ISSN 1392 - 1207. MECHANIKA. 2012 Volume 18(6): 657-665

The phase portrait of the vibro-impact dynamics of two mass particle motions along rough circle

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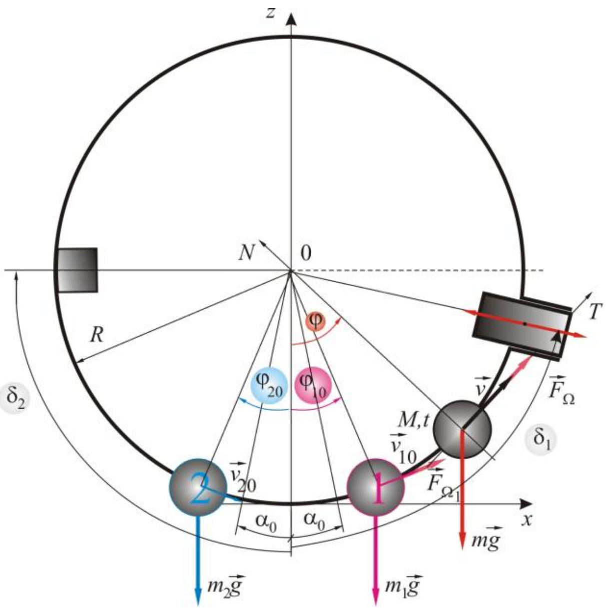
crossref http://dx.doi.org/10.5755/j01.mech.18.6.3159

1. Introduction

Nonlinear phenomena in the presence of certain discontinuity represent the area of interest of numerous researchers from all over the world. Theoretical knowledge of vibro-impact systems (see references [1-3]) are of particular importance to engineering practice because of the wide application of vibro-impact effects, used for the realization of the technological process. The analysis of mathematical pendulum with and without “turbulent” attenuation and papers published by Katica (Stevanovic) Hedrih [4, 5] related to the heavy mass particle motion along the rough curvilinear routes are the basis of this work. Based on the original works from the area of non-linear mechanics, or vibro-impact systems by the authors: František Peterka [6-8], Katica (Stevanovic) Hedrih [9], and the others, and the previous works of the authors of this paper [10-15] in which the authors analyzed several variants of vibro-impact system with one degree of freedom, based on the oscillator moving along a rough circle, sliding Coulomb-type friction and limited elongation, in this paper the vibro-impact system with two degrees of freedom, based on forced oscillations of two heavy mass particles, mass m_1 and m_2 moving along rough circle in vertical plane, sliding Coulomb-type friction and limited elongation is studied (Fig. 1).

The elongation limiter is set on the right. The limiter position is determined by the angle δ_1, measured from the equilibrium position of the mass particles, i.e. from the vertical line crossing the centre of the circular line. The system consists of two mass particles, m_1 and m_2, exposed to the effect of gravity. These mass particles are moving along rough circle in vertical plane on which the two sided impact limiters of elongation (constraints) were placed. The limiter position is determined by the angle δ, measured from the equilibrium position of the mass particles, i.e. from the vertical line crossing the centre of the circular line. The limiter set on the right side from the equilibrium position, defined by the angle δ_1 is stable. The first mass particle is affected by the external periodic force $F_1(t) = F_{\Omega_1} = F_{10}\cos\Omega_1 t$, where F_{10} is the corresponding force amplitude, and Ω_1 is the frequency of external force. The angular elongations of the first and second mass particles in an arbitrary moment t were marked by φ_1, φ_2 respectively, and measured from the equilibrium position. At the initial moment of time the material points were on the distances φ_{10}, φ_{20} from the equilibrium position 0-0, and were given the initial angular velocities, $\dot{\varphi}_{10}$, $\dot{\varphi}_{20}$.



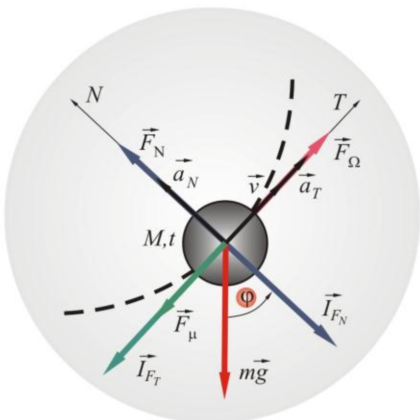


Fig. 1 System with two elongation limiters, based on oscillator with two pellets a – initial position mass particles, b – force diagram

The task is to consider the properties of forced oscillation of the first and second mass particles in a circular rough line with limited elongations, so the system becomes vibro-impact with one sided limited angular elongation. The differential equations of motion of the mass particles are requested for each interval of motion from impact to impact, from collision to collision, and the interval of motion when the friction force direction alternation appears associated with the direction alternation of angular velocity of motion of a mass particle, and also velocity alternation as a consequence of the mass particle impact into the angular elongation limiter and mutual impact of the mass particles.

Differential equations are matched to the initial motion conditions, system elongation limitation conditions, the mass particles impact conditions, and alternation conditions of friction force direction. Also, it was necessary to determine the impact conditions of both mass particles separately, the phase trajectory equations in phase planes and the mass particles collision conditions in ideally elastic impacts. Determine after how many impacts the system will stop behaving as vibro-impact system?

