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Minimum volume of trusses at shakedown – mathematical models and new solution algorithms

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1. Introduction

Adapted to variable repeated load elastic-plastic structure satisfies strength conditions and it is safe in respect to cyclic-plastic collapse. Usually, optimal project of the structure, obtained neglecting stiffness constraints, does not satisfy serviceability requirements [1, 2]. For trusses, not only strength and stiffness, but also stability constraints [3] should be included into mathematical models of minimum volume problem of elastic-plastic trusses at shakedown.

Geometry of a truss (lengths L_j of bars, $j=1,2,...,n$, $j \in J$), yield limits σ_{yj} of their material, also variable repeated load are prescribed. Load $\boldsymbol{F}(t)$ is characterized only by its lower and upper load variation bounds $\boldsymbol{F}_{inf}$, $\boldsymbol{F}_{sup}$ ($\boldsymbol{F}_{inf} \le \boldsymbol{F}(t) \le \boldsymbol{F}_{sup}$, here t is time). Loading history is unknown.

In this paper the following shakedown optimization problem is under consideration: the truss of minimum volume $V=\sum_j L_j A_j$ ($j \in J$) satisfying strength, stiffness and stability conditions is to be found (here A_j are cross-sectional areas). Stability constraints are related with recommendations of Eurocode 3, when admissible forces of compressive bars are obtained by reduction of their material yield limit σ_y. In other words saying, vector of limit forces $\boldsymbol{N}_0$ ($N_{0j} = \sigma_{yj} A_j$, $j \in J$) is substituted by new one $\boldsymbol{N}_{0,cr}$. Improved algorithm of truss minimum volume problem solution is proposed.

2. Mathematical models of analysis problem

Adapted to cyclic loading $\boldsymbol{F}_{inf} \le \boldsymbol{F}(t) \le \boldsymbol{F}_{sup}$ the truss responds in a purely elastic manner, but stress-strain state of the structure, depends on loading history. For truss undergone plastic strains it is rational to introduce residual forces $\boldsymbol{N}_r$, strains $\boldsymbol{\Theta}_r$ and nodal displacements $\boldsymbol{u}_r$ ($\boldsymbol{\Theta}_r = \boldsymbol{D}\boldsymbol{N}_r + \boldsymbol{\Theta}_p$, here $\boldsymbol{D}$ is quasi-diagonal flexibility matrix of truss elements, $\boldsymbol{\Theta}_p$ is vector of plastic strains). Determination of truss residual displacements $\boldsymbol{u}_r$ (usually stiffness of truss is ensured by restriction of nodal displacements) is quite difficult problem of dissipative system mechanics [4, 5]. It becomes more difficult, when load $\boldsymbol{F}(t)$ is characterized only by its lower and upper load variation bounds $\boldsymbol{F}_{inf}$, $\boldsymbol{F}_{sup}$. In that case it is possible to find only variation bounds $\boldsymbol{u}_{r,inf}$, $\boldsymbol{u}_{r,sup}$ of residual displacements $\boldsymbol{u}_r(t)$ ($\boldsymbol{u}_{r,inf} \le \boldsymbol{u}_r(t) \le \boldsymbol{u}_{r,sup}$) [6, 7].

Analysis problem, i.e. determination of $\boldsymbol{N}_r$, $\boldsymbol{u}_r$, $\boldsymbol{\Theta}_p$ at shakedown, can be solved, when not only geometry of the truss and limit forces $\boldsymbol{N}_0$ of its bars, but also load variation bounds $\boldsymbol{F}_{inf}$, $\boldsymbol{F}_{sup}$ are known. Vector of residual forces $\boldsymbol{N}_r$ is found due to the solution of analysis problem static formulation (minimum complementary deformation energy principle [2,7,8]
minimize

$$0.5\, \boldsymbol{N}_r^T \boldsymbol{D} \boldsymbol{N}_r \tag{1}$$

subject to

$$\boldsymbol{A}\boldsymbol{N}_r = \boldsymbol{0} \tag{2}$$

$$\left.\begin{array}{l} \boldsymbol{f}_{max} = \boldsymbol{N}_0 - \boldsymbol{N}_r - \boldsymbol{N}_{e,max} \ge \boldsymbol{0} \\ \boldsymbol{f}_{min} = \boldsymbol{N}_{0,cr} + \boldsymbol{N}_r + \boldsymbol{N}_{e,min} \ge \boldsymbol{0} \end{array}\right\} \tag{3}$$

Statically admissible residual forces $\boldsymbol{N}_r = \left(N_{r1}, N_{r2}, ..., N_{rn}\right)^T$ satisfy equilibrium Eqs. (2) and yield conditions (3) (here $\boldsymbol{A}$ is the $m \times n$ equilibrium matrix). In the quadratic programming problem (1)–(3) vectors of truss elastic force extreme values $\boldsymbol{N}_{e,max}$, $\boldsymbol{N}_{e,min}$ ($\boldsymbol{N}_{e,max} \le \boldsymbol{N}_e(t) \le \boldsymbol{N}_{e,min}$) are known

$$\left.\begin{array}{l} \boldsymbol{N}_{e,\max} = \boldsymbol{\alpha}_{sup}\boldsymbol{F}_{sup} - \boldsymbol{\alpha}_{inf}\boldsymbol{F}_{inf} \\ \boldsymbol{N}_{e,\min} = -\boldsymbol{\alpha}_{sup}\boldsymbol{F}_{inf} + \boldsymbol{\alpha}_{inf}\boldsymbol{F}_{sup} \end{array}\right\} \tag{4}$$

