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Solving the problem of pipeline freezing with respect to external heat exchange

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1. Introduction

Engineering systems in buildings, in particular, heating, cold and hot water supply systems, as well as the system supplying heat to air heaters in air supply ventilation units and external utility lines, for the most part, use water as operating fluid. However, there is a risk of pipeline freezing during cold winter seasons due to sudden gradual drops in temperature and in the event of replacing heating systems of heat supply under emergency operating conditions. The volume naturally increases in the freezing process, leading to increase in pressure within the pipe and subsequent destruction thereof Gordon, J. [1] and Gilpin, R. [2].

The freezing process is usually considered in cases where water does not move in the pipe and the temperature decreases with time [3].

The solution of the aforementioned problem is directly related to the solution of the Stefan J. [4].

In addition to using theoretical-analytical and numerical methods, such as variation methods of solving the problem of water freezing in pipes and freezing front movement [5, 6] based on compiling a heat balance equation or solving a heat conduction equation, experimental research in carried out [7, 8].

In addition to the above, various effects are taken into account, including uneven distribution of ice in the cross-section of the pipe [9], the flow of liquid along the pipe caused by natural convection inevitably occurring during freezing. The latter has been found to have little effect on the freezing process.

Despite the numerous studies mentioned above, of great practical interest is the case where water moves inside the pipe, i.e. where forced convection occurs. The water flow and the release of heat naturally slow down the freezing process. However, practice shows that pipelines may freeze at low temperatures.

Various engineering methods are used to prevent the process, including laying external pipeline networks to an appropriate depth and installing heat insulation; in certain cases intense forced water circulation in pipes is created, resulting in the release of heat due to internal friction.

However, the likelihood of accidents cannot be completely excluded in any situation, and therefore, the issue pertaining to the rate of water freezing in pipe-lines should be studied. It should be noted that mathematical modelling of hydraulic and thermal modes of heating system does not take the issue of freezing into account Gilpin R. [10].

The complexity of solving the aforementioned problem lies in size of the areas that vary with time where the temperature field is examined. Physical properties of this fluid, such as the thermal conductivity coefficient, heat capacity, and density change sharply when passing through the moving boundary; moreover, at the boundary, heat release occurs due to fluid transitions, in our case, ice turning into water, which complicates solving the problem even more. No exact solution for the general case has yet been found. There are known specific solutions, for example, a number of approximate solutions have been obtained for temperature distribution in the case of plane interface in a semi-infinite fluid,; it should be noted that the McDonald A., at al [3] solves the problem for the pipeline, but for simplification purposes it is assumed that external surface temperature of the pipe is known and constant which corresponds to first-type boundary conditions.

The purposes of this study are to 1) formulate the problem of freezing of moving water in pipes under sharp temperature drops taking into account the flowing of water in pipes; 2) determine the rate of freezing, which is important in order to estimate the time required to eliminate the damage inevitably done in the process; 3) study the ways of reducing the likelihood of accidents.

2. Statement of the problem and mathematical model

In practice, the simplified solution where the temperature at the pipe boundary is specified as the environment temperature can be considered sufficiently ac-curate only in the case of underground, particularly, channel-free laying, where pipes come in direct contact with the soil, in which case the soil temperature at the pipeline laying level can be considered to be the temperature at the pipe boundary.

At the same time, situations may emerge, primarily with respect to internal networks, where pipelines are laid using open laying methods (Fig. 1), and there-fore, the nature of heat exchange with the environment on the surface thereof must be taken into account, which, in turn, requires switching to third-type boundary conditions.

In this case, we will solve the problem of heat transfer in zone-inhomogeneous medium.

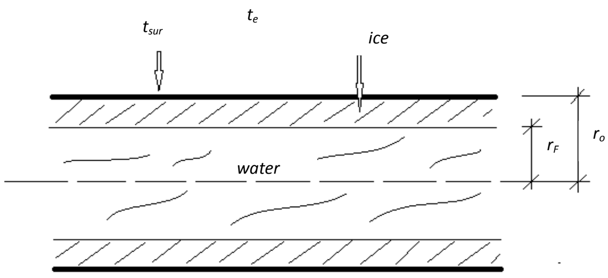

Fig. 1 Ice layer formation inside the pipe that is surrounded by air

Let's compile for the aforementioned case a relevant system of differential and algebraic equations of heat balance, heat exchange, and the heat flow released due to internal friction [11]:

$$\frac{a_{1,2}}{r}\frac{\partial}{\partial}\left(r\frac{\partial t_{1,2}}{\partial r}\right)=\frac{\partial t_{1,2}}{\partial \tau}. \quad (1)$$

The boundary condition on the pipe surface takes the following form:

$$q_l = 2\pi r_o \alpha\left(t_e - t_{sur}\right). \quad (2)$$

For the moving boundary inside the pipe, where $r = r_F$, it can be expressed as $t = 0^0$:

$$\rho_2 r_{ps}\frac{dr_F}{d\tau}=\lambda\frac{\partial t}{\partial r}, \quad (3)$$

where $t_{1,2}$ are the temperatures in frozen and non-frozen areas; t_e is the outside air temperature; t_{sur} is the pipe surface temperature; $a_{1,2}$, m^2/s - the thermal diffusivity in the said areas; r_0, m is the pipeline radius; r_F, m is the freezing front radius changing with time; ρ_2 - water density, kg/m^3; λ is the heat conductivity of ice, W/mK; α, W/m^2K – heat transfer coefficient.

Solving the (1)-(3) equation system allows us to identify temperature fields in both solid and liquid areas – the way it was done by Parfentieva N.A. [12] and Poots G. [13].

However, our main task is to determine the rate of freezing, therefore, we are not addressing the problem of identifying the temperature fields. Still, if we know the position of the freezing front and boundary conditions, we can determine the temperatures in the two areas.

Let us consider the expression for linear density of heat flow q_l, W/m, passing through the freezing front surface:

$$q_l = -\frac{2\pi\lambda t_{sur}}{ln\left(r_F'\right)} \quad (4)$$

The expression includes $r'_F = r_F/r_o$ – the freezing front dimensionless radius of the current relative to the pipeline radius r_o, m.

The introduction of the dimensionless radius of the freezing front will allow in the future doing the calculations for any radius of the pipe.

We assume that the freezing point of water is zero degrees, and ignore the difference between the outer and inner pipe diameters in this case. Based on the Stefan condition, taking into account that the specific surface area of the phase boundary per 1 metre of pipeline equals $2\pi r_F$, the same density value of the heat flow q_l can be written as:

$$q_l = 2\pi\rho r_{ps} r_F \frac{dr_F}{d\tau} - q_{l.pf} = 2\pi\rho r_{ps} r_o^2 r_F' \frac{dr_F'}{d\tau} - q_{l.pf}. \quad (5)$$

The value $q_{l.pf}$, W/m, represents the linear density of the heat flow released in the process of water moving inside the pipeline due to viscous friction.

It can easily be demonstrated that the said parameter can be expressed as:

$$q_{l.pf} = \pi R r_o^2 r_F'^2 w, \quad (6)$$

where w is the speed of flowing water , m/s; and R are specific friction pressure losses, Pa/m. The following expression holds true for hydraulically smooth pipes [14]:

$$R = 3.2\cdot 10^4 \frac{w^{1.79}}{\left(2r_o r_F' \cdot 10^{-3}\right)^{1.29}}. \quad (7)$$

And finally, on the outer surface of the pipeline, the surface heat exchange condition holds true:

$$q_l = 2\pi r_o \alpha\left(t_{out} - t_{sur}\right). \quad (8)$$

The overall heat exchange coefficient α may, at the first approximation, be considered constant along the pipe length, as, subject to one and the same pipe diameter, it primarily depends on the temperature difference ($t_{out} - t_{sur}$), and the said difference should not significantly change, as, according to the statement of the problem, $t_{out} = const$, and at the beginning of the freezing t_{sur} will also be substantially constant and equal to the temperature of the phase transition.

From (8) and (4) we can receive:

$$t_{sur} = \frac{t_{out}}{1 - 1/\left[Bi\, ln\left(r_F'\right)\right]}, \quad (9)$$

then the result substitutes into (4) and after that into (5) as a left part, and into the right part (5) – the expression (6) taking into account (7), and then we get:

$$q_l = \frac{-2\pi\lambda t_{out}}{ln\left(r_F'\right) - 1/Bi} = \pi r_o^2 r_F' \left[2\rho r_{ps}\frac{dr_F'}{d\tau} - A\frac{w^3 r_F'}{r_F'^{1.28}}\right], \quad (10)$$

where A is cited in (11). This expression follows from (7). Value B shows the impact of the liquid friction on heat flow q_l. It can be shown that, subject to (6)-(7):

$$B = \frac{Aw^{2.79} r_o^2}{\lambda t_{out}}, \text{ where } A = \frac{3.3\cdot 10^4}{\left(2r_o \cdot 10^{-3}\right)^{1.29}}. \quad (11)$$

Then it is possible to get in a non-dimensional form:

$$\frac{1}{1/Bi - ln\left(r_F'\right)} = r_F' \frac{dr_F'}{dFo} - \frac{B}{r_F'^{5.28}}, \quad (12)$$

when one may express $dr_F^{'}$ and finally obtain the final solution as an integral (13).

$$Fo^{'} = -\int_{1}^{r_F^{'}} \frac{r_F\left(1/Bi - ln\left(r_F^{'}\right)\right)dr_F}{1 + B\left(1/Bi - ln\left(r_F^{'}\right)\right)/r_F^{'4.87}} . \qquad (13)$$

Parameter *Fo'* represents a modified Fourier criterion (non-dimensional time), and *Bi* complex represents a non-dimensional Biot criterion characterising the ratio between the external heat exchange and internal heat conduction. They are defined in this case by the following expression:

$$Fo^{'} = \frac{\lambda t_{out}\tau}{\rho r_{ps} r_o^2} ;\ Bi = \frac{\alpha r_o}{\lambda} , \qquad (14)$$

where τ is time, s, calculating from the beginning of freezing.

The integral (13), with certain simplifications, can provide the approximate dependence of moving boundary on time, but we find it reasonable to use numerical methods to obtain such dependence.

3. Results and discussion

From the Fig. 2 shows calculation results - the value of *Bi* was considered to be equal to 0.67, which corresponds to real-life heat exchange conditions for the pipe with an outer diameter of 325 mm. Dependence diagrams of the freezing radius at various values of *B* are shown in Fig. 2 in solid lines.

The increase in value of *B* leads to the increase in *Fo'* with the given value of freezing front radius, which is physically quite obvious, as the heat released due to friction should slow down the process.

The dashed line shows the dependence of *Fo'* on r_F at $Bi \to \infty$ and $B = 0$, i.e. not taking into account the heat released due to friction and under first-type boundary conditions for the purposes of comparison with the analytical solution contained.

Under the first-type boundary conditions, as we can see, we get a significantly lower freezing time value, although the nature of the dependence remains the same.

It is obvious that when solving similar problems to obtain numerical time values for practical use, third-type boundary conditions should be taken for the outer pipe surface.

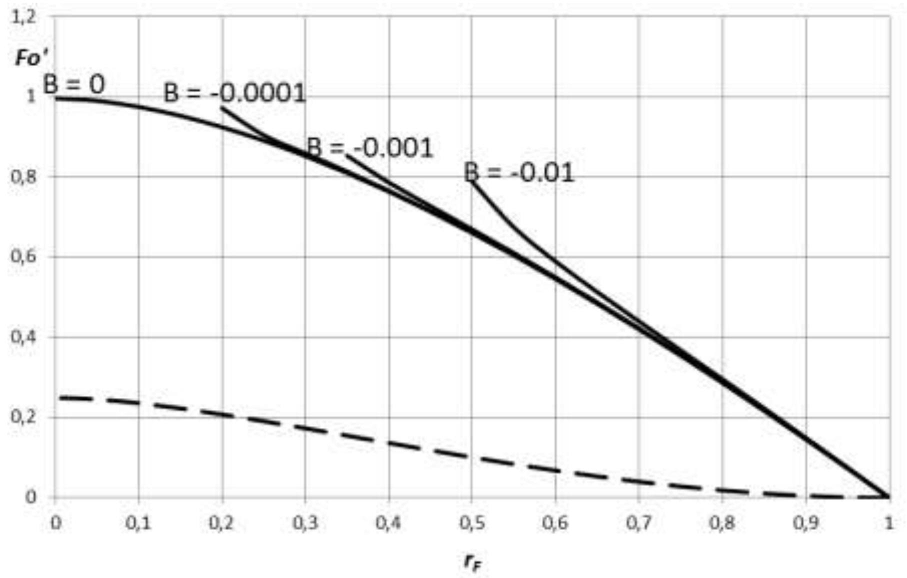

Fig. 2 Dependence of *Fo'* value on r_F according to equation (13) at various values of *B* and *Bi* parameters

4. Conclusions

It is easily seen that at $Bi < \infty$ the freezing occurs more slowly due to the presence of additional resistance to heat exchange on the outer pipeline surface. At the same time, for each *Bi* a certain limit value of *B* is maintained, at which $Fo' \to \infty$, meaning physical termination of further freezing as a consequence of external heat loss compensation with internal heat release occurring due to friction: the heat released due to friction is equal to the heat transferred to the air by time unit. Therefore, to obtain correct calculations, friction should by all means be taken into account. Obviously, as the value of *Bi* goes down with decreasing r_o, pipes of smaller diameter are in a better position than larger ones, and considering that the value of *B* also increases, the effect is enhanced. Therefore, we conclude that to ensure additional protection against freezing, it is advisable to use reduced diameter pipelines, thus increasing the resistance to external heat exchange and share of frictional heat.

This conclusion is not obvious - large pipe diameter must protect the fluid from the complete freezing but from our calculations, we can conclude that the friction plays an important role in this process, so the flow of the fluid in the tube should be Puazeil flow, wherein the heat release is going around the pipe section. It is important for the above numerical calculations and comparison the results with $Bi \to \infty$ when heat release is ignored by the internal friction, causes an error significantly greater than 100%.

The simplified formulation of the problem leads to the qualitative conclusions, but the solution cannot be used for specific engineering calculations. The introduction of generalized variables allows to use obtained solutions for any parameters: tube diameter, volume flow rate, surface properties, etc. Additionally solution can be used when driving a liquid with non-water properties.

Thus, the problem of pipeline freezing at temperatures lower than temperatures of phase transition in the presence of moving liquid inside them has been solved. The obtained solution can be used in calculations not only with respect to pipeline freezing, but also for axially symmetric structures made of moisture permeable materials.

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SOLVING THE PROBLEM OF PIPELINE FREEZING WITH RESPECT TO EXTERNAL HEAT EXCHANGE

S u m m a r y

The paper proposes a solution to the problem of freezing of heating pipelines in engineering systems in buildings in emergency situations.

A mathematical process model is proposed, wherein heat emissions induced by viscous friction and phase transition are taken into account. Formulas for estimating the rate of freezing have been obtained. Calculation results obtained by using numerical methods are presented. Recommendations on the proper choice of pipe-line diameters to reduce the probability of liquid freezing therein are provided as well.

Keywords: pipeline, freezing front, frictional heat, Stefan condition, Biot criteria.

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