2. Differential equation of oscillations of a mass particle moving along rough circle

The observed vibro-impact system has two degrees of freedom, so the corresponding governing nonlinear differential equations of motion presented as

$$\ddot{\varphi}_1 \pm \dot{\varphi}_1^2 tg\alpha_0 + \frac{g}{R\cos\alpha_0}\sin(\varphi_1 \pm \alpha_0) = = \frac{F_{10}}{m_1 R}\cos\Omega_1 t \begin{cases} for \ \dot{\varphi}_1 > 0 \\ for \ \dot{\varphi}_1 < 0 \end{cases} \tag{1}$$

$$\ddot{\varphi}_2 \pm \dot{\varphi}_2^2 tg\,\alpha_0 + \frac{g}{R\cos\alpha_0}\sin(\varphi_2 \pm \alpha_0) = 0 \begin{cases} for \ \dot{\varphi}_2 > 0 \\ for \ \dot{\varphi}_2 < 0 \end{cases} \tag{2}$$

for $\mu = tg\alpha_0$ is sliding Coulomb-type friction coefficient, φ_1, φ_2 are generalized coordinates for monitoring motion of the first and second mass particles.

This system of double differential non-linear equations is coupled by initial motion conditions:

a) the first mass particle (pellet 1), in further text is marked with subscript 1

$$\varphi_{1(0)} = \varphi_{10} \quad \text{and} \quad \dot{\varphi}_{1(0)} = \dot{\varphi}_{10} \ ; \tag{3}$$

b) the second mass particle (pellet 2), in further text is marked with subscript 2

$$\varphi_{2(0)} = \varphi_{20} \quad \text{and} \quad \dot{\varphi}_{2(0)} = \dot{\varphi}_{20} \ . \tag{4}$$

At the initial moment of motion, the mass particles were given the positive initial angular velocity $(\dot{\varphi}_1 > 0, \dot{\varphi}_2 > 0)$.

For the complete description of the observed vibro-impact system are needed to be set, and also matching of limitation conditions angular elongations and impacts to the elongations limiters.

$$\left.\begin{aligned} &\varphi_{1ul_i} = \delta_1 \ \text{ or } \ \varphi_{1ul_i} = 2\pi - \delta_2 \\ &\varphi_{2ul_i} = \delta_1 \ \text{ or } \ \varphi_{2ul_i} = \delta_2 \\ &\dot{\varphi}_{odl_i} = -k\dot{\varphi}_{ul_i}, \ i = 1,2,3,...,n \end{aligned}\right\} \tag{5}$$

where k is coefficient of collision (impact), within the interval from $k = 0$ for ideal plastic collision to $k = 1$ for ideal elastic collision (impact), and n is the number of impacts until the system is returned into the equilibrium position.

The differential equation of motion of the second pellet (2) can be solved in analytical form, so its first integer is phase trajectory equation in form of

$$\dot{\varphi}_2^2(\varphi) = \frac{2g}{(1+4tg^2\alpha_0)R\cos\alpha_0} \times \times\left[\cos(\varphi_2 \pm \alpha_0) - 2tg\,\alpha_0 \sin(\varphi_2 \pm \alpha_0)\right] + Ce^{\mp 2\varphi_2 tg\alpha_0},$$

where C is integration constant depending on the initial motion conditions.

For graphic presentations of the phase trajectories in the individual motion intervals of the second mass particle we use software package MathCad 14.

The differential equation of motion of the first heavy mass particle (1) cannot be solved explicitly (in a closed form). For its approximate solution the software package WOLFRAM Mathematica 7 is used. The results are checked by using software package MATLAB R2008a.

3. Motion analysis of the vibro-impact system

The operational system of the mobile angular elongation limiter, which is positioned on the right side from the equilibrium position is based on the fact that the system is pulled by the impact of the pellet, and returned to the initial position by the impact of the pellet into the elongation limiter set on the left side from the equilibrium position. This system creates the motion of:

The first mass particle, mass m_1, within the interval from the impact to the second mass particle , mass m_2, to the impact into elongation limiter set on the right side (δ_1), or to the impact into angular elongation limiter set on the left side ($2\pi - \delta_2$), or, to the first mass particle motion alternation (when it happens). There is a possibility for the first pellet to have an impact into elongation limiter to the left ($2\pi - \delta_2$), then itreaches the alternation point and hits again into the same elongation limiter.

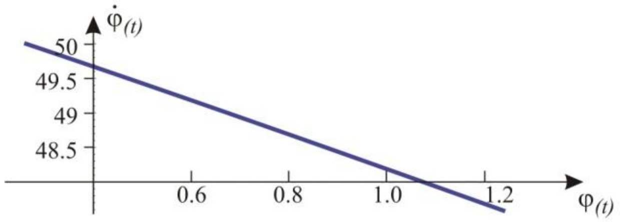
Motion the second mass particle, mass m_2 is in the interval from the impact with the first mass particle , mass m_1, to the impact into angular elongation limiter set on the left (δ_2), or to the impact into angular elongation limiter set on the right (δ_1), i.e. to the second mass particle motion direction alternation (when it happens). There is a possibility that the second mass particle has an impact into elongation limiter to the left (δ_2) reaches the alternation point, and hits again into the same elongation limiter.

For the determination of phase portrait branches of the first and second heavy mass particle individually, the motion of the heavy mass particles along rough circle line is divided into corresponding motion intervals and subintervals.