Here $\boldsymbol{\alpha}$ is influence matrix of elastic axial forces $\boldsymbol{\alpha} = \boldsymbol{\alpha}_{sup} + \boldsymbol{\alpha}_{inf}$ ($\boldsymbol{\alpha} = \boldsymbol{K}\boldsymbol{A}^T\boldsymbol{\beta}$, $\boldsymbol{K} = \boldsymbol{D}^{-1}$, $\boldsymbol{\beta} = \left(\boldsymbol{A}\boldsymbol{K}\boldsymbol{A}^T\right)^{-1}$). Without losing generality, it is assumed that $\boldsymbol{F}_{inf} \ge \boldsymbol{0}$, $\boldsymbol{F}_{sup} \ge \boldsymbol{0}$. Vectors $\boldsymbol{N}_{e,max}$, $\boldsymbol{N}_{e,min}$ represent locus apexes of forces $\boldsymbol{N}_e(t)$. Thus, all combinations of elastic forces from load $\boldsymbol{F}_{inf}$, $\boldsymbol{F}_{sup}$ are evaluated in the yield conditions (3) ($-N_{0,cr,j} \le N_{j,min}$, $N_{j,max} \le N_{0,j}$, $j \in J$), but here a particular loading history is not considered. Functions $f_{max,j} \ge 0$, $f_{min,j} \ge 0$ ($\boldsymbol{f}_{max} = \left(f_{max,j}\right)^T$, $\boldsymbol{f}_{min} = \left(f_{min,j}\right)^T$), $j \in J$ are convex, the matrix $\boldsymbol{D}$ is positively defined, therefore optimal solution $\boldsymbol{N}_r^*$ of analysis problem (1)–(3) is global.

A possible failure of bars under compression because of buckling is evaluated by introducing reduced limit axial force vector $\boldsymbol{N}_{0,cr}$ in yield conditions $\boldsymbol{f}_{min} = \boldsymbol{N}_{0,cr} + \boldsymbol{N}_r + \boldsymbol{N}_{e,min} \geq \boldsymbol{0}$. Components $N_{0,cr,j}$ ($j \in J$) of vector $\boldsymbol{N}_{0,cr}$ are determined according to the recommendations of Eurocode 3

$$N_{0,cr,j} = \varphi_j N_{0,j} \tag{5}$$

here

$$\varphi_j = \frac{1}{\Phi_j + \left[\Phi_j^2 - \bar{\lambda}_j^2\right]^{0.5}} \tag{6}$$

where

$$\Phi_j = 0.5\left(1 + a\left(\bar{\lambda}_j - 0.2\right) - \bar{\lambda}_j^2\right),$$
$$\bar{\lambda}_j = \frac{\lambda_j}{\lambda_{1j}}\sqrt{\beta_A} = \frac{\lambda_j}{\pi\left[E_j / \sigma_{y,j}\right]^{0.5}}\sqrt{\beta_A}, \quad j \in J.$$

Here E_j is elasticity modulus of j-th bar; $\lambda_j = L_j / i_j$ is bar slenderness, i_j is radius of gyration. In the case of bar under pure compression $\beta_A = 1$, the value of imperfection factor a depends on the shape of cross-sections and the properties of applied material ($a = 0.21$ for hot rolled pipes). A possible failure because of buckling of the system with bars under tension and compression is not evaluated when $\boldsymbol{N}_{0,cr} = \boldsymbol{N}_0$ ($\varphi_j = 1$, $j \in J$).

Mathematical model (1)–(3) can be rewritten simpler
minimize

$$0.5\ \boldsymbol{N}_r''^T \boldsymbol{B}\,\boldsymbol{D}\,\boldsymbol{B}^T \boldsymbol{N}_r'' = 0.5\ \boldsymbol{N}_r''^T \tilde{\boldsymbol{D}}\,\boldsymbol{N}_r'' \tag{7}$$

subject to

$$\left.\begin{aligned} \boldsymbol{f}_{max} &= \boldsymbol{N}_0 - \boldsymbol{B}^T \boldsymbol{N}_r'' - \boldsymbol{N}_{e,max} \geq \boldsymbol{0} \\ \boldsymbol{f}_{min} &= \boldsymbol{N}_{0,cr} + \boldsymbol{B}^T \boldsymbol{N}_r'' + \boldsymbol{N}_{e,min} \geq \boldsymbol{0} \end{aligned}\right\} \tag{8}$$

here $\boldsymbol{N}_r = \boldsymbol{B}^T \boldsymbol{N}_r''$, $\boldsymbol{N}_r = \left(\boldsymbol{N}_r', \boldsymbol{N}_r''\right)^T$, $\boldsymbol{A}'\boldsymbol{N}_r' + \boldsymbol{A}''\boldsymbol{N}_r'' = \boldsymbol{0}$, where matrix

$$\boldsymbol{B} = \left[-\boldsymbol{A}''^T\left(\boldsymbol{A}'^T\right)^{-1}, \boldsymbol{I}\right] \tag{9}$$

Vector of residual forces $\boldsymbol{N}_r''^*$ is the optimal solution of analysis problem (7), (8).

