ISSN 1392 - 1207. MECHANIKA. 2011. 17(4): 418-422

A sequencing approach of models in mixed-model assembly lines

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crossref http://dx.doi.org/10.5755/j01.mech.17.4.573

1. Introduction

Manufacturers nowadays are increasingly facing the challenge of providing a rich product variety at very low cost. This typically requires the implementation of cost efficient, flexible production systems [1, 2]. Often, so called mixed-model assembly lines are employed. However, the diversity of mixed-model lines makes a thorough sequence planning essential for exploiting the benefits of assembly line production [3 - 5]. To sequence mixed-models in assembly lines some criteria have been considered in the literature [6, 7], two main sequencing objectives are the Just-In-Time(which pursues the constant rate of part usage) and the leveling of work load. The criterion of minimizing work overload is more meaningful for some small-medium manufacturing companies.

The impacts of idle time and over time on the production costs are different in actual production,, so it is necessary to figure them separately. The company can estimate the production costs of waste on the assembly line accurately by the introduction of the cost factors. Our approaches are focused on the sequences of mixed-model assembly line that affect this aspect. Accordingly a sequence planning approach based on PSO is devised, and an immune mechanism is introduced into it. According to antibody affinity and concentration calculation, replace the particles timely in order to maintain the population diversity and prevent premature convergence and particles into local extreme. Finally, the algorithm is proved to be effective and superior through simulation examples. Although our study is inspired by a single real-world case, the underlying problem setting is highly relevant in practice.

2. Problem definition

A traditional assembly line consists of multiple stations arranged along some kind of transportation systems [8]. As shown in Fig. 1, the station in this paper is considered as a time-service window, where time is the length of the station. If the worker can not return to the left-hand border before the next workpiece has arrived, this finally results to a work overload whenever the operations of a workpiece can not be finished within the station's boundaries, and the operation time by other workers out of the line can be called overtime. On the other hand, if the next workpiece hasn't arrived after the worker return to the left-hand border, the time for the worker to wait for the workpiece can be called idletime.

In the proposed sequencing model we assume: The planning horizon is divided into R production cycles (with $r = 1, 2, 3, \ldots, R$) in a and for each model $m \in M$ the demand dm at the end of the planning horizon is given and has to be met. It follows that the sum over model demands is equal to the number of production cycles available

$$R = \sum_{m=1}^{M} d_m$$

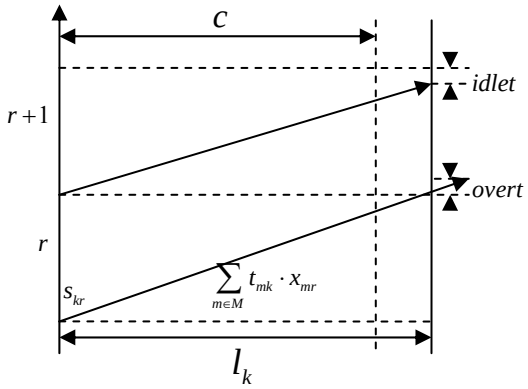

Fig. 1 Movement of workers in stations

The assignment decision is represented by binary variables $x_{mr} \in \{0,1\}$, $\forall m \in M, r = 1, 2, ..., R$, which indicate whether a copy of model is m is produced in cycle r. According to the definition of idletime and overtime on the assembly line, the sequencing model of minimizing total idletime and overtime cost can be described as follow

$$\min \quad f(k,r,m) = \sum_{k=1}^{K}\sum_{r=1}^{R}(\alpha\ idlet_{kr} + \beta\ overt_{kr}) \tag{1}$$

$$s.t. \quad s_{k,r+1} \geq s_{kr} + \sum_{m\in M} t_{mk}\ x_{mr} - C + idlet_{kr} \tag{2}$$

$$s_{kr} + \sum_{m\in M} t_{mk}\ x_{mr} - overt_{kr} \leq l_k \tag{3}$$

$$x_{mr} \in \{0,1\},\ \forall m \in M, r = 1, 2, \ldots\ R \tag{4}$$

$$\sum_{m\in M} x_{mr} = 1,\ \sum_{r=1}^{R} x_{mr} = d_m \tag{5}$$

$$s_{kt} \geq 0,\ \ s_{k1} = 0 \tag{6}$$

The objective function (1) minimizes the total cost on the line, and the $idlet_{kr}$ and $overt_{kr}$ indicate the idletime and overtime of the product r on the station k. α and β are the cost trade-off parameters, which can be used to adjust the impact from different time cost, satisfied that $\alpha + \beta = 1$. The constraints (2) guarantee that processing of a model copy in cycle $r + 1$ by station k can not start before this station has completed the preceding unit in cy-

cle r. Work is restricted to the stations' borders by constraints (3). Constraints (4) and (5) ensure that there must be a definite product in each sequence position, the total quantity demand of the product must be met. Some parameters are initialized in constraints (6).