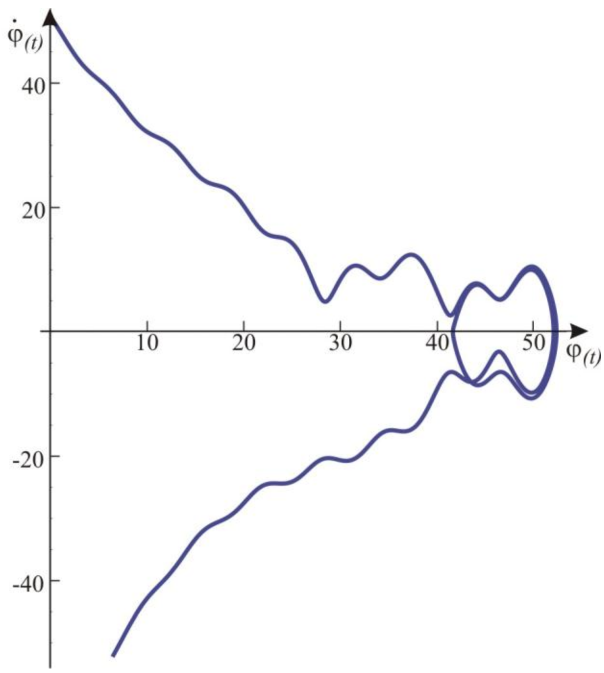
The first pellet – the first motion interval represents the interval from the initial time until the first impact of the pellet 1 into the angular elongation limiter on the right side.

The first motion interval of the first mass particle corresponds to the differential Eq. (1) of motion for $\dot{\varphi}_1 > 0$.

The impact conditions are $t = t_{ul_1 -}$, $\varphi_1\left(t_{ul_1 -}\right) = \delta_1$, $\dot{\varphi}_1\left(t_{ul_1 -}\right) = \dot{\varphi}_{1ul_1 -}$.



a



b

Fig. 2 Phase trajectory curve for the first mass particle in the first motion interval: a - in time t = 0.02 s, b - in time t = 12 s

Phase trajectory $\dot{\varphi}_{11} = f\left(\varphi_1\right)$ in the first motion interval (that will be used for the determination of the velocity of the mass particle impact into the angular elongation limiter) defined by using the software package Wolfram Mathematica 7 (also used for all other graphic presentations) is presented in Fig. 2.

The parameter values are: $\delta_1 = \frac{\pi}{4}[\text{rad}]$, $R = 0.5[\text{m}]$, $\varphi_{10} = \frac{\pi}{12}[\text{rad}]$, $\dot{\varphi}_{10} = 50\left[\frac{\text{rad}}{\text{s}}\right]$, $\alpha_0 = 0.05$, $m_1 = 0.2[\text{kg}]$, $g = 9.81\left[\frac{\text{m}}{\text{s}^2}\right]$, $F_{10} = 1.2[\text{N}]$ and $\Omega_1 = 3.3\left[\frac{\text{rad}}{\text{s}}\right]$.

The angular velocity of the first mass particle into elongation limiter $\left(\dot{\varphi}_{1ul_1}\right)$ is read from the phase trajectory, presented in Fig. 2, *a*. The time interval of the first heavy mass particle impact into elongation limiter $\left(t_{1ul_1}\right)$ is determined by using software package MATLAB R2008a. Both values ($\dot{\varphi}_{1ul_1}$ and t_{1ul_1}) are taken for the angle where the elongation limiter is positioned.

The second pellet – the first motion interval represents the interval from the initial moment until the first collision of the pellet 2 to the pellet 1.

The first motion interval of the second mass particle corresponds to the differential Eq. (2) of motion for $\dot{\varphi}_2 > 0$, matching the initial conditions (2).

The phase trajectory $\dot{\varphi}_{21} = f\left(\varphi_2\right)$ in the first motion interval is presented in Fig. 3.

The parameter values are: $\varphi_{20} = -\frac{\pi}{12}[\text{rad}]$, $\dot{\varphi}_{20} = 6\left[\frac{\text{rad}}{\text{s}}\right]$, $R = 0.5[\text{m}]$, $\alpha_0 = 0.05$, $m_2 = 0.2[\text{kg}]$, $g = 9.81\left[\frac{\text{m}}{\text{s}^2}\right]$.

The second pellet's phase trajectory, shown in Fig. 3, b, points on the periodic motion of the second pellet.

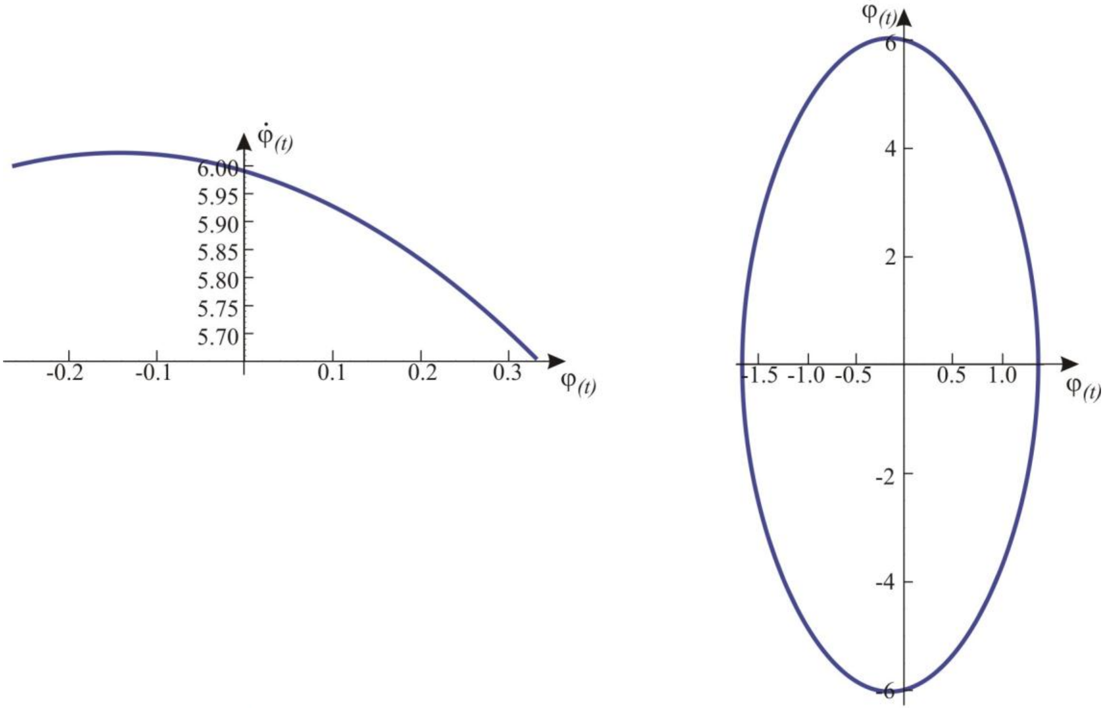
For the further study it was necessary to define the position of the pellet 2 when the pellet 1 reaches the elongation limiter. The position of the second pellet $\varphi_2\left(t_{1ul_1}\right)$ in time t_{1ul_1} is determined by using MATLAB-u R2008a.

After the definition of the position $\varphi_2\left(t_{1ul_1}\right)$ angular velocity of pellet 2 at the moment when from pellet 1 reaches the elongation limiter can be read from graphic presentation of the phase trajectory $\dot{\varphi}_{21} = f\left(\varphi_2\right)$ (presented in Fig. 3, a).

The mass particles have an impact in the second motion interval of the first mass particle and in the first motion interval of the second mass particle.

The first mass particle – the second motion interval is an interval from the first impact into elongation limiter to the first heavy mass particles collision.

The second motion interval of the first mass particle corresponds to the differential equation of motion in form of (1) for $\dot{\varphi}_1 < 0$, matched to the initial conditions of motion $t_1 = t_{1ul_1+}$, $\varphi_1\left(t_{1ul_1+}\right) = \delta_1$, $\dot{\varphi}_1\left(t_{1ul_1+}\right) = \dot{\varphi}_{1odl_1} = -\dot{\varphi}_{1ul_1-}$.