Dual problem to the (7), (8) is stated as follows
maximize

$$\left\{-0.5\,\boldsymbol{N}_r''^T \tilde{\boldsymbol{D}}\,\boldsymbol{N}_r'' - \boldsymbol{\lambda}_{max}^T\left(\boldsymbol{N}_0 - \boldsymbol{N}_{e,max}\right) - \boldsymbol{\lambda}_{cr}^T\left(\boldsymbol{N}_{0,cr} + \boldsymbol{N}_{e,\min}\right)\right\} \tag{10}$$

subject to

$$-\boldsymbol{B}\boldsymbol{\Theta}_p = \tilde{\boldsymbol{D}}\,\boldsymbol{N}_r'', \quad \boldsymbol{\Theta}_p = \boldsymbol{\lambda}_{max} - \boldsymbol{\lambda}_{cr} \tag{11}$$
$$\boldsymbol{\lambda}_{max} \geq \boldsymbol{0}, \quad \boldsymbol{\lambda}_{cr} \geq \boldsymbol{0} \tag{12}$$

Unknowns of the problem (10)–(12) are residual axial forces $\boldsymbol{N}_r''$ and plasticity multipliers $\boldsymbol{\lambda}_{max}$, $\boldsymbol{\lambda}_{cr}$ ($\boldsymbol{\Theta}_p = \boldsymbol{\Theta}_{p,0} + \boldsymbol{\Theta}_{p,cr}$, $\boldsymbol{\Theta}_{p,0} = \boldsymbol{\lambda}_{max}$, $\boldsymbol{\Theta}_{p,cr} = -\boldsymbol{\lambda}_{cr}$). Conditions (11) are compatibility equations of residual strains $\boldsymbol{\Theta}_r$ (they can be obtained from geometrical equations $\boldsymbol{D}\,\boldsymbol{N}_r + \boldsymbol{\lambda}_{max} - \boldsymbol{\lambda}_{cr} - \boldsymbol{A}^T\boldsymbol{u}_r = \boldsymbol{0}$ by the elimination of residual displacements $\boldsymbol{u}_r$). The optimal solution $\boldsymbol{N}_r''^*$, $\boldsymbol{\lambda}_{max}^*$, $\boldsymbol{\lambda}_{cr}^*$ of the problem (10)–(12) is obtained without considering the loading history (full vector $\boldsymbol{N}_r^*$ is determined applying matrix (9); residual displacements $\boldsymbol{u}_r^*$ are obtained analogously). Nevertheless, a particular loading history $\boldsymbol{F}(t)$ ($\boldsymbol{F}_{inf} \leq \boldsymbol{F}(t) \leq \boldsymbol{F}_{sup}$) exists, which leads the structure to shakedown with $\boldsymbol{N}_r^*$, $\boldsymbol{u}_r^*$ and $\boldsymbol{\lambda}_{max}^*$, $\boldsymbol{\lambda}_{cr}^*$. When the sign of object function (10) is changed to opposite one, mathematical model (10)–(12) corresponds the principle of minimum total potential energy.

The appearance of plastic strains $\boldsymbol{\Theta}_p = \boldsymbol{\lambda}_{max} - \boldsymbol{\lambda}_{cr}$ is related with the rule (complementary slackness conditions) $\boldsymbol{\lambda}_{max}^T\boldsymbol{f}_{max} = 0$, $\boldsymbol{\lambda}_{cr}^T\boldsymbol{f}_{min} = 0$, $\boldsymbol{\lambda}_{max} \geq \boldsymbol{0}$, $\boldsymbol{\lambda}_{cr} \geq \boldsymbol{0}$.

During shakedown process local unloading phenomenon (non-holonomic plasticity) of truss bars is frequent occurrence. It occurs when during plastic deformation process $f_{max,j} = 0$, $\lambda_{max,j} > 0$ (when $\lambda_{max,j} f_{max,j} = 0$) and in optimal solution of the problem (10)–(12) it is obtained that $f_{max,j} > 0$ and $\lambda_{max,j}^* > 0$, $j \in J$. Technically notion of unloading (when $f_{cr,j} > 0$ and $\lambda_{cr,j}^* > 0$, $j \in J$) is possible to apply also for truss bars under compression.

Complementary slackness conditions of mathematical programming

$$\left.\begin{aligned} &\boldsymbol{\lambda}_{max}^T\left(\boldsymbol{N}_0 - \boldsymbol{N}_r - \boldsymbol{N}_{e,max}\right) = 0 \\ &\boldsymbol{\lambda}_{cr}^T\left(\boldsymbol{N}_{0,cr} + \boldsymbol{N}_r + \boldsymbol{N}_{e,\min}\right) = 0 \\ &\boldsymbol{\lambda}_{max} \geq \boldsymbol{0},\ \boldsymbol{\lambda}_{cr} \geq \boldsymbol{0} \end{aligned}\right\} \tag{13}$$

are included into problem (10)–(12). Conditions (13) do not allow direct evaluation of the unloading phenomenon of bars and non-monotonic variation of residual displacements $\boldsymbol{u}_r(t)$ during shakedown process. That is why, variation bounds of residual displacements $\boldsymbol{u}_{r,inf}$, $\boldsymbol{u}_{r,sup}$ ($\boldsymbol{u}_{r,inf} \leq \boldsymbol{u}_r(t) \leq \boldsymbol{u}_{r,sup}$) will be applied in stiffness constraints of truss minimum volume problem.

3. Kuhn-Tucker conditions and truss analysis problem

Problem (7), (8) in terms of mathematical programming theory could be written as follows
minimize

$$\left\{\mathcal{F}(\boldsymbol{x}) = \frac{1}{2}\boldsymbol{x}^T\tilde{\boldsymbol{D}}\,\boldsymbol{x} \ \middle|\ \boldsymbol{x} \in \mathcal{L}\right\} \tag{14}$$

Here $\mathcal{L} = \left\{\boldsymbol{x} \mid \varphi_z(\boldsymbol{x}) \geq 0 \text{ for } z = 1, 2, ..., \zeta,\ z \in Z\right\}$ is an admissible set of variables $\boldsymbol{x}$. The global solution $\boldsymbol{x}^* \in \mathcal{L}$

minimizes an object function $\mathcal{F}\left(\boldsymbol{x}^*\right)$.

Kuhn-Tucker conditions for optimal solution $\boldsymbol{x}^*$ of convex mathematical programming problem (14) read [9]

$$\nabla F\left(x^*\right) - \sum_z \lambda_z \nabla\varphi_z\left(x^*\right) = 0 \tag{15}$$

$$\lambda_z \varphi_z\left(\boldsymbol{x}^*\right) = 0, \quad \lambda_z \geq 0, \quad z \in Z \tag{16}$$

Kuhn-Tucker conditions for the problem (7), (8) obtain the following form

$$-\boldsymbol{B}\boldsymbol{\Theta}_p = \widetilde{\boldsymbol{D}}\boldsymbol{N}_r''^*, \quad \boldsymbol{\Theta}_p = \boldsymbol{\lambda}_{max} - \boldsymbol{\lambda}_{cr} \tag{17}$$

$$\boldsymbol{\lambda}_{max}^T \boldsymbol{f}_{max} = 0, \; \boldsymbol{\lambda}_{cr}^T \boldsymbol{f}_{min} = 0, \; \boldsymbol{\lambda}_{max} \geq \boldsymbol{0}, \; \boldsymbol{\lambda}_{cr} \geq \boldsymbol{0} \tag{18}$$

Full equation system, characterizing stress-strain state of the structure at shakedown, is obtained by integration of relations (8) and (17), (18). This equation system results

$$\boldsymbol{N}_r = \boldsymbol{N}_{r,0} + \boldsymbol{N}_{r,cr} = \boldsymbol{G}\boldsymbol{\lambda}_{max} - \boldsymbol{G}\boldsymbol{\lambda}_{cr} = \\ = \boldsymbol{G}\left(\boldsymbol{\lambda}_{max} - \boldsymbol{\lambda}_{cr}\right) = \boldsymbol{G}\boldsymbol{\Theta}_p \tag{19}$$

$$\boldsymbol{u}_r = \boldsymbol{H}\left(\lambda_{max} - \lambda_{cr}\right) = \boldsymbol{H}\boldsymbol{\Theta}_p \tag{20}$$

$$\left.\begin{array}{l} \boldsymbol{\lambda}_{max}^T \boldsymbol{f}_{max} = 0, \; \boldsymbol{\lambda}_{cr}^T \boldsymbol{f}_{min} = 0, \; \boldsymbol{\lambda}_{max} \geq \boldsymbol{0}, \; \boldsymbol{\lambda}_{cr} \geq \boldsymbol{0} \\ \boldsymbol{f}_{max} \geq \boldsymbol{0}, \; \boldsymbol{f}_{min} \geq \boldsymbol{0} \end{array}\right\} \tag{21}$$

Here $\boldsymbol{G}$ and $\boldsymbol{H}$ are influence matrixes of residual forces $\boldsymbol{N}_r$ and displacements $\boldsymbol{u}_r$:

$$\boldsymbol{G} = \boldsymbol{K}\left(\boldsymbol{A}^T\boldsymbol{\alpha}^T - \boldsymbol{I}\right), \quad \boldsymbol{H} = \boldsymbol{\alpha}^T \tag{22}$$

Influence matrixes $\boldsymbol{G}$ and $\boldsymbol{H}$ can be obtained by means of distortion [10]. Finally, it is possible obtain

$$\boldsymbol{N}_r''^* = -\widetilde{\boldsymbol{D}}^{-1}\boldsymbol{B}\boldsymbol{\Theta}_p^* = \boldsymbol{G}''\left(\boldsymbol{\lambda}_{max}^* - \boldsymbol{\lambda}_{cr}^*\right) \tag{23}$$

where matrix $\boldsymbol{G}'' = -\widetilde{\boldsymbol{D}}^{-1}\boldsymbol{B}$ is sub-matrix of influence matrix $\boldsymbol{G}$.

4. Kuhn-Tucker conditions and Rosen optimality criterion

In this paper Rosen project gradient method [9] is applied for numerical truss experiments. Here gradient $\nabla\mathcal{F}\left(\boldsymbol{x}\right)$ of object function $\mathcal{F}\left(\boldsymbol{x}\right)$ is projected on the boundary of admissible field $\mathcal{L}$ (problem (14)). Vector $\boldsymbol{x}^*$ is the optimal solution if satisfies Rosen algorithm optimality criterion

$$\left\{\boldsymbol{I} - \nabla^T\varphi\left(\boldsymbol{x}^*\right)\left(\nabla\varphi\left(\boldsymbol{x}^*\right)\nabla^T\varphi\left(\boldsymbol{x}^*\right)\right)^{-1}\nabla\varphi\left(\boldsymbol{x}^*\right)\right\}\cdot \\ \nabla\mathcal{F}\left(\boldsymbol{x}^*\right) = \boldsymbol{0} \tag{24}$$

$$\left(\nabla\varphi\left(\boldsymbol{x}^*\right)\nabla^T\varphi\left(\boldsymbol{x}^*\right)\right)^{-1}\nabla\varphi\left(\boldsymbol{x}^*\right)\nabla\mathcal{F}\left(\boldsymbol{x}^*\right) \geq \boldsymbol{0} \tag{25}$$

Here $\nabla\varphi\left(\boldsymbol{x}^*\right) = \dfrac{\partial\varphi\left(\boldsymbol{x}^*\right)}{\partial\boldsymbol{x}}$ are gradients of problem (14) active constraints, i.e. satisfied as equalities $\varphi_i\left(\boldsymbol{x}\right) = 0$, $z \in Z$. In the equation set (24), (25) relations (25) are plasticity multipliers

$$\left.\begin{array}{l} \boldsymbol{\lambda} = \left(\nabla\varphi\left(\boldsymbol{x}^*\right)\nabla^T\varphi\left(\boldsymbol{x}^*\right)\right)^{-1}\nabla\varphi\left(\boldsymbol{x}^*\right)\nabla\mathcal{F}\left(\boldsymbol{x}^*\right) \\ \boldsymbol{\lambda} \geq \boldsymbol{0} \end{array}\right\} \tag{26}$$

associated with active yield conditions $\varphi_z\left(\boldsymbol{x}^*\right) = 0$, $z \in Z$ (plasticity multipliers, corresponding to non-active yield conditions are equal to zero). Therefore complementary slackness conditions $\boldsymbol{\lambda}^T\boldsymbol{\varphi}\left(\boldsymbol{x}^*\right) = 0$ of mathematical programming are satisfied for all $z \in Z$. Thus, non-active yield conditions are not included in relations (24), (25). Relations (24) are strain compatibility Eq. (11) and all system (24), (25) is Kuhn-Tucker conditions for problem (14).

It is advisable to apply Rosen algorithm, because solving static analysis problem formulation (7), (8) optimal solution of dual problem is also obtained (like with known Simplex method). That is especially important for shakedown analysis problem solution with non-linear yield conditions [11].

5. About residual displacements of trusses at shakedown

When shakedown safety factor is $s > 1$, it is possible to determine only variation bounds $\boldsymbol{u}_{r,inf}$, $\boldsymbol{u}_{r,sup}$ of residual displacements $\boldsymbol{u}_r\left(t\right)$, ($\boldsymbol{u}_{r,inf} \leq \boldsymbol{u}_r\left(t\right) \leq \boldsymbol{u}_{r,sup}$) of the truss at shakedown (load variation bounds $\boldsymbol{F}_{inf}$, $\boldsymbol{F}_{sup}$ are known). There are many different precision techniques for residual displacement bounds $\boldsymbol{u}_{r,inf}$, $\boldsymbol{u}_{r,sup}$ calculation of adapted structure [4, 5, 6]. Their comparative review is possible to find in the work of Lange-Hasen [12]. In the research [8] was proposed a method of fictitious structure for displacement bound $\boldsymbol{u}_{r,inf}$, $\boldsymbol{u}_{r,sup}$ calculation

maximize (minimize)

$$\widetilde{\boldsymbol{H}}_i\,\widetilde{\boldsymbol{\lambda}} = \begin{bmatrix} u_{ri,sup} \\ u_{ri,inf} \end{bmatrix}, \quad i = 1,2,\ldots,m \tag{27}$$

subject to

$$-\widetilde{\boldsymbol{B}}_\lambda\,\widetilde{\boldsymbol{\lambda}} = \boldsymbol{N}_r''^*, \quad \widetilde{\boldsymbol{\lambda}} \geq \boldsymbol{0} \tag{28}$$

$$\widetilde{\boldsymbol{\lambda}}^T\widetilde{\boldsymbol{N}}_0 \leq \widetilde{D}_{max} \tag{29}$$

here $\boldsymbol{N}_r''^*$ is the vector of residual forces obtained by shakedown analysis problem (7), (8) solution, $\widetilde{\boldsymbol{N}}_0$ is the vector of limit forces of fictitious structure discrete model, $\widetilde{D}_{max}$ is maximal magnitude of dissipated energy during shakedown process (vector of plasticity multipliers $\widetilde{\boldsymbol{\lambda}} = \left(\widetilde{\boldsymbol{\lambda}}_{max}, \; \widetilde{\boldsymbol{\lambda}}_{cr}\right)^T$ is compatible with $\widetilde{\boldsymbol{N}}_0$). Upper bound of the dissipated energy $\widetilde{D}_{max}$ can be also calculated by Koiter's suggested formula [13]. The fictitious structure method allows determining more exactly the residual displacement variation bounds $\boldsymbol{u}_{r,inf}$, $\boldsymbol{u}_{r,sup}$ compared with global Koiter's conditions.