3. The immunity particle swarm optimization algorithm

The original PSO maintains a population of particles, let M be the size of swarm. For each particle i, its status can be shown as follow

Position : $x_i^t = \left(x_{i1}^t,\ x_{i2}^t, \cdots,\ x_{id}^t \right)^T$

Velocity : $v_i^t = \left(v_{i1}, v_{i2}, \cdots, v_{id} \right)^T$

Individual best position : $p_i^t = \left(p_{i1}^t, p_{i2}^t, \cdots, p_{iD}^t \right)^T$

Group best position : $p_g^t = \left(p_{g1}^t, p_{g2}^t, \cdots, p_{gD}^t \right)^T$

So the position x_i on $t + 1$ can be updated in the following manner

$$v_{id}^{t+1} = \omega v_{id}^t + c_1 r_1 \left(p_{id}^t - x_{id}^t \right) + c_2 r_2 \left(p_{gd}^t - x_{id}^t \right) \tag{7}$$

$$x_{id}^{t+1} = x_{id}^t + v_{id}^{t+1} \tag{8}$$

The inertia weight ω is employed to control the impact of the previous history of velocities on the current velocity, thus to influence the trade-off between global and local exploration abilities of the particles. r_1, r_2 represent uniform random numbers between 0 and 1. c_1, c_2 are two positive constants, called the cognitive and social parameter, respectively. PSO algorithm is easily to be applied, and have the features of fast convergence, but it's also easily trapped into local extreme point at the same time. The search accuracy is not high, and the convergence will become slower at the time of late evolution. A new algorithm is applied based on the original particle swarm optimization algorithm, the information processing framework in the immune system is used to improve the performance of the algorithm when the particle may be premature convergence in local optimum too early, which will be applied to solve the mixed-model assembly line sequencing problem.

In the biological immune system, antibodies produced by the lymphocytes to recognize and resist the attack of various antigens. Feasible solution of target problem can be seen as the antibodies in the immune system, and the antigen will be the optimal solution. Affinity is used to indicate the level of similarity between antibody and antigen, described as follow

$$Affinity(x_i) = 1/1 + f \tag{9}$$

f in formula (9) is the objective function value. The smaller f is the higher level of antibody affinity in solving the minimization problem, which also shows that the particle is nearer away from the optimal particle.

In order to distinguish the level of difference between two antibodies, the degree of similarity needs to be computed, which can be indicted by the different fitness function value between antibodies. The similarity between x_i and x_j that expressed as $g(x_i, x_j)$ can be defined as formula (10)

$$g\left(x_i, x_j \right) = abs\left(fitness(x_i) - fitness(x_j) \right) \tag{10}$$

The antibody concentration is an important indicator of the measure of antibody diversity [9]. The concentration of antibody i have relations with the similarity between antibody i and other antibodies in the system [10]. The threshold value of the similarity is expressed as Distance, so the concentration of antibody i can be figured as the ration of the antibody number between the antibodies whose similarity are smaller than that of antibodies i and total antibodies. As the number of total antibodies is N, so the concentration of antibody i can be figured as formula (11)

$$D\left(x_i \right) = count\left(\sum_{j=1}^{N} g\left(x_i, x_j \right) \le Distance \right) / N \tag{11}$$

In the immune system, the high concentration of low-affinity antibodies should be suppressed, while low concentrations of high-affinity antibodies should be promoted. To ensure the effectiveness and diversity of antibodies, part of the antibodies with high concentration and low-affinity should be eliminated, and the corresponding number of new antibodies will be produced to replace randomly. Selection probability will be decided by both the concentration and affinity

$$P_g(x_i) = 1 - affinity(x_i) / \sum_{i=1}^{N} affinity(x_i) \tag{12}$$

$$P_d(x_i) = D(x_i) / \sum_{i=1}^{N} D(x_i) \tag{13}$$

$$P_s\left(x_i \right) = \alpha P_g\left(x_i \right) + \left(1 - \alpha \right) P_d\left(x_i \right) \tag{14}$$

P_g is the selection probability based on affinity, and P_d is the selection probability based on concentrations, α is a random number between 0 and 1. Formula (14) shows that the replacing rate will increase with lower affinity and higher concentration. P_r is the pre-set rate of replacing, antibody i will be replaced when $P_s\left(x_i \right) \ge P_r$, on the other hand, i will be preserved. The method jointed with immune system not only makes antibodies to retain a high degree of individual adaptation, but also maintains the diversity of antibodies, avoids falling into local optimal, improves the ability of global search.