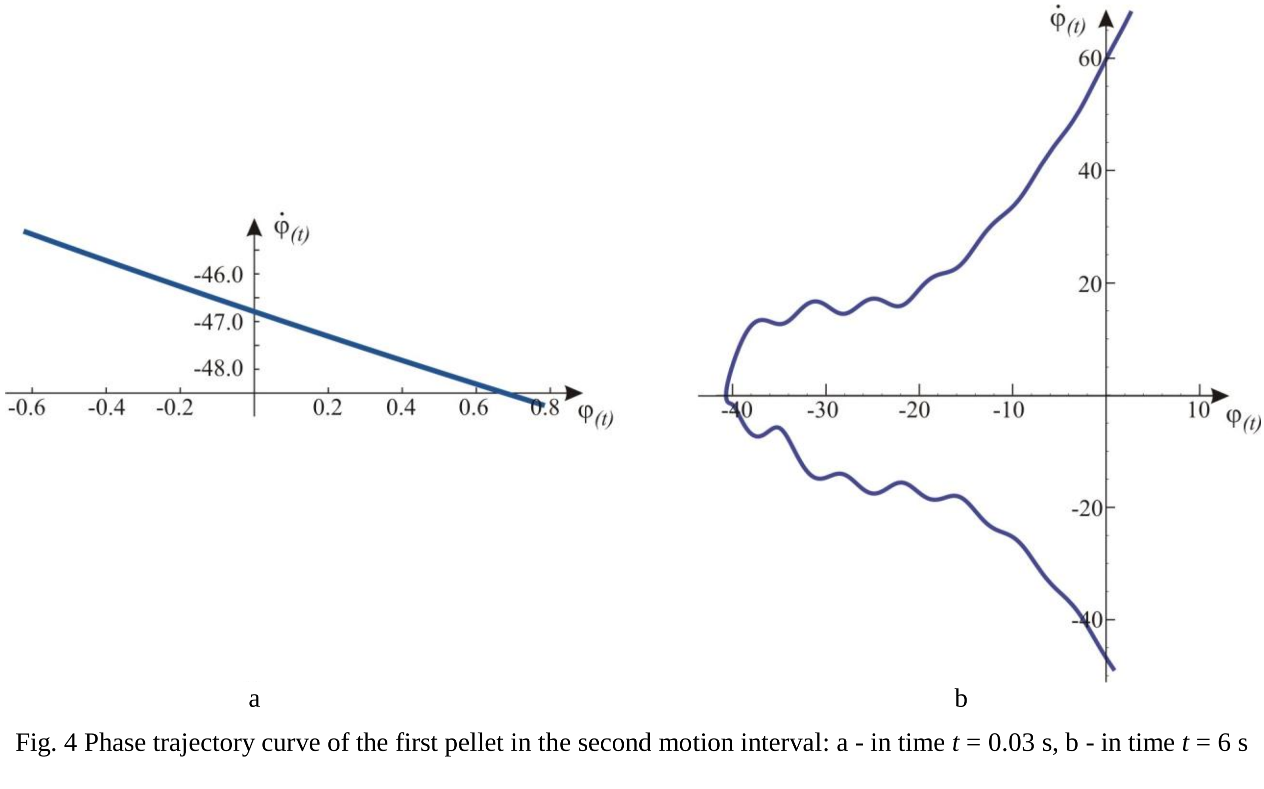


a b

Fig. 3 Phase trajectory curve of the second mass particle in the first motion interval: a - in time t = 0.1 s, b - in time t = 10 s

Phase trajectory $\dot{\varphi}_{12} = f\left(\varphi_1\right)$ in the second motion interval is presented in Fig. 4.

Further analysis is based to the definition of time interval in which the first collision occurs. After the definition of the moment of the first impact t_{sud_1} the angle φ_{sud_1} is determined as a basis for further motion analysis of the pellets.



a b

Fig. 4 Phase trajectory curve of the first pellet in the second motion interval: a - in time t = 0.03 s, b - in time t = 6 s

The initial data for the determination of the time interval t_{sud_1} are:

a) for the first pellet $\varphi_1\left(t_{1ul_1}\right)=\delta_1$, $\dot{\varphi}_1\left(t_{1ul_1}\right)=\dot{\varphi}_{1odl_1}=-\dot{\varphi}_{1ul_1}$, $a_{T,12}=R\ddot{\varphi}_1$;

b) for the second pellet $\varphi_2\left(t_{1ul_1}\right)$, $\dot{\varphi}_2\left(\varphi_2\left(t_{1ul_1}\right)\right)$, $a_{T,21}=R\ddot{\varphi}_2$.

NOTE: In the previous expressions the indexes *ij* present: *i* - pellets number; *j* - motion interval number.

The condition for the time t_{sud_1} definition is

$$\left.\begin{array}{l} s=s_{12}\left(t_{sud_1}\right)+s_{21}\left(t_{sud_1}\right) \quad \text{or} \\ \left(\delta_1-\varphi_2\left(t_{1ul_1}\right)\right)=\varphi_{12}\left(t_{sud_1}\right)+\varphi_{21}\left(t_{sud_1}\right) \end{array}\right\} \tag{6}$$

Time t_{sud_1} is determined from the relation

$$t_{sud_1}^2+\frac{2\left(\dot{\varphi}_{1ul_1}+\dot{\varphi}_2\left(\varphi_2\left(t_{1ul_1}\right)\right)\right)}{\left(\ddot{\varphi}_{12}+\ddot{\varphi}_{21}\right)}t_{sud_1}-\frac{2\left(\delta_1-\varphi_2\left(t_{1ul_1}\right)\right)}{\left(\ddot{\varphi}_{12}+\ddot{\varphi}_{21}\right)}=0 \tag{7}$$

The accelerations $\ddot{\varphi}_{12}$ and $\ddot{\varphi}_{21}$ were approximately determined (with sufficient accuracy) as the median value of the average accelerations in the sub-intervals of the observed interval. In this case the interval $\left(\delta_1-\varphi_2\left(t_{1ul_1}\right)\right)$ is divided into six equal sub-intervals.

For the obtained value of t_{sud_1} with the corresponding program files from MATLAB R2008a for the second motion interval of the first pellet and the first motion interval of the second pellet (values must match) the angle of the first impact φ_{sud_1} is determined.

Values for the angle φ_{sud_1} can be used for the determination of angular velocities of the pellets 1 and 2 immediately before the first impact from the phase trajectories for the second motion interval of the first pellet (Fig. 4, *a*) and the first motion interval of the second pellet (Fig. 3, *a*) i.e. $\dot{\varphi}_{1sud_1,ul}$ and $\dot{\varphi}_{2sud_1,ul}$.

The mass centres of particles are positioned on the rough circle line, i.e. the impact centres are positioned on the same axes. This is about central impact.

The expressions for explicit definition of the angular velocities immediately after the impacts with using Law of momentum and Newton's hypothesis about the relation of relative angular velocities of the mass particles are

$$\left.\begin{array}{l} \dot{\varphi}_{2sud_1,odl}=\dfrac{m_1\left(1+k\right)}{m_1+m_2}\dot{\varphi}_{1sud_1,ul}-\dfrac{m_2-km_1}{m_1+m_2}\dot{\varphi}_{2sud_1,ul} \\ \dot{\varphi}_{1sud_1,odl}=\dfrac{km_2-m_1}{m_1+m_2}\dot{\varphi}_{1sud_1,ul}+\dfrac{m_2\left(k+1\right)}{m_1+m_2}\dot{\varphi}_{2sud_1,ul} \end{array}\right\} \tag{8}$$

The generalized coordinate φ_{sud_1} where the first impact appears and velocities of the pellets immediately after the collision $\dot{\varphi}_{1sud_1,odl}$, $\dot{\varphi}_{2sud_1,odl}$ are the initial conditions of motion of the pellets in the following motion intervals.

The motion analysis of the observed vibro-impact system is conducted up to the twelfth impact of pellets 1 and 2. It should be mentioned that until the fourth impact of the pellets, the pellets are moving in zone from δ_1 to δ_2. From the fourth to the twelfth impact, the pellets are moving in zone from δ_1 to $2\pi-\delta_2$. After the twelfth impact of the pellets, the motion zone is divided, so the first pellet is moving in zone $\left(\delta_1\right)$ - $\left(2\pi-\delta_2\right)$, and the other pellet is moving within the zone $\left(\delta_1\right)$ - $\left(\delta_2\right)$. The first pellet influenced by the external single frequency force after the eighth impact into elongation limiter at the coordinate $\left(2\pi-\delta_2\right)$ does not have a strength to cross the limit π, i.e. the alternation point is positioned on the distance $t_{1,alt_i}>\pi$, that points out that the pellet will be still after several impacts at the coordinate $2\pi-\delta_2$. After the alternation point in zone $\left(\delta_1\right)$ - $\left(\delta_2\right)$, the second pellet has only two impacts into mobile elongation limiter set at the coordinate δ_1 which is not pulled inside at those moments. After the second impact, the second pellet continues to move without impacts and in several motion intervals returns into equilibrium position $\varphi_2=0$. The second pellet completed thirteen impacts into elongation limiter, three of them into stable, and ten of them into mobile elongation limiter.

The graphic visualization of the motion analysis, performed for the observed vibro-impact system based on oscillator moving along rough circle line, composed of two ideally smooth pellets is shown in Fig. 5 and Fig. 6. The phase portrait of the pellet 1 is shown in Fig. 5, and phase portrait of the pellet 2 is shown in Fig. 6.

4. Conclusions

Non-linearity of the observed vibro-impact system is due to the discontinuity of angular velocities of the mass particles moving along rough circle line. The discontinuities of angular velocities occur at the moment of impact of mass particle 1 into angular elongation limiters at the coordinate δ_1 and $\left(2\pi-\delta_2\right)$, at the moment of direction alternation of motion of the mass particles 1 and 2 (when it happens), causing the alternation of angular velocity direction and friction force alternation, and at the moment of impact (collision) of mass particles. This non-linearity is described for both mass particles by the system of regular non-linear differential equations, particularly by the second member, representing angular velocity square of the generalized coordinate $\dot{\varphi}_1^2, \dot{\varphi}_2^2$. That corresponds to the case of turbulent attenuation. It should be mentioned that in the observed vibro-impact system with two degrees of freedom we have trigger constrained singularities, i.e. we have bifurcation phenomena of the equilibrium positions due to the influence of the sliding Coulomb's friction force and the alternations of angular velocities direction of the mass particles.

For the individual motion intervals of the mass particles the differential equations of motion with matched

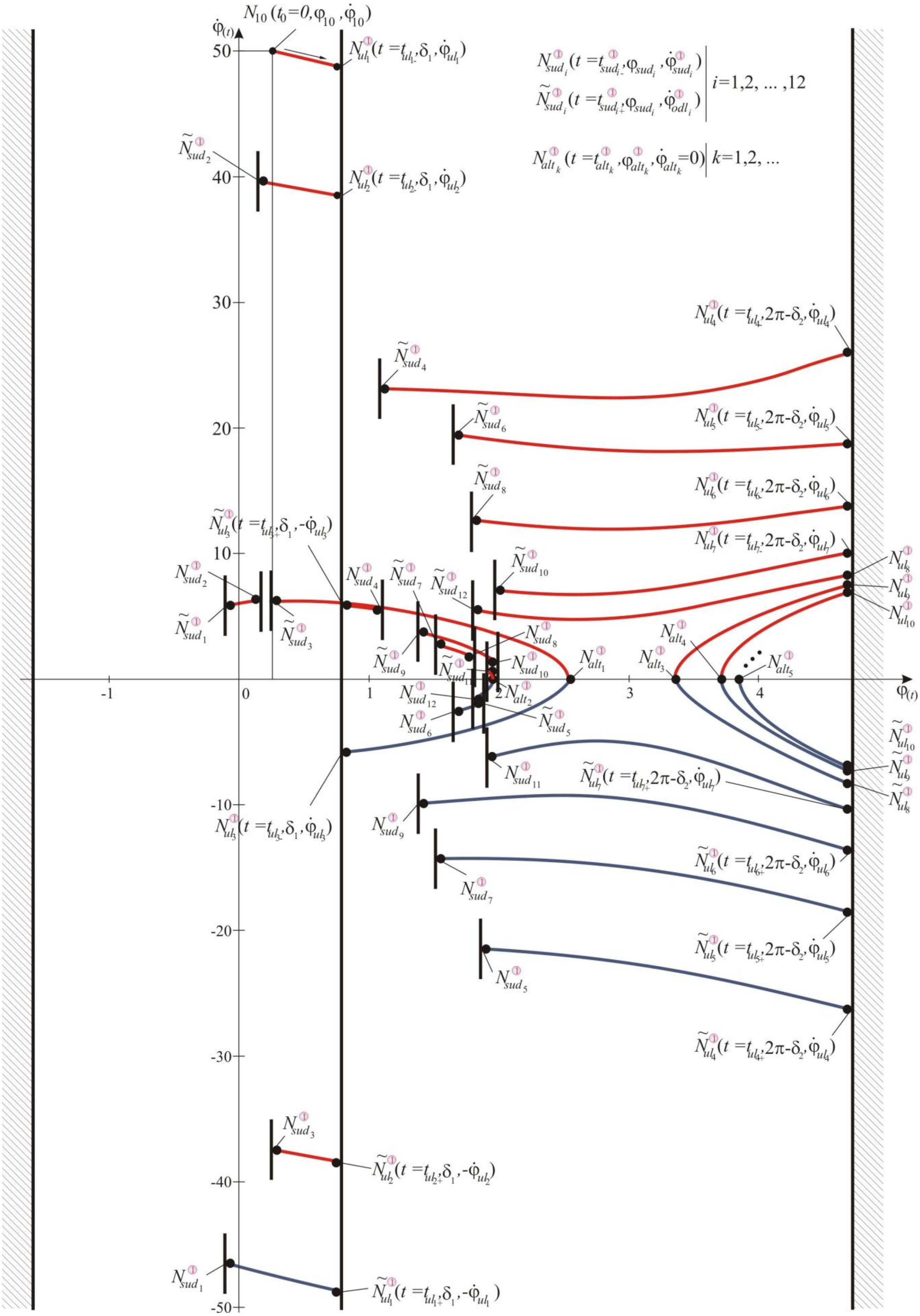


Fig. 5 Phase portrait of the pellet 1 (as a part of an oscillator) moving along rough circle line, with sliding friction force $\mu = tg\alpha_0$, with limited elongations in plane $(\varphi_1, \dot{\varphi}_1)$ under the influence of external single frequency force F_{Ω_1}

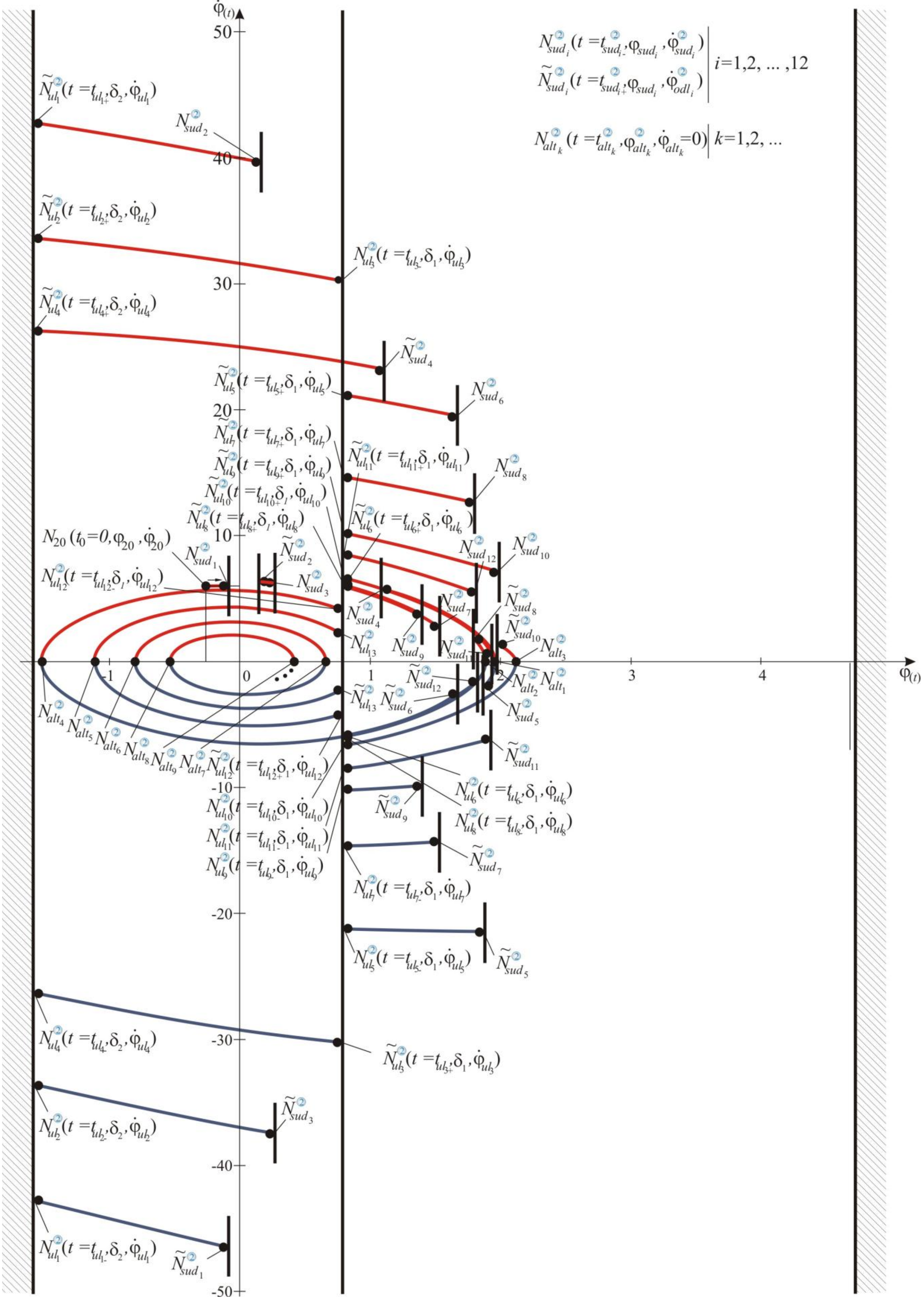


Fig. 6 Phase portrait of the pellet 2 (as a part of an oscillator) moving along rough circle line, with sliding friction force $\mu = tg\alpha_0$, with limited elongations in plane $\left(\varphi_2, \dot{\varphi}_2\right)$

initial motion conditions are written in this paper, related to the positions of the mass particles at the moment of impact into elongation limiters, at the moment of motion direction alternation and at the moment of collision of the mass particles.

It should be noted, that with mobile elongation limiter set in positions “on” and “off”, in the coordinate δ_1, after the twelfth impact of the pellets their zones of motion are separated. The first mass particle is calmed down at the position defined by the coordinate $\left(2\pi - \delta_2\right)$, and the second mass particle is back into the equilibrium position $\left(\varphi_2 = 0\right)$.

It should be noticed the methodology of the determination of time and position of the mass particles in the moment of collision. The outgoing velocities of the mass particles after the impacts are determined analytically and time of the impact, the position of the mass particles at the moment of collision, and the ingoing velocities are determined numerically. By the numerical solutions of the differential equations of motion (MATLAB R20008a and Wolfram Mathematica 7) by using the initial motion conditions, the graphic visualization of oscillations of the mass particles in the observed vibro-impact system with two degrees of freedom is given.

The phase portraits of mass particle 1 and mass particle 2 are obtained by the combination of analytical and numerical results in the procedure of producing of graphic interpretations of phase trajectories in the individual motion intervals of the mass particles, with the application of software programs MATLAB R20008a and Corel Draw 12. In these phase portraits there the phenomena of non-linearity of vibro-impact system with two degrees of freedom are clearly visible.

Acknowledgment

Parts of this research were supported by the Ministry of Sciences and Technology of Republic of Serbia through Mathematical Institute SANU Belgrade Grant ON174001 Dynamics of hybrid systems with complex structures. Mechanics of materials and Faculty of Technical Sciences University of Priština residing in Kosovska Mitrovica.

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DVIEJŲ KONCENTRUOTŲ MASIŲ VIBROSMŪGINIO JUDĖJIMO APSKRITIMINE NELYGIA TRAJEKTORIJA FAZINIS PORTRETAS

R e z i u m ė

Straipsnyje analizuojama, kaip juda vibrosmūginė sistema, susidedanti iš dviejų laisvės laipsnių švytuoklės, apskritimine nelygia trajektorija vertikalioje plokštumoje veikiant išorinei pastovaus dažnio jėgai. Ryšių netolygumą sukelia Kulono slydimo trinties koeficientas $\mu = tg\,\alpha_0$. Švytuoklė susideda iš dviejų masių – rutuliukų, kurių laisvas judėjimas apribotas dviem kampinio posūkio atramomis. Nesmūginis masių judesys, veikiant išorinei pastovaus dažnio jėgai, suskaidytas į atitinkamus intervalus, aprašomas dviem judesio diferencialinėmis lygtimis, kurios priskiriamos netiesinių homogeninių antros eilės diferencialinių lygčių grupei. Tiriamos vibrosmūginės sistemos judesio diferencialinės lygtys išspręstos naudojant programinius paketus. Tirtų vibrosmūginių sistemų analitiniai ir skaitmeniniai rezultatai yra judesio grafinio vizualizavimo, kuris ir yra šių analitinių tyrimų tikslas, pagrindas. Sukurta smūgio, vykstančio tam tikroje padėtyje apibrėžtame laiko intervale, įvertinimo metodologija.

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THE PHASE PORTRAIT OF THE VIBRO-IMPACT DYNAMICS OF TWO MASS PARTICLE MOTIONS ALONG ROUGH CIRCLE

S u m m a r y

The paper is based on the analysis motion of vibro-impact system based on oscillator with two degrees of freedom moving along the circular rough line in vertical plane under the influence of external single frequency force. Non-ideality of the bonds originates of the sliding Coulomb's type friction coefficient $\mu = tg\,\alpha_0$. The oscillator is composed of two mass particles-pellets, whose free motion is limited by two angular elongation limiter. Non-impact motion of the mass particles under the action of external single frequency force, divided into appropriate intervals, is described by two differential equations of motion which belong to a group of ordinary non-linear homogeneous second order differential equations. The differential equations of motion of the observed vibro-impact system are solved by using software packages. The combination of analytical and numerical results for the specific kinetic parameters of the observed vibro-impact systems is the basis for graphic visualization of motion which was the subject of this analytical research. The original contribution of this paper is in the form of established methodology of the process of determining time interval and position at the moment of collision.

Keywords: Two mass particles, rough circle, two impact limiters, pellet, vibro-impact, phase trajectory branches, graphical presentation, single frequency force.

Received August 01, 2011
Accepted November 15, 2012