6. Mathematical models of truss minimum volume problem

A project of minimum volume truss is determined by solving the following problem
minimize

$$\sum_j L_j A_j \tag{30}$$

subject to

$$\left.\begin{aligned} \boldsymbol{f}_{max}(A) &= \boldsymbol{N}_0 - \boldsymbol{G}\boldsymbol{\Theta}_p - \boldsymbol{N}_{e,max} \geq \boldsymbol{0} \\ \boldsymbol{f}_{min}(A) &= \boldsymbol{N}_{0,cr} + \boldsymbol{G}\boldsymbol{\Theta}_p + \boldsymbol{N}_{e,min} \geq \boldsymbol{0} \end{aligned}\right\} \tag{31}$$

$$\left.\begin{aligned} \boldsymbol{N}_0 &= \left(N_{0j}\right)^T, & \boldsymbol{N}_{0,cr} &= \left(N_{0j,cr}\right)^T \\ N_{0,j} &= \sigma_{yj} A_j, & N_{0,j,cr} &= \varphi_j \sigma_{yj} A_j \end{aligned}\right\} \tag{32}$$

$$A_j \geq A_{j,min}, \quad j \in J \tag{33}$$

$$\boldsymbol{\Theta}_p = \boldsymbol{\lambda}_{max} - \boldsymbol{\lambda}_{cr} \tag{34}$$

$$\boldsymbol{\lambda}_{max}^T \boldsymbol{f}_{max} = 0, \ \boldsymbol{\lambda}_{cr}^T \boldsymbol{f}_{min} = 0, \ \boldsymbol{\lambda}_{max} \geq \boldsymbol{0}, \ \boldsymbol{\lambda}_{cr} \geq \boldsymbol{0} \tag{35}$$

$$\boldsymbol{u}_{r,min} \leq \boldsymbol{u}_{r,inf}, \quad \boldsymbol{u}_{r,sup} \leq \boldsymbol{u}_{r,max} \tag{36}$$

Load variation bounds $\boldsymbol{F}_{inf}$, $\boldsymbol{F}_{sup}$ are prescribed, so in the mathematical model (30)–(36) extreme forces $\boldsymbol{N}_{e,max}$, $\boldsymbol{N}_{e,min}$ are known functions from $\boldsymbol{F}_{inf}$, $\boldsymbol{F}_{sup}$.

Unknowns of the non-linear mathematical programming problem (30)–(36) are cross-sectional areas A_j, $j \in J$ of truss elements and vectors of plasticity multipliers $\boldsymbol{\lambda}_{max}$, $\boldsymbol{\lambda}_{cr}$. Lower bound of cross-sectional areas $A_{j,min}$ is included into constructive constraints (33) $A_j \geq A_{j,min}$. Formulas (35) represent complementary slackness conditions of mathematical programming (13). Structure stiffness constrains (36) are realized via restriction of nodal displacements ($\boldsymbol{u}_{r,min}$, $\boldsymbol{u}_{r,max}$ are prescribed lower and upper variation bounds of residual displacements $\boldsymbol{u}_r$). It is not difficult to introduce elastic displacements $\boldsymbol{u}_e$ into stiffness constraints (36) applying influence matrix of displacements $\boldsymbol{\beta}$ and load vectors $\boldsymbol{F}_{inf}$, $\boldsymbol{F}_{sup}$: $\boldsymbol{u}_{min} \leq \boldsymbol{u}_{r,inf} + \boldsymbol{u}_{e,inf}$, $\boldsymbol{u}_{r,sup} + \boldsymbol{u}_{e,sup} \leq \boldsymbol{u}_{max}$. Vectors $\boldsymbol{u}_{e,inf}$, $\boldsymbol{u}_{e,sup}$ are determined according to the formulas analogical to (4). Difficulties of problem (30)–(36) solution are related to direct dependence of influence matrixes $\boldsymbol{\alpha}$, $\boldsymbol{\beta}$, $\boldsymbol{H}$ and $\boldsymbol{G}$ from design variables A_j, $j \in J$.

Mathematical model (30)–(36) can be applied also for the determination of minimal volume of elastic systems, adopting that $\boldsymbol{\Theta}_p = \boldsymbol{0}$
minimize

$$\sum_j L_j A_j$$

subject to

$$\sigma_{yj} A_j - N_{ej,max} \geq 0, \ \ \varphi_j \sigma_{yj} A_j + N_{ej,min} \geq 0$$

$$A_j \geq A_{j,min}, \quad j \in J$$

$$\boldsymbol{u}_{e,min} \leq \boldsymbol{u}_{e,inf} = \boldsymbol{\beta}_{inf} \boldsymbol{F}_{sup} - \boldsymbol{\beta}_{sup} \boldsymbol{F}_{inf}$$

$$\boldsymbol{u}_{e,sup} = \boldsymbol{\beta}_{sup} \boldsymbol{F}_{sup} - \boldsymbol{\beta}_{inf} \boldsymbol{F}_{inf} \leq \boldsymbol{u}_{e,max}$$

here influence matrix of displacement $\boldsymbol{\beta}(A) = \boldsymbol{\beta}_{inf} + \boldsymbol{\beta}_{sup}$ depends on cross-sectional areas A_j, $j \in J$. If $\boldsymbol{N}_{e,max}$, $\boldsymbol{N}_{e,min}$ are calculated from different effects (load combinations, changer of temperature, distortions), just discussed mathematical model becomes useful for practical design.

Discussion comes back to mathematical model (30)–(36). Minimal magnitude of object function (30) is obtained neglecting possible loss of bar stability if the factor of yield stress reduction $\varphi_i = 1$ ($j \in J$) in the yield conditions $\boldsymbol{f}_{min} = \boldsymbol{N}_{0,cr} + \boldsymbol{G}\boldsymbol{\Theta}_p + \boldsymbol{N}_{e,min} \geq \boldsymbol{0}$. The project of minimum volume truss would be obtained according to the conditions of cyclic-plastic collapse, if both, stiffness (36) and stability, constrains were neglected. That could be a failure because of alternating plasticity or incremental collapse. In those cases shakedown theory of elastic-plastic structure cannot be applied.

Stiffness constraints (36), requiring solution of problems (27)–(29), show that the main non-linear truss optimization problem (30)–(36) is not a classical non-linear mathematical programming problem. It should be solved step-by-step. That is why it is useful to change the solution of minimum volume problem (30)–(36) into the solution of two separate problems. *The first problem* is obtained by substituting the stiffness constraints (36) into not so strict ones $\boldsymbol{u}_{r,min} \leq \boldsymbol{H}(\boldsymbol{\lambda}_{max} - \boldsymbol{\lambda}_{cr}) \leq \boldsymbol{u}_{r,max}$, i.e.
minimize

$$\sum_j L_j A_j \tag{37}$$

subject to constraints (31)–(35) and

$$\boldsymbol{u}_{r,min} \leq \boldsymbol{H}(\boldsymbol{\lambda}_{max} - \boldsymbol{\lambda}_{cr}) \leq \boldsymbol{u}_{r,max} \tag{38}$$

In that case classical non-linear mathematical programming problem is obtained. Its optimal solution is A_j^*, $j \in J$, $\boldsymbol{\lambda}_{max}^*$, $\boldsymbol{\lambda}_{cr}^*$. *The second problem* is the problem of residual displacement variation bound $\boldsymbol{u}_{r,inf}$, $\boldsymbol{u}_{r,sup}$ determination (27)–(29). It is solved only after optimal solution A_j^*, $j \in J$, $\boldsymbol{\lambda}_{max}^*$, $\boldsymbol{\lambda}_{cr}^*$ of the problem (37), (38) is obtained. Generally, the second problem is solved, when unloading phenomenon of truss bars occurs.

7. Solution algorithms

In this research calculations of numerical examples of minimal volume truss were performed applying mathematical models of problems (37), (38) and (27)–(29). Rosen gradient method [9] was used for problem solution and mathematical model (37), (38) is transformed into the following one

minimize

$$\left\{ \sum_j L_j A_j + \boldsymbol{\lambda}_{max} \left(\boldsymbol{N}_0 - \boldsymbol{G}\boldsymbol{\Theta}_p - \boldsymbol{N}_{e,max} \right) + \right.$$
$$\left. + \boldsymbol{\lambda}_{cr} \left(\boldsymbol{N}_{0,cr} + \boldsymbol{G}\boldsymbol{\Theta}_p + \boldsymbol{N}_{e,min} \right) \right\} \tag{39}$$

subject to

$$\boldsymbol{f}_{max} = \boldsymbol{N}_0 - \boldsymbol{G}\boldsymbol{\Theta}_p - \boldsymbol{N}_{e,max} \geq \boldsymbol{0} \tag{40}$$

$$\boldsymbol{f}_{min} = \boldsymbol{N}_{0,cr} + \boldsymbol{G}\boldsymbol{\Theta}_p + \boldsymbol{N}_{e,min} \geq \boldsymbol{0} \tag{41}$$

$$\left. \begin{array}{l} \boldsymbol{N}_0 = \left(N_{0j} \right)^T, \quad \boldsymbol{N}_{0,cr} = \left(N_{0j,cr} \right)^T \\ N_{0,j} = \sigma_{yj} A_j, \quad N_{0,j,cr} = \varphi_j \sigma_{yj} A_j \end{array} \right\} \tag{42}$$

$$A_j \geq A_{j,min}, \quad j \in J \tag{43}$$

$$\boldsymbol{\Theta}_p = \boldsymbol{\lambda}_{max} - \boldsymbol{\lambda}_{cr}, \quad \boldsymbol{\lambda}_{max} \geq \boldsymbol{0}, \quad \boldsymbol{\lambda}_{cr} \geq \boldsymbol{0} \tag{44}$$

$$\boldsymbol{u}_{r,min} \leq \boldsymbol{H} \left(\boldsymbol{\lambda}_{max} - \boldsymbol{\lambda}_{cr} \right) \leq \boldsymbol{u}_{r,max} \tag{45}$$

The problem (39)–(45) is equivalent to the problem (37), (38), only its practical realization applying Rosen algorithm is simpler (according to authors experience) than the problem (37), (38) solution.

Different approaches were proposed by Tin-Loi and Ferris [14] for minimum weight problem solution with complementary slackness conditions, they were using penalty and parametric methods.

As it was mentioned earlier the solution of minimum volume problem (30)-(36) is changed into the solution of two separate problems: the first problem (39)-(45) and the second problem (27)-29). From solution algorithm scheme (Fig. 1) it is possible to see the necessity of iterative calculation of the minimum volume problem (39)–(45): stiffness matrix $\boldsymbol{K}$ (herewith influence matrixes $\boldsymbol{\alpha}$, $\boldsymbol{\beta}$, $\boldsymbol{H}$ and $\boldsymbol{G}$) depends on design variables A_j, $j \in J$. Matrix $\boldsymbol{K}$ is assumed as constant in each stage of problem (39)–(45) calculation, when optimal solution $\widetilde{A}_j$, $j \in J$, $\boldsymbol{\lambda}_{max}$, $\boldsymbol{\lambda}_{cr}$ is obtained. In other words, during problem (39)–(45) solution applying Rosen algorithm initial matrixes $\boldsymbol{\alpha}$, $\boldsymbol{G}$, $\boldsymbol{H}$ and reduction factors φ_j, $j \in J$ remain constant with in each stage (Fig. 1). Optimal solution of the problem (39)–(45) $\widetilde{A}_j^*$, $j \in J$, $\boldsymbol{u}_{r,sup}$, $\boldsymbol{\lambda}_{cr}^*$ is obtained at the end of the last stage, when condition $\left| A_j^0 - \widetilde{A}_j \right| \leq \delta$ is satisfied (δ is required precision). After the optimal solution determination it is necessary to check if stiffness constraints (36) $\boldsymbol{u}_{r,min} \leq \boldsymbol{u}_{r,inf}$, $\boldsymbol{u}_{r,sup} \leq \boldsymbol{u}_{r,max}$ are satisfied. In other words, the determination of residual displacement variation bounds $\boldsymbol{u}_{r,inf}$, $\boldsymbol{u}_{r,sup}$, i.e. the solution of the

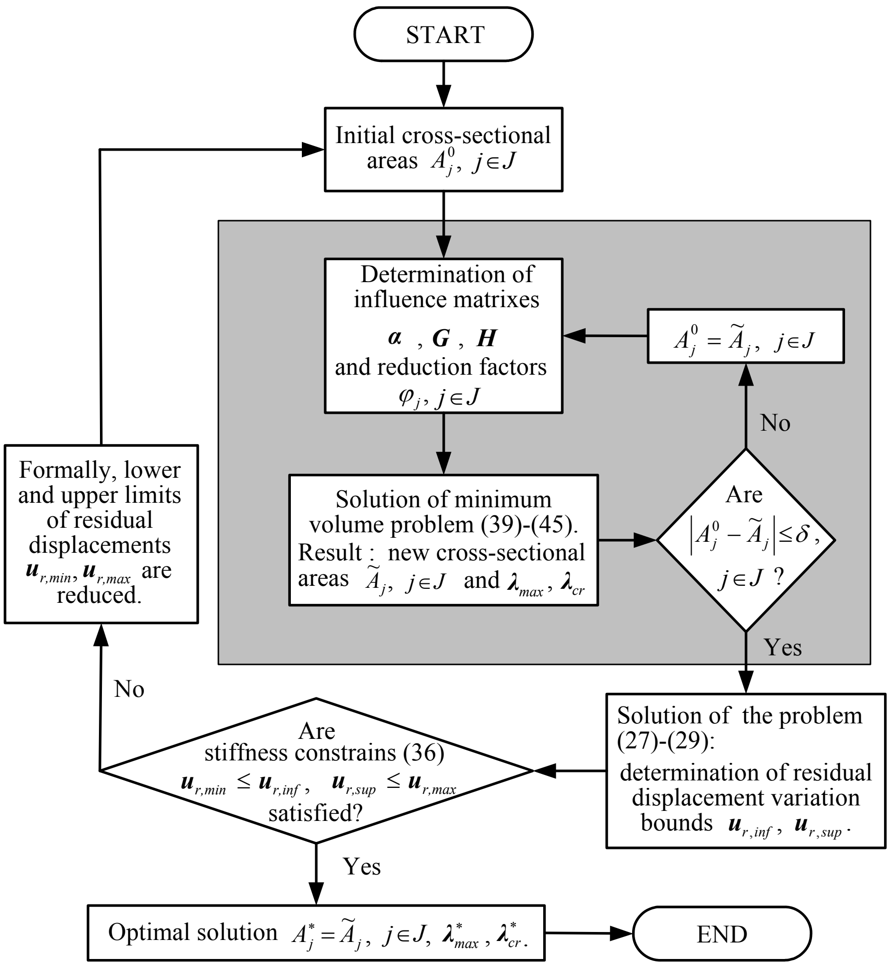

Fig. 1 Flowchart of the proposed solution algorithm

problem (27)–(29), is required. If constraints (36) are violated, formally admissible value (it can be $u_{ri,min}$, $u_{ri,max}$, $i = 1,2,...,m$) of the most violated stiffness constraint is reduced. Later it is returned to the beginning of the first problem (39)–(45) solution, as it is shown in Fig 1. Strictly saying, according to the proposed solution algorithm of truss minimum problem (30)–(36) local optimal solution (that is the result of $u_{ri,min}$, $u_{ri,max}$, $i = 1,2,...,m$ changing technique) is obtained. The algorithm ensures good convergence of optimal solution during calculation process, when strength, stiffness and stability constraints are included into conditions of the problem (30)–(36). Worse convergence of minimum volume problem solution is then, when only strength and stiffness constraints are evaluated. However, most researchers are satisfied that volume minimization in fact does not proceed although cross-sectional areas of separate elements (bars) are alternating varying.

In order to improve convergence, authors of the paper propose more sensitive calculