

# Vibration characteristics of periodic structure with micro octagon-like unit cell

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## 1. Introduction

Nowadays, periodic composites and structures with damping performances have been widely applied to engineering fields, such as the mechanical field, aerospace industry or electronic industry and so on. Developed composite damping structures indicate that various damping requirements can be met through simple composition of several damping materials. However, a set of systemic and effective theories and methods, which can guide the design, have not come into being yet [1]. Recently, researches on vibration characteristics of periodic composites and structures have attracted many attentions around the world [2]. In the early 50 s, composite damping structures made of viscoelastic materials have been studied. In China, the study about this started in the 70 s [3]. One of the light composite structures is called periodic truss sandwich structure, which is considered as the most promising internationally [4]. The lattice sandwich structure is introduced by professor Evans and the others recently. It looks like a spatial latticed truss structure, and it has many kinds of topological structures including tetrahedron, pyramidal and diamond [5]. The length of truss is usually 10 ~ 100 mm [6]. It has many excellent properties such as light weight, energy absorption, noise reduction and so on. Wang Haiqiao et al. reviewed the status of various damping materials and discussed advanced damping materials and advanced damping technology [7]. Chae-Hong Lim et al. studied the fabrication of sandwich panels with periodic cellular metal cores and discussed its mechanical performances [8]. Ragulskis, K. et al. investigated vibration characteristics of a three-layered polymeric film, in which the upper and lower layers are stiff and do not deform in the transverse direction, while the internal layer can deform in the transverse direction [9]. Vaicaitis, R et al. studied nonlinear dynamic response and vibration control capabilities of Electrorheological materials based adaptive sandwich beam [10]. Mei et al. researched the damping characteristic of composite material with periodic microtruss structure [11, 12].

In this paper, we establish two periodic structure models with micro octagon-like unit cell: one is with a heavy sphere in the unit cell, another is without the sphere. By analyzing and comparing the vibration characteristics of the two models, it can be concluded that the damping characteristics of the model with a heavy sphere in the unit cell is better; meanwhile, properly choosing the weight of the sphere can make the band-gap starting frequency and cut-off frequency drop sharply and the band gap range be-

come wider, in this way, the effect of vibration alleviation can be improved much better within a given frequency range.

## 2. Theory

Damping characteristics are related to dynamic loads, and dynamic loads usually contain harmonic load, impact load, sudden load, and random load. In this paper, the harmonic load is used, which can provide the continuous dynamic performances or damping characteristics of structures in accordance with load frequencies. Generally, the technology to analyze structures under harmonic load is called as Harmonic Analysis in literature.

In order to build the computational models, Finite Element Method (FEM) is applied to discretize the structure. After discretizing the computational models, assembling element matrixes and applying boundary conditions, the dynamic equations of the structure can be found, which governs the vibration of the structure. Here, it is assumed that  $M$ ,  $C$  and  $K$  stand for structural mass matrix, structural damping matrix and structural stiffness matrix and that  $b$ ,  $x$  be used to denote the harmonic load vector, and harmonic nodal displacement vector, respectively.

For the sake of simplicity, matrix  $A$  is defined as

$$A = K - \omega^2 M + i\omega C \quad (1)$$

where  $\omega$  is circular frequency,  $i$  is imaginary unit. According to the theory of FEM for structure analysis, matrix  $K$ ,  $M$ ,  $C$ , thus  $A$  can be supposed to be symmetric matrixes.

Thus,  $x$  satisfies the following complex linear equation with symmetric matrix  $A$

$$Ax = b \quad (2)$$

For every given load frequency  $f$  or circular frequency  $\omega$ , all the nodal complex displacement in the structure, which describes the amplitude and phase of nodal vibration, can be obtained by solving Eq. 2. In this study, Eq. 2 is solved by the preconditioned conjugate gradient method [13], which is applicable to solve system of large scale complex linear equations. The method is described simply as follows:

1. give initial value  $x_0$  and allowable tolerance  $\varepsilon$ ;

2. compute  $r_0 = b - Ax_0$ ,  $z_0 = \text{Jacobi}_n(A^{-1}r_0)$ ,

$$p_1 = z_0, \hat{r}_0 = r_0^*, \hat{p}_1 = p_1^*;$$

3. iterate for subscript  $k$  to compute

$$\alpha_k = \left( \hat{r}_{k-1}, z_{k-1} \right) / \left( \hat{p}_k, Ap_k \right), \quad x_k = x_{k-1} + \alpha_k p_k,$$

$$\hat{r}_k = \hat{r}_{k-1} - \alpha_k Ap_k, \quad \hat{r}_k = \hat{r}_{k-1} - \alpha_k A^* p_k,$$

$$z_k = \text{Jacobi}_n \left( A^{-1} \hat{r}_k \right), \quad \beta_k = \left( \hat{r}_k, z_k \right) / \left( \hat{r}_{k-1}, z_{k-1} \right),$$

$$p_{k+1} = z_k + \beta_k p_k, \quad p_{k+1} = z_k^* + \beta_k^* p_k;$$

4. if  $\left( \hat{r}_k, z_k \right) / \left( \hat{r}_0, z_0 \right) < \varepsilon$ , output  $x_k$ , otherwise

$k = k+1$  and go to step 3.

where  $z_k = \text{Jacobi}_n \left( A^{-1} \hat{r}_k \right)$  means to use Jacobi iterative method to solve linear equations  $Az_k = r_k$ , and the iterative number is  $n$ , and superscript \* stands for complex conjugate operator.

### 3. Numerical examples

In order to study the vibration characteristics of periodic structure with micro octagon-like unit cell, two models are established here.

A. Model 1 - periodic structure model without a sphere in the octagon-like unit cell

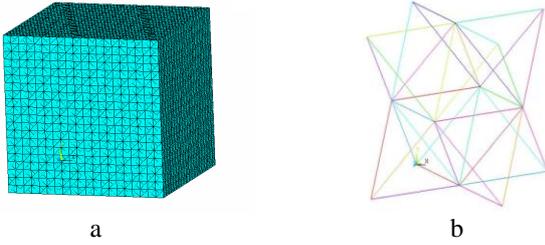


Fig. 1 Periodic structure Model 1: a) periodic structure, b) unit cell

The model is a cube with periodic micro octagon-like structures, as shown in Fig. 1, a. Fig. 1, b is the micro octagon-like structure, which is the unit cell of the model. In the unit cell, each edge is a rod with 70.7 mm length and cross-sectional area 25 mm<sup>2</sup>, which is assumed to be made of aluminium with density  $\rho = 2800 \text{ kg/m}^3$ , modulus of elasticity  $E = 7E+10 \text{ Pa}$  and Poisson's ratio  $\nu = 0.3$ ; and all the other areas are filled with rubber, which parameters are density  $\rho = 1000 \text{ kg/m}^3$ , modulus of elasticity  $E = 5E+6 \text{ Pa}$ , Poisson's ratio  $\nu = 0.47$  and damping loss factor 0.05.

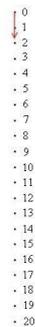


Fig. 2 Observation nodes

During the simulating process, the displacement excitation is applied to the central point on the top surface

of the model. We simulate displacement transfer function - frequency curves of some observation nodes, which are chosen from the top surface to the bottom surface of the model, as shown in Fig. 2, here node No. 0 is the central point of the top surface, to which a harmonic excitation is applied. The simulation results are illustrated in Fig. 3, where the two coordinate axes of the rectangular coordinate system are frequency and displacement transfer function, respectively. The displacement transfer function is expressed in the form of  $20 \log(U_o/U_i)$ , where  $U_o$  is the displacement of node No. 0, and  $U_i$  is the displacement of the other observation nodes. In Fig. 3, a, curve 1, curve 2 and curve 3 stand for displacement transfer function-frequency curves of nodes No. 1, No. 5 and No. 10, respectively. In Fig. 3, b, curve 1, curve 2 and curve 3 denote the simulation results of nodes No. 10, No. 15 and No. 20, respectively.

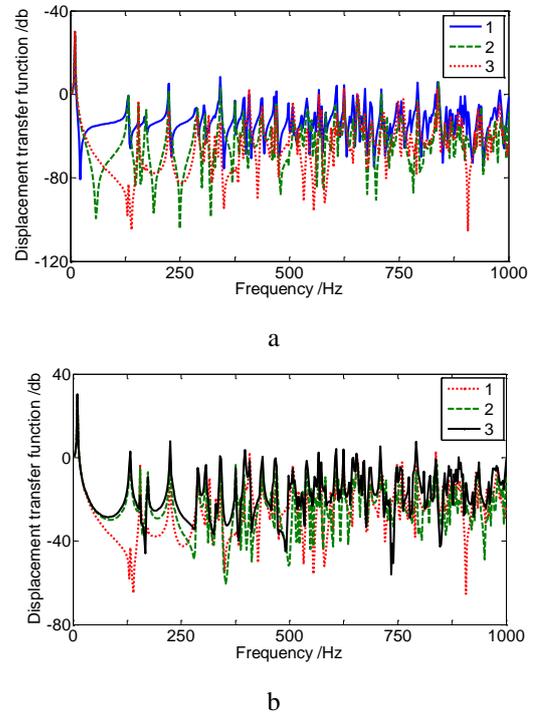


Fig. 3 Simulation results of model 1: a) 1 - No. 1; 2 - No. 5; 3 - No. 10; b) 1 - No. 10; 2 - No. 15; 3 - No. 20

B. Model 2 - periodic structure model with a sphere in the octagon-like unit cell

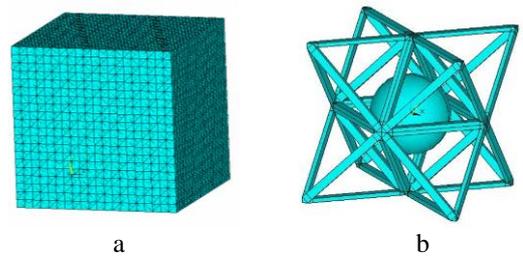


Fig. 4 Periodic structure model 2: a) periodic structure; b) unit cell

Encouraged by the fact that dynamic damping systems possess high damping characteristics, we further

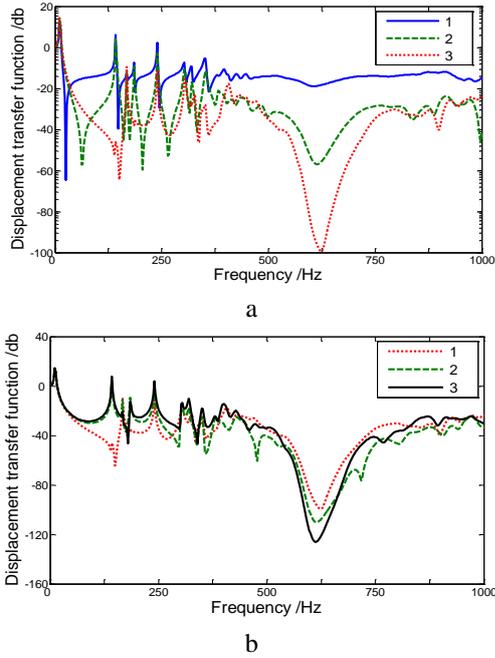


Fig. 5 Simulation results of model 2: a) 1 - No. 1; 2 - No. 5; 3 - No. 10; b) 1 - No. 10; 2 - No. 15; 3 - No. 20

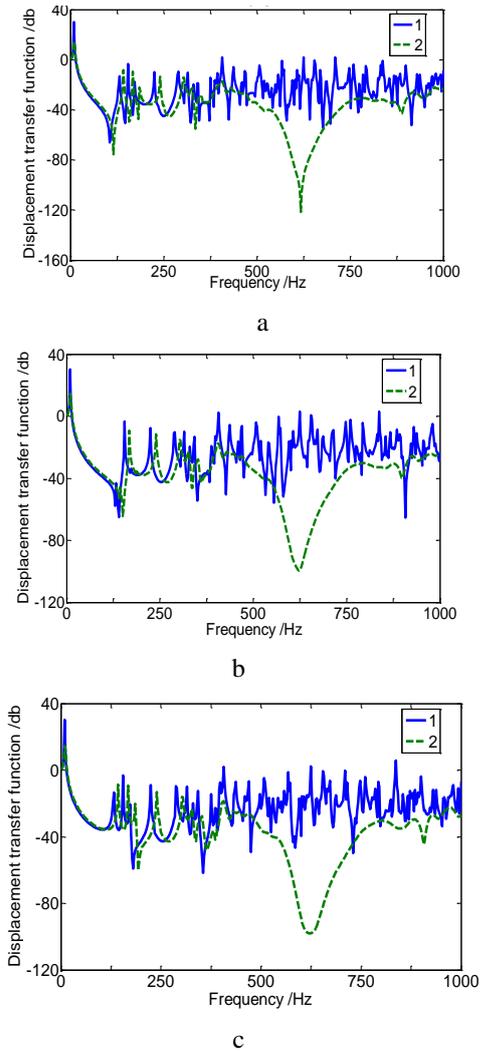


Fig. 6 Comparison between model 1 and model 2: a) 1 - model 1; 2 - model 2; b) 1 - model 1; 2 - model 2; c) 1 - model 1; 2 - model 2

modify the first model. A sphere is applied to the center of every unit cell, as illustrated in Fig. 4. Here, the sphere acts as the mass ball of a mass-spring system, and the rubber acts as the spring of the mass-spring system. The sphere is 0.15 kg. The displacement transfer function-frequency curves of the chosen 5 nodes are illustrated in Fig. 5. Here, Fig. 5, a is the simulation results of the nodes No. 1, No. 5 and No. 10, and Fig. 5, b is the simulation results of the nodes No. 10, No. 15 and No. 20.

#### 4. Discussion

##### A. Comparison between model 1 and model 2

By comparing the simulation results of model 1 in Fig. 3 with the results of model 2 in Fig. 5, we can conclude that model 2 has the better performances of energy absorbing and shock absorption. In this section, the displacement transfer function-frequency curves of the two models are compared in Fig. 6. Here, Fig. 6, a-c are simulation results of nodes No. 9, No. 10 and No. 11, respectively, and curve 1 is for model 1 and curve 2 is for model 2.

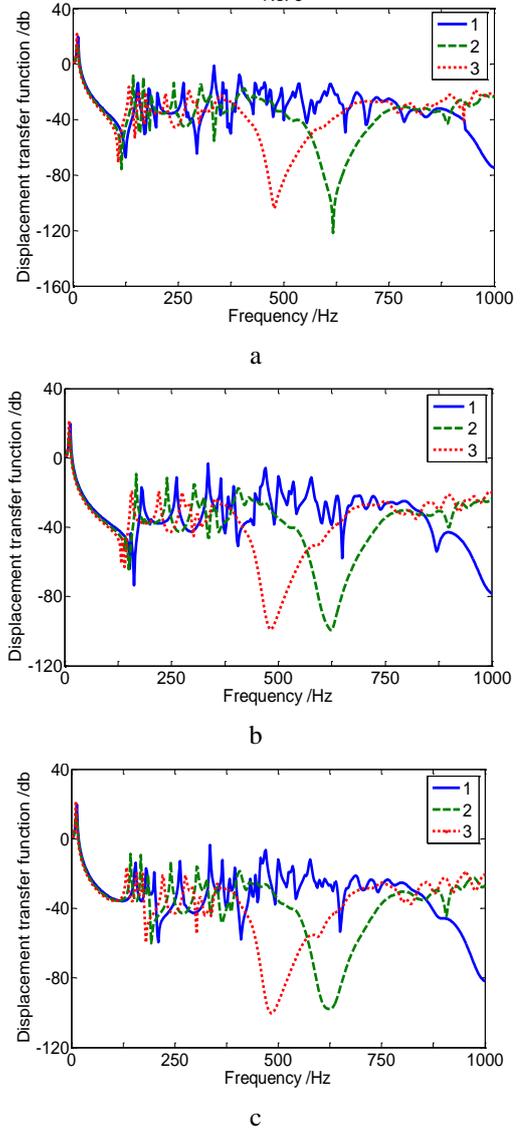


Fig. 7 Comparison among model 2: a) 1 - 0.05 kg; 2 - 0.15 kg; 3 - 0.25 kg; b) 1 - 0.05 kg; 2 - 0.15 kg; 3 - 0.25 kg; c) 1 - 0.05 kg; 2 - 0.15 kg; 3 - 0.25 kg

## B. Comparison among model 2 with different weight sphere in unit cell

I have concluded that model 2 has the better performances of energy absorbing and shock absorption. However, whether can the weight of the sphere affect the vibration characteristics of the structure?

In this section, we offered three numerical examples of model 2 with different weight spheres in unit cell, and the simulation results are compared in Fig. 7. Here, Fig. 7, a-c are simulation results of nodes No. 9, No. 10 and No. 11, respectively; and curve 1 is for sphere = 0.05 kg, curve 2 is for sphere = 0.15 kg and curve 3 is for sphere = 0.25 kg. By analyzing the vibration characteristics of the three numerical examples, we can find out that when the sphere is a little heavier, the band-gap starting frequency and cut-off frequency can drop sharply, the band gap range can become wider, and the effect of alleviation of vibration can be better. It demonstrates that suitably choosing the weight of the sphere can further improve the effect of vibration alleviation of the structure.

## 5. Conclusions

We build two periodic structure models: one is with a sphere in the micro octagon-like unit cell, another is without the sphere. By comparing and analyzing, we can draw the conclusions.

1. The periodic structure model with a sphere in the micro octagon-like unit cell has the advantage of vibration alleviation;

2. When the weight of the sphere in the unit cell is chosen properly, the band-gap starting frequency and cut-off frequency can drop sharply, the band gap range can become wider, and the effect of vibration alleviation can be improved much better within a given frequency range.

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PERIODINĖS KONSTRUKCIJOS SU  
AŠTUONKAMPIU MIKROELEMENTU SVYRAVIMO  
CHARAKTERISTIKOS

## Re z i u m ė

Įvertindami tai, kad šio tipo dinaminės sistemos turi didelį slopinimo koeficientą, straipsnyje tyrinėjome sferos, patalpintos periodinės konstrukcijos vienetiniame elemente, svorio efekto įtaką konstrukcijos svyravimo charakteristikoms. Pirmiausia buvo sukurti du periodinės konstrukcijos modeliai su aštuonkampiu mikroelementu: vienas su sunkia sfera elemente, kitas be sferos. Abiejų modelių vienetiniame elemente visos briaunos pagamintos iš aliuminio, o visos kitos erdvės užpildytos guma. Antra, svyravimų problemai spręsti yra pritaikytas pradinio jungtinio gradiento metodas, todėl poslinkių perdavimo funkcijos – dažnio kreivės yra nustatytos modelio mazguose. Galiausiai yra pateikta keletas skaitinių pavyzdžių su sfera vienetiniame elemente ir be jos. Imitavimo rezultatai parodė, kad periodinės konstrukcijos modelis su sfera vienetiniame elemente yra pranašesnis dėl svyravimų slopinimo,

be to, tinkamai panaudojus sferos svorį vienetiniame elemente, pradinių dažnių juosta ir stabdymo dažniai gali gerokai sumažėti, o juostos intervalas paplatėti ir svyravimų slopinimo efektas nustatyto dažnio intervalo viduje gali žymiai pagerėti.

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#### VIBRATION CHARACTERISTICS OF PERIODIC STRUCTURE WITH MICRO OCTAGON-LIKE UNIT CELL

##### S u m m a r y

Encouraged by the fact that dynamic damping systems possess high damping characteristics, the paper aims at researching the effect of weight of the sphere embedded into the unit cell of periodic structures on the vibration characteristics of the structure. Firstly, two periodic structure models with micro octagon-like unit cell are built: one is with a heavy sphere in the unit cell, another is

without the sphere. In the unit cell of the two models, all the edges are made of aluminum and all the other areas are filled with rubber. Secondly, the preconditioned conjugate gradient method is applied to solve the vibration problem, thus the displacement transfer function-frequency curves of nodes in models are obtained. Finally, several numerical examples with or without sphere in the unit cell are presented. Simulation results show the periodic structure model with a sphere in the unit cell has the advantage of vibration alleviation; in addition, when the weight of the sphere in the unit cell is chosen properly, the band-gap starting frequency and cut-off frequency can drop sharply, the band gap range can become wider, and effect of vibration alleviation can be improved much better within a given frequency range.

**Keywords:** vibration characteristics, periodic structure with micro octagon-like unit cell.

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