{\Pi}_{0,min} \le \boldsymbol{\Pi}_0 \le \boldsymbol{\Pi}_{0,max} \tag{12}$$

where:
$$\Pi_{1k,min}(\Pi_{0k}) \le \Pi_{1k} \le \Pi_{1k,max}(\Pi_{0k}) \tag{13}$$

$$\boldsymbol{\Pi}_k \equiv \{\Pi_{0k}, \Pi_{1k}\} \in \Omega_s \subset \mathbb{R}^2(0; +\infty) \tag{14}$$

$$I_{y,k} = I_y(\boldsymbol{\Pi}_k), \quad k \in K_{31} \tag{15}$$

$$s \in (HE, IPE, RHS), \quad k \in K \tag{16}$$

The optimized parameters $\boldsymbol{\Pi}_0$ and the plastic multipliers $\boldsymbol{\lambda}_j$, $j \in J$ are the unknowns of the problem (6) - (16). The mathematical model is composed of the nonlinear objective function (6) and the conditions-constraints: the linear inequalities (7), (9); the linear stiffness constraints (10), (11); the structural constraints (12). The aim of the optimization problem is to find the optimal distribution of the elements' cross-section parameters $\boldsymbol{\Pi}_0$ of the structure, subjected to VRL and dead loads with such additional conditions of biparametrical optimization principle: during optimization the biparameters $\boldsymbol{\Pi}_0$-$\boldsymbol{\Pi}_1$ satisfie the nonlinear boundary conditions (13) of characteristics' fields Ω_s (14) of steel profile assortments (Fig. 2). In addition, the approximation (15) for evaluation the inertia moments $I_{y,k}$ of cross-sections of the frame elements are used [17].

The vector of parameters $\boldsymbol{\Pi}_0$ must ensure reliability requirements, which implicates not only elastic but also plastic strains. Thus, minimization is performed, when the configuration of the whole structure, physical characteristics of the material, the loading and the displacement vectors $\boldsymbol{u}_{min}$, $\boldsymbol{u}_{max}$ of admissible variation bounds at the structure nodes are known (the latter are used for stiffness constraints' verification).

After considering the general character of the problem (6) - (16), the introduced denotations are discussed in details. In the mathematical model (6) - (16) $\boldsymbol{L}$ is the length vector of the structure's elements; $\boldsymbol{A}(\cdot)$ is the vector-function of cross-section geometry conversion into the area of cross-section, when $\Pi_{0k} \in (A_{0k}, W_{pl,y0k})$; $\boldsymbol{\varphi}_j$ is the vector of values of yield conditions under the j-th point in the locus; $\boldsymbol{u}_{e,inf}$, $\boldsymbol{u}_{e,sup}$ are the vectors of the lower and upper values of elastic displacements in the structure subjected to VRL (1), (2); $\boldsymbol{u}_{e,c}$, $\boldsymbol{S}_{e,c}$ are the vectors of elastic displacements and elastic internal forces in the structure subjected to the dead load; $\boldsymbol{\Pi}_{0,min}$, $\boldsymbol{\Pi}_{0,max}$ are the vectors of the lower and upper values of the cross-section parameters. In addition, such expressions of matrices are used

$$[Z]=[\Phi][G],\ [Y]\equiv\left[\boldsymbol{Y}_j \middle| j\in J\right]=[\Phi]\left[\tilde{S}_e\right] \tag{17}$$

$$\left[\tilde{S}_e\right]=\left[\boldsymbol{S}_{e,j}+\boldsymbol{S}_{e,c}\middle| j\in J\right] \tag{18}$$

where $[\Phi]$, $[B]$ are the configuration matrices of the yield conditions of the structure; $[H]$, $[G]$ are the influence matrices of residual displacements and internal forces. The vectors of residual displacements and residual internal forces are as follows

$$\boldsymbol{u}_r=[\beta][A][K][\Phi]^T\boldsymbol{\lambda}=[H]\boldsymbol{\lambda} \tag{19}$$

$$\boldsymbol{S}_r=\left([K][A]^T[H]-[K][\Phi]^T\right)\boldsymbol{\lambda}=[G]\boldsymbol{\lambda} \tag{20}$$

where $[A]$ is the matrix of coefficients of equilibrium equations of the structure. Finally, state of the structure is described by the total elastic and residual internal forces, strains and displacements

$$\begin{cases}\boldsymbol{S}_j=\boldsymbol{S}_{e,c}+\boldsymbol{S}_{e,j}+\boldsymbol{S}_r \\ \boldsymbol{\theta}_j=\boldsymbol{\theta}_{e,c}+\boldsymbol{\theta}_{e,j}+\boldsymbol{\theta}_r\end{cases} j\in J \tag{21}$$

$$\begin{cases}\boldsymbol{u}_{inf}=\boldsymbol{u}_{e,c}+\boldsymbol{u}_{e,inf}+\boldsymbol{u}_r \\ \boldsymbol{u}_{sup}=\boldsymbol{u}_{e,c}+\boldsymbol{u}_{e,sup}+\boldsymbol{u}_r\end{cases} \tag{22}$$