4. The application of immunity particle swarm algorithm in mixed-model assembly sequencing problem

A random number between 0 and 1 is used to code each particle, sequenced by the particle size of each dimension, so we can indicate a corresponding product sequence type according to predefined initial product sequence.

Assume that the product A, B, C three products, ration of 1:2:3 in a product sequence cycle. The dimension is the number of products put into a production cycle,

$Dim = 6$. The initial product sequence is A-B-B-C-C-C, so the way of encoding and decoding can be defined as Table 1 .

In Table 1, the value of particle X_i is used for particle swarm optimization iterative process, whose location and velocity is updated, while the decoded sequence is used to calculate the corresponding function. The assembly sequencing problem can be converted to a continuous problem with the way of encoding. The method not only show the characteristics that the particle swarm optimization algorithm is easy to implement and fast convergence in solving the problem of continuous function, but also revert to the practical problems to choose the best solution for the function.

The basic optimization process of immunity particle swarm optimization designed in this paper can be shown as follows.

STEP 1: Particle population parameters initialization.

Determine the number of particles popsize and dimension *Dim*, the max velocity v_{max}, the cognitive and social parameter c_1, c_2, the start value and end value of inertia weight ω_{start} and ω_{end} , max number of alternative T_{max}. The threshold value of the similarity distance, and the pre-set rate of replacing P_r.

STEP 2: Initial particles location xi and velocity vi randomly.

Compute particle fitness, set current location as individual extreme P_{best}, set the location of particle with smallest fitness as global extreme G_{best}. Make the number of not optimizing of G_{best} $n = 0$.

Table 1

Encoding and decoding of particles

Particle Xi	0.72	0.03	0.19	0.75	0.14	0.12
InitialSequence	1(A)	2(B)	3(B)	4(C)	5(C)	6(C)
Sorting	5	1	4	6	3	2
DecodingSequence	C	A	C	C	B	B

STEP 3: Update particle state.

Update the location and velocity of each particle according to formula (7) and (8). The speed is limited under pre-set range. Update the value of cognitive and social parameter, inertia weight. To improve the algorithm's global search ability and convergence performance, a method of inertia weight update based on the strategy of linear differential decreasing is develop in this paper, shows as formula (15).

$$\frac{d\omega(t)}{dt} = \frac{2(\omega_{start} - \omega_{end})}{t_{max}^2} \times t \tag{15}$$

STEP 4: Update individual extreme P_{best} and global extreme G_{best}. Judge whether G_{best} is optimized, Yes, set G_{best} to the particle location, $n = 0$, otherwise, $n = n + 1$.

STEP 5: Determine whether it's time to update particles with antibody replacement mechanism. If $n = N$, immunity process starts, turn to STEP 6, otherwise, turn to STEP 8.

STEP 6: Calculate the affinity and concentration of each particle according to formula (10), (11), determine the selection rate P_g based on affinity and selection rate P_d based on the concentration.

STEP 7: Replace particles based on immunity mechanism. Determine replace rate of each particle according formula (14), if $P_s(x_i) \geq P_r$, replace particles with new particles produced randomly, return to STEP 3.

STEP 8: Judge whether stopping criterion is satisfied. If the current iteration number reaches T_{max}, or the satisfied value of the problem is achieved, iteration stops, results output, otherwise, turn to STEP 3.

5. Experimental results

Four similar parts of different products are produced in a mixed-model assembly line in an automobile parts manufacturing plant, distinguished by A, B, C, D. Demand for the four products are 1200, 1200, 1800 and 2400 pieces. Each workstation operation time of product shows in Table 2.

Table 2

Product time

Product number	Work station number					
	1	2	3	4	5	6
A	50	65	120	70	86	0
B	70	80	75	40	113	41
C	100	97	0	135	0	72
D	95	55	90	34	91	135

The product cycle time $C = 77$ min, length of work station $L = 80$ min. Determine minimum product cycle is {2:2:3:4} according to the rate among the demands of each product. The cost trade-off parameters are designed based on the situation of company, $\alpha = 0.4$, $\beta = 0.6$. In the optimization process, maximum number of iterations $T_{max} = 500$, size of popsize Popsize = 10, the value of cognitive and social parameter c_1, c_2 are updated with the strategy of linear adaptive according to the number of iterations, range from 0.5 to 2.5. The stare value of inertia weigh $\omega_{start} = 0.9$, end value $\omega_{end} = 0.4$, maximum velocity $v_{max} = 5$, the threshold value of the similarity Distance = 1, pre-set replacing rate $P_r = 0.4$. In order to illustrate that the optimization algorithm is more efficient and effective in solving mixed-model assembly line sequencing problem, try to solve the problem with immunity PSO, original PSO and traditional heuristic method [9]. Results shown in Table 3 are the mean value after calculating 50 times repeatedly. Compared with other methods, immunity PSO in this paper gets better results, with the lowest total idle-over time cost.

Table 2

Simulation results

Methods	Sequencing	Objective function value
Immunity PSO	DBDCACACDBD	922.8
Original PSO	ACDBDBDCDCA	927.9
Reciprocal of product ratio	DCDABCDABCD	938.8

As Fig. 2, the dotted line stands for immunity PSO, while the solid line stands for original PSO. It's seen

that both two algorithms showed inherent characteristics of convergence of particle swarm optimization in the early iterations (50 generations ago), but the original PSO presented the phenomenon of prematurity and converges in local optimum too early at 100 generation, and the local optimum wasn't revolted until 500 generation. The immunity PSO in this paper not only inherited the advantages of fast convergence of original PSO, but also jumped out of the local optimum by the immunity information processing adaptive mechanisms at middle of iteration (100-200 generation), which makes better solution than that of original PSO.

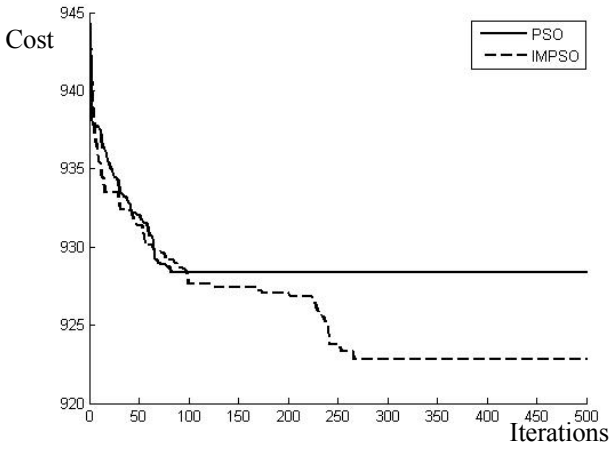

Fig. 2 Comparison of two particle swarm optimization process

6. Conclusions

Mixed-model assembly line sequencing problem is crucial for the line efficiency. An improved particle swarm optimization algorithm was proposed to solve sequencing problem in order to minimize the total idle-over cost. In order to avoid prematurely trapped in local optimal and the satisfactory solution can not be obtained, original PSO was optimized with the information processing mechanism in immune system. Compute affinity and concentration of each particle and replace in time. Through the analysis of simulation example and comparison with original PSO, the immunity PSO was proved better in avoiding premature convergence and value of objective functions, and it can solve the mixed-model assembly line sequencing problem effectively and quickly.

Acknowledgment

This research is supported by the Shanghai Natural Science Foundation (No.10ZR1431700).

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LAIPSNIŠKO PRIARTĖJIMO MODELIS NAUDOJIMAS MIŠRAUS TIPO SURINKIMO LINIJOSE

R e z i u m ė

Laipsniškas skirtingų modelių planavimas mišraus tipo surinkimo linijose lemia jų efektyvumą. Šiame straipsnyje formalizuojama nagrinėjamo modelio priartėjimo problema, kas leidžia iki minimumo sumažinti prastovų ir viršvalandžių bendrąją kainą. Šiai problemai optimizuoti buvo panaudotas supaprastintas detalių spiečiaus optimizavimo (PSO) algoritmas. Norint išvengti ankstyvos detalių konvergencijos, algoritme buvo panaudotas imuniteto mechanizmas. Detalės buvo išdėstytos laiko atžvilgiu, išlaikant jų įvairovę pagal panašumą ir tankumą, taip, kad nepatektų į vietinį optimumą. Be to, sprendimai, gauti darant šias prielaidas, buvo palyginti su tradiciniu PSO algoritmu. Rezultatai parodė, kad ši naujovė labai padeda spręsti mišrių modelių surinkimo linijose priartėjimo problemas.

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A SEQUENCING APPROACH OF MODELS IN MIXED-MODEL ASSEMLY LINES

S u m m a r y

Sequence planning of different models for a mixed-model assembly line is crucial for its efficiency. This paper formalized this model sequencing problem based on minimizing the total cost of idle time and overtime. An adapted Particle Swarm Optimization (PSO) algorithm was proposed to optimize the problem. To avoid early convergence of the particles, an immunity mechanism was introduced into the algorithm. The particle was replaced in time to keep the diversity according to the particle affinity and consistency, and so to avoid being trapped into local optimum. Furthermore, the solutions yielded by these approaches were compared to the traditional PSO algorithm, and the results showed that this novel approach has a lot of advantages for solving the sequencing problems in mixed-model assembly lines.

Received March 31, 2011
Accepted August 22, 2011