To simplify the solution of the NLP problem, the nonlinear complementary slackness *Kuhn-Tucker* conditions can be moved into the objective function (6) [25].

3. Design algorithm

The design algorithm of the elastic-plastic steel structures is implemented in the SAOSYS system developed in MATLAB environment by the authors. A new analysis module EPSOptim-SD was created for designing such structures. Further, we will describe the main parts of the algorithm (Fig. 3).

Preliminary calculations. Before starting the solution of the structure's optimization problem (6) - (16), these preliminary operations and calculations were performed: 1. the finite element model of the structure was prepared; 2. the matrices of coefficients of equilibrium equations $[A_k]$, $k\in K$ of the separate finite elements and assemblage matrix $[A]$ of the whole structure were created; 3. the external VRL and dead loads of the structure were collected to the respective vectors and matrices $\boldsymbol{F}_{inf}$, $\boldsymbol{F}_{sup}$, $[F]$, $\boldsymbol{F}_c$ of the bounds and combinations of loads; 4. the vectors of the admissible bounds of displacements $\boldsymbol{u}_{min}$, $\boldsymbol{u}_{max}$ were prepared; 5. the total length vector $\boldsymbol{L}$ of the sets of structural elements and the element length vector $\boldsymbol{L}_{max}$ composed of the longest elements in the sets were created; 6. the edge values' vectors $\boldsymbol{\Pi}_{0,min}$, $\boldsymbol{\Pi}_{0,max}$, $\boldsymbol{\Pi}_{1,min}$, $\boldsymbol{\Pi}_{1,max}$ of the leading and driven biparameters were created.

Solving the optimization problem. To solve the optimization problem (6) - (16), we use an iterative approximation and begin with the highest geometrical values of the vectors $\boldsymbol{\Pi}_0=\boldsymbol{\Pi}_{0,max}$, $\boldsymbol{\Pi}_1=\boldsymbol{\Pi}_{1,max}$.

Step 1: the design parameters $\boldsymbol{\Pi}_0$ and $\boldsymbol{\Pi}_1$ are assigned to the respective cross-sections of the finite elements (initialization of the elements' cross-sections).

Step 2: the interpolation procedure of the theoretical inertia moments $I_{y,k}=I_y(\cdot)$, $k\in K_{31}$ of cross-sections is performed with respect to dispersion [17].

Step 3: the stiffness matrices $[K_k]$, $k\in K$ of the elements are recalculated, and the stiffness matrix $[K]$ of the whole structure is assembled.

Step 4: the influence matrices of the elastic internal forces $[\alpha]$ and elastic displacements $[\beta]$ are calculated.

Step 5: with reference to the conditions (3) the positive and negative members of the matrix $[\beta]$ are selected. The vectors of elastic displacements of the lower and upper values $\boldsymbol{u}_{e,inf}$, $\boldsymbol{u}_{e,sup}$ of the structure subjected to VRL are calculated. The vetor of elastic displacements $\boldsymbol{u}_{e,c}$ of the structure subjected to the dead load $\boldsymbol{F}_c$ is calculated.

Step 6: the matrices of yield-strength conditions $[\Phi_k]$, $[B_k]$, $k\in K$ of separate elements are recalculated and the whole structure matrices $[\Phi]$ and $[B]$ are assembled.

Step 7: the influence matrices of residual displacements and internal forces $[H]$, $[G]$ (19), (20) and derivative matrices $[Z]$, $[Y]$ (17) of the mathematical model (6) - (16) are prepared.

Step 8: the routine P1 solves one iteration of nonlinear mathematical programming optimization problem (6)–(16). If the procedure of solving is successful (i.e. optimal solution is found), we have a new vector $\boldsymbol{\Pi}_0^*$ of parameters and a new vectors $\boldsymbol{\lambda}_j^*$, $j\in J$ of plastic multipliers for every point of the locus. If the solution fails (i.e. the admissible point and optimal solution are not found), the leading geometry vector $\boldsymbol{\Pi}_0$ is increased recurrently:

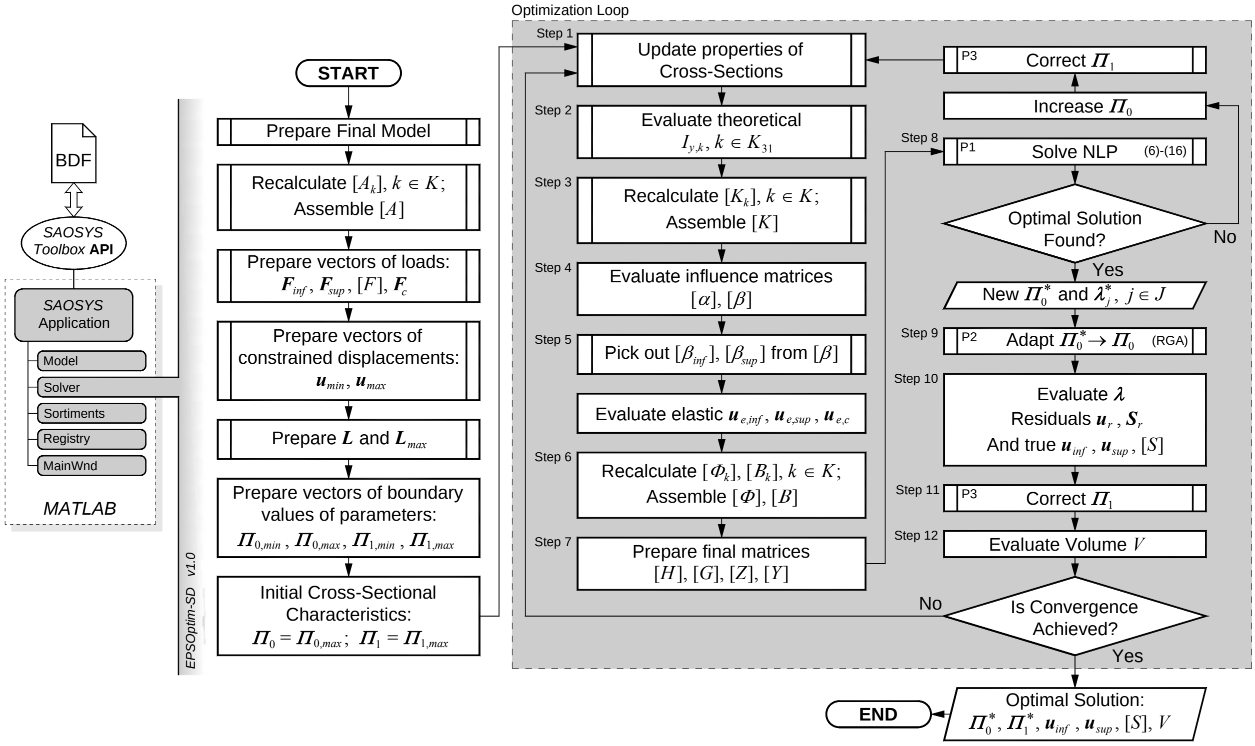

Fig. 3 The algorithm of optimal shakedown design of steel frame structures (SAOSYS/EPSOptim-SD solver)

$$\Pi_{0k} = \begin{cases} \frac{1}{2}\left(\Pi'_{0k} + \Pi_{0k}\right), & \frac{\Pi'_{0k} - \Pi_{0k}}{\Pi'_{0k}} \ge \varepsilon \\ \xi \Pi_{0k}, & \text{otherwise} \end{cases} \quad k \in K \qquad (23)$$

where $\Pi'_{0k} \equiv \Pi_{0k}^{*,\,prev}$ is the leading geometry vector of the previous successful iteration; ε is the relative threshold of recurrent Π_{0k} increase; ξ is the partial ratio of Π_{0k} direct increase. The routine P3 corrects the driven geometry vector of cross-sections $\boldsymbol{\Pi}_1$; then, we return to *Step 1*.

Step 9: the routine P2 performs the adaptation procedure of the leading geometry vector $\boldsymbol{\Pi}_0^*$ with respect to the strategy Recurrent-General-Aero (RGA) of optimization and convergence control.

Step 10: the vectors of the total plastic multipliers $\boldsymbol{\lambda}$, the residual displacements $\boldsymbol{u}_r$ and the residual internal forces $\boldsymbol{S}_r$ are calculated. The boundary vectors of true displacements $\boldsymbol{u}_{inf}$, $\boldsymbol{u}_{sup}$ and the matrix of combinations of true internal forces $[S]$ are calculated with reference to all points of the locus.

Step 11: the routine P3 performs a correction procedure of the driven geometry vector $\boldsymbol{\Pi}_1$ [17].

Step 12: the structure's volume V is calculated. This iterative process is performed until the convergence condition of the problem is satisfied.

4. Reconstructed SAOSYS system of structural modeling, analysis and optimal design

The system JWM SAOSYS Toolbox v0.47 for MATLAB environment is presented as a prototype of toolbox software for numerical analysis. Actually the system is intended for the analysis and optimal design of steel structures by the finite element method. The previous version of the architecture SAOSYS v0.42 was based on the database principle [15, 17]. When some improvements were made, it became a structurized hierarchical model of fully object-oriented components (Fig. 4). The reconstructed architecture of the system embraces: 1. the databases of system registers and steel profiles' assortments (*DBs*); 2. a general model of structure geometry and finite elements (*ModelSpace*); 3. the solving modules of analysis and optimal design problems (*Solvers*); 4. the graphical user interface (*GUI*). The above reconstruction of SAOSYS enabled us to apply new integration and development facilities widely used in information technologies now. In addition to the structural part of SAOSYS system, the graphical user interface was created, which can be used in structural modeling, the control of problem solution and graphical interpretation of the results of analysis (Fig. 1).

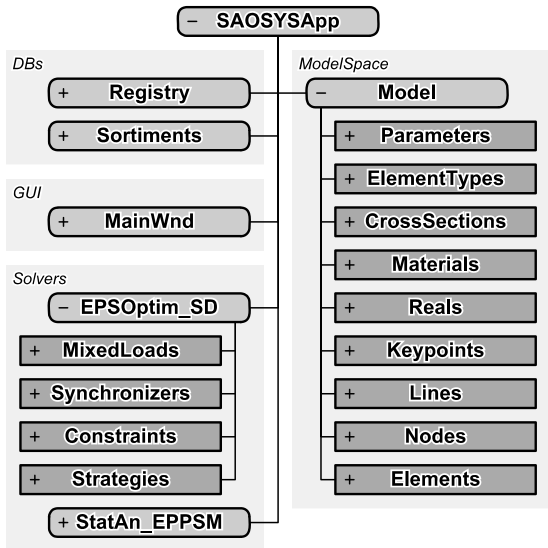

Fig. 4 Object model of partially expanded JWM SAOSYS Toolbox v0.47 architecture

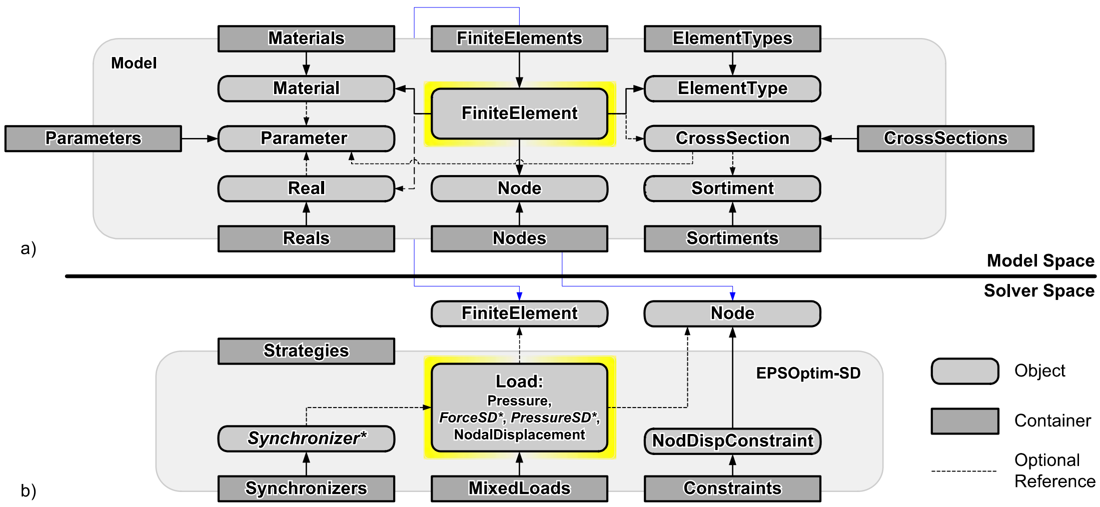

Fig. 5 Design pattern relationships: a) ideology of the finite element model in SAOSYS v0.47; b) the structure of EPSOptim-SD solver and its collaboration with the finite element model

A new general object model of the structure and the collaboration ideology of the components in the reconstructed SAOSYS v0.47 is shown in design pattern (Fig. 5, a). A general object model of the structure defined in this way gives the complete information about the structure and is ready for being directly used in the SAOSYS solution modules [30].

Also a new solution module EPSOptim-SD (Elastic-Plastic Structural Optimization at Shakedown) is specially created for design of the elastic-plastic steel structures subjected to VRL. The main components of this module (Fig. 5, b) are as follows: 1. the collection of loads *MixedLoads* is intended for collecting the information about loads (i.e. nodal and distributed loads, dead and varying repeated loads), acting on the structure's model (*Model*); 2. the synchronizers (*Synchronizers*) are intended for uniting and synchronizing separate VRLs into the load sets, which may be treated as a single effect (e.g. the modeling of wind effects); 3. the collector of information about stiffness requirements of the structure (the constrains of node displacements and deflections) (*Constraints*); 4. designed strategies (*Strategies*) of optimization, adaptation and convergence control of biparameters $\boldsymbol{\Pi}_0$ - $\boldsymbol{\Pi}_1$ in iterative algorithm.

5. A numerical example

Design structure. One-span industrial building frame is designed (Fig. 6): elastic-plastic stage; the case of varying repeated load. The frames are placed along the building at a distance of $L = 9.0$ m. The elements' material is steel S275: $E = 210$ GPa, $f_y = 275$ MPa.

The frame is subjected to three varying repeated loads ($\mu = 3$): 1. snow from the left $[s_{L,inf} = 0] \le s_L(t) \le [s_{L,sup} = 18.720]$ kN/m; 2. snow from the right $[s_{R,inf} = 0] \le s_R(t) \le [s_{R,sup} = 18.720]$ kN/m; 3. the united and synchronized wind loads from the left and the right (a set of effects):

$$[w_{L,inf}^{(1)} = -0.842] \le w_L^{(1)}(t) \le [w_{L,sup}^{(1)} = 1.685] \text{ kN/m;}$$

$$[w_{R,inf}^{(1)} = -1.685] \le w_R^{(1)}(t) \le [w_{R,sup}^{(1)} = 0.842] \text{ kN/m.}$$

In addition, the frame is subjected to the dead load of roofing $g_c = 2.340$ kN/m. The own weight of the structure is not evaluated.

The frame is modeled by using equilibrium finite elements (Fig. 6). It consists of 11 nodes, 17 finite elements and $n_{\Pi 0} = 8$ design parameters R_1 - R_8 (i.e. element cross-sections). The columns R_1 are designed from HE type profiles. The truss top chord R_2 is designed from IPE type of profiles, and the bottom chord and the grid $R_{3\div 8}$ – from RHS profiles.

The stiffness conditions of the structure are as follows: $|u_{7x,max\text{-}min}| \le 0.050$ m, $(u_{9y,max}, u_{10y,max}) \le 0.077$ m.

The structure of finite elements is modeled directly referring to the object-oriented application programming interface (API) of the system SAOSYS (Fig. 4). Thus, in MATLAB environment, the initial data and a batch file (BDF) is created for execution [30].

Optimization problem. A general characteristics of the optimization problem (6) - (16) of mathematical programming are as follows: nonlinear objective function; unknowns $n_x = 2^\mu \cdot n_\lambda + n_{\Pi 0} = 2^3 \cdot 130 + 8 = 1048$; linear constraints inequalities $n_{lieq} = 2092$. The optimization problem belongs to a group of nonlinear and non-convex mathematical programming problems (NLP) [8].

The results obtained. Structural design was performed by using an iterative procedure. In general, 16 iterations were made (Fig. 7). As a result, optimal biparameters $\boldsymbol{\Pi}_0^*$-$\boldsymbol{\Pi}_1^*$ of theoretical cross-sections were found. The profiles closest to them are presented in the table. The volume of the designed structure is $V = 0.2041$ m^3. It is worth noticing that discrete optimization of the structure, which was not described in this paper, is also required to find the optimal discrete solution [13, 29].

The displacements of the structure $u_{7x,inf} = -0.045$ m, $u_{7x,sup} = 0.045$ m, $u_{9y,sup} = u_{10y,sup} = 0.077$ m (Fig. 8) show that stiffness requirements for the nodes of the structure are satisfied. An envelope diagram of bending moments $M(t)$ of the structure subjected to VRL effect shows the distribution character of bending moments with reference to all points of the locus (Fig. 9). The diagram of strength reserve of structural elements (Fig. 7) shows the location of plastic hinges. The elements $E\{7, 9, 11, 16\}$ are designed under strength reserve state (i.e. plastic flow, or stability loss in the case of LINK11 can be observed). The strength reserve of the elements $E\{8, 13, 14\}$ is below 5.0 % (the plastic hinges have not been formed yet).

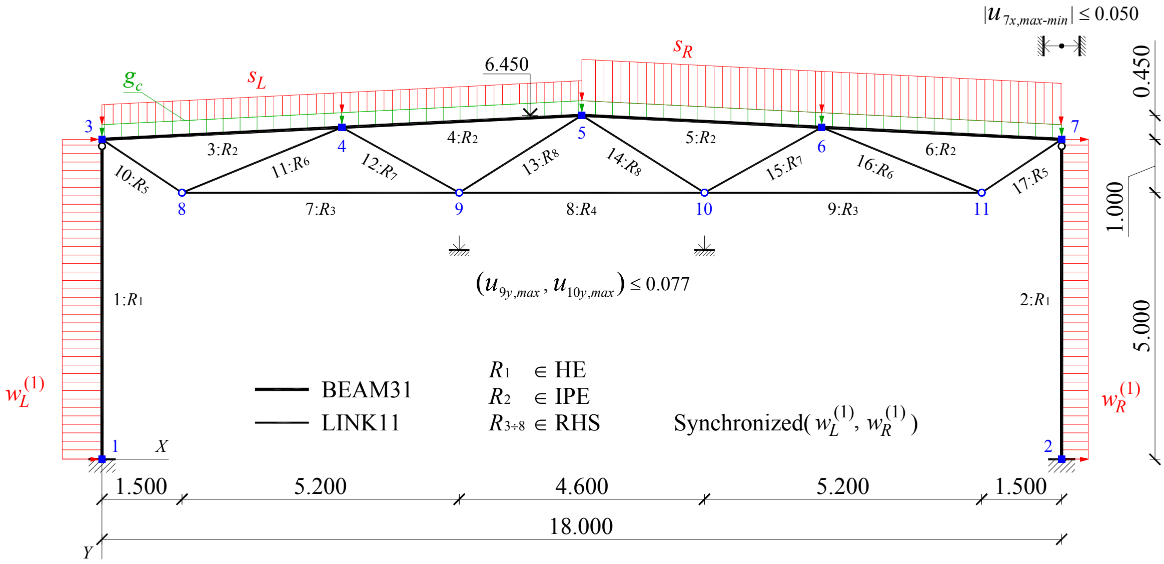

Fig. 6 A structural model discretized by BEAM31 and LINK11 finite elements

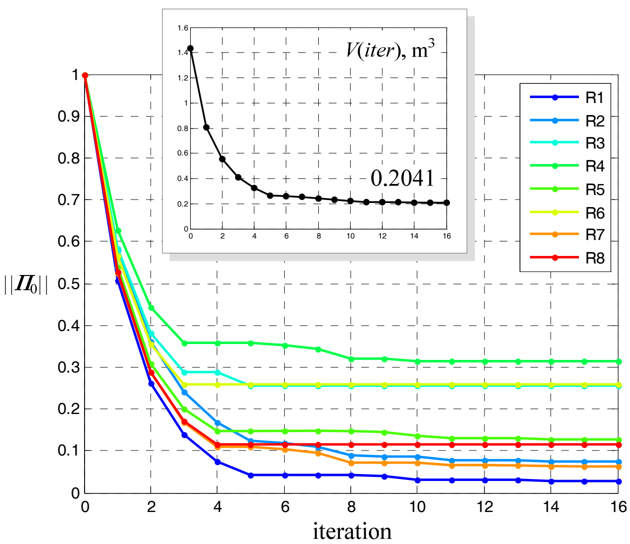

Fig. 7 Variation of $||\Pi_0||$ and volume V in iterative calculations

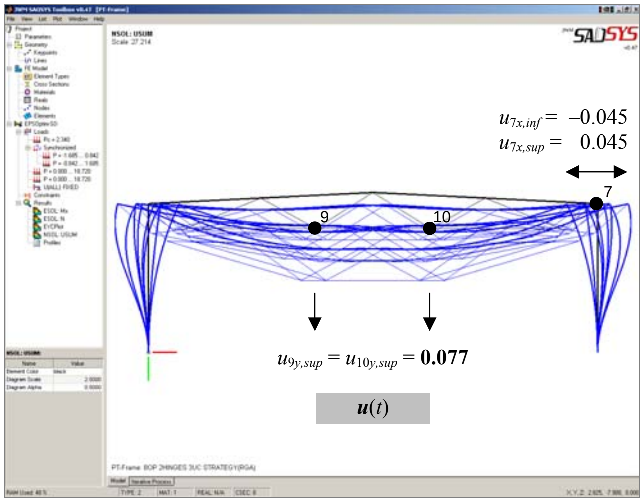

Fig. 8 The diagram $\boldsymbol{u}(t)$ of displacements of the structure subjected to VRL [m]

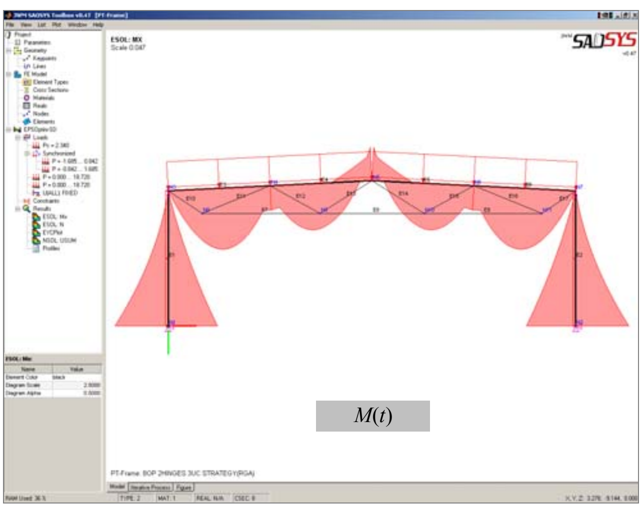

Fig. 9 The envelope diagram of bending moments $M(t)$ of the structure subjected to VRL

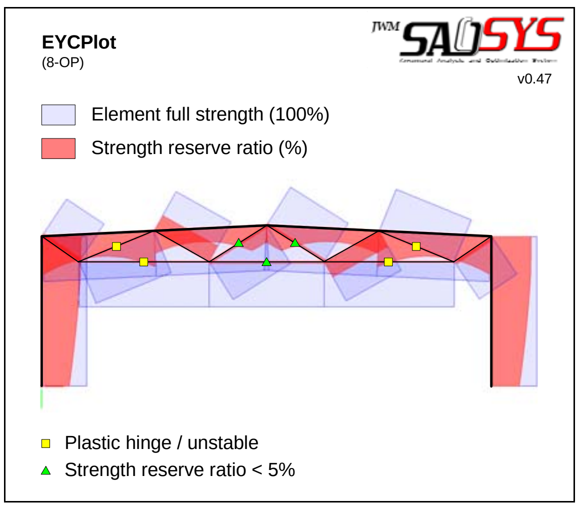

Fig. 7 Strength reserve diagram of structural elements

Table

The calculated optimal theoretical cross-sections

R_i	$\boldsymbol{\Pi}_0^*$	$\boldsymbol{\Pi}_1^*$	Closest profiles
	$W_{pl,y0k}\cdot 10^{-4}$ m^3	$A_k\cdot 10^{-3}$ m^2	
1	5.255	5.585	HE: 240AA, 240AA, 220AA
2	5.253	4.554	IPE: A300, 270, O240
	$A_{0k}\cdot 10^{-3}$ m^2	$I_{min,k}\cdot 10^{-7}$ m^4	RHS:
3	1.886	5.534	40×80×3, 30×70×6, 50×70×3.5
4	2.290	8.687	40×90×3.5, 40×80×5, 60×80×3.5
5	0.923	1.173	30×42×4, 35×50×2, 30×45×3.5
6	1.956	30.860	60×120×5, 120×80×4, 40×120×7
7	0.833	0.952	30×45×2.5, 20×45×3.5, 30×42×3
8	0.855	4.299	30×70×4, 40×60×5, 40×70×3.5

6. Conclusions

1. The complementary slackness conditions of mathematical programming does not allow evaluation of possible unloading phenomena at cross-sections of the structure and nonmonotonic variation of residual displacements. Thus, the optimal shakedown design problem is not a traditional mathematical programming problem, i.e. during the solution process, it is necessary to check stiffness conditions to determine lower and upper bounds of residual and elastic displacements.

2. The biparametric optimization principle of the admissible fields Ω of geometric characteristics of the discrete assortment profiles (the optimized leading geometry $\boldsymbol{\Pi}_0$ and the controlled driven geometry $\boldsymbol{\Pi}_1$) allows the design of the elements depending on the dispersion of geometric characteristics of the profile sets in assortments.

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BIPARAMETRIS PLIENINIŲ KONSTRUKCIJŲ PROJEKTAVIMAS PRISITAIKOMUMO SĄLYGOMIS

R e z i u m ė

Sudarytas prisitaikančiųjų strypinių plieninių konstrukcijų minimalaus tūrio nustatymo biparametrio optimizavimo uždavinio matematinis modelis. Atsižvelgiama ne tik į prisitaikomumo ir stabilumo (santvarinė dalis) ribojimo sąlygas, bet ir į konstrukcijos standumo reikalavimus (ribojant įlinkius arba poslinkius). Straipsnyje konstrukcijų tūrio optimizavimo uždavinys sudaromas remiantis deformuojamo kūno mechanikos ekstreminiais energiniais principais, tampriųjų plastinių konstrukcijų prisitaikomumo ir matematinio programavimo teorijomis. Diskretizacijai naudojami pusiausvirieji strypiniai baigtiniai elementai su įrąžų interpoliacijos funkcijomis. Remiantis optimizuojamų biparametrų principu, elementai projektuojami iš sortimentinių plieninių profiliuočių HE, IPE, RHS, atsižvelgiant į profiliuočių geometrinių charakteristikų sklaidą sortimentuose. Biparametris plieninių konstrukcijų projektavimas realizuojamas autorių MATLAB aplinkoje sukurtąja įrankių sistema JWM SAOSYS Toolbox v0.47. Straipsnyje pristatomas naujas EPSOptim-SD skaičiavimo modulis. Sistemos SAOSYS galimybės atskleidžiamos pramoninio rėmo su stiprumo, stabilumo ir standumo norminiais apribojimais projektavimo pavyzdžiu. Netiesiniai optimizacijos uždaviniai sprendžiami atsižvelgiant tik į mažų poslinkių prielaidas.

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BIPARAMETRIC SHAKEDOWN DESIGN OF STEEL FRAME STRUCTURES

S u m m a r y

The paper presents a mathematical model created for solving the biparametric optimization problem of minimal volume design steel frame structures at shakedown. The shakedown and stability (for a part of the truss) constraints-conditions as well as the structure's stiffness requirements (i.e. the restriction of displacements and deflections) are evaluated. Extreme energy principles of the deformable body mechanics, as well as shakedown and mathematical programming theories of elastic-plastic structures are used in the work for creating the structure's volume optimization problem. Discretization is based on equilibrium finite elements with interpolation functions of internal forces. The elements are designed using HE, IPE, RHS steel profile assortments and considering dispersion of geometrical characteristics of profile assortment sets by principle of design biparameters. Biparametric design of steel structures is realized by using the tool system JWM SAOSYS Toolbox v0.47 created by the authors in MATLAB environment. A new analysis module EPSOptim-SD is also presented. The possibilities of the system SAOSYS are demonstrated by a numerical example of industrial building frame design with standard strength, stability and stiffness constraints. The assumption of small displacements is adopted in optimization of nonlinear problems.

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БИПАРАМЕТРИЧЕСКОЕ ПРОЕКТИРОВАНИЕ СТАЛЬНЫХ КОНСТРУКЦИЙ В УСЛОВИЯХ ПРИСПОСОБЛЯЕМОСТИ

Р е з ю м е

Построена математическая модель задачи бипараметрической оптимизации минимального объема стальных стержневых конструкций в условиях приспособляемости. Учитывается не только условия ограничения приспособляемости, но и условия жесткости (ограничения на перемещения или прогибы). Приведенная задача рассматривается на основе экстремальных энергетических принципов механики твердого тела и основных положений теории приспособляемости упругопластических конструкций с привлечением математического программирования. Дискретизация конструкций проводится стержневыми равновесными конечными элементами с функциями интерполяции внутренних сил. Используя принцип оптимизируемых бипараметров, элементы проектируются из стальных профилей HE, IPE, RHS с учетом разброса дискретных характеристик в ассортиментах. Бипараметрическое проектирование стальных конструкций реализовано в среде MATLAB, при помощи созданной авторами прикладноинструментальной системой JWM SAOSYS Toolbox v0.47. Представляется новый модуль EPSOptim-SD для решения такого типа задач. Возможности системы SAOSYS раскрываются на примере проектирования однопролетной рамы производственного здания с ограничениями приспособляемости и жесткости. Исследования проведены с учетом предположения о малых перемещениях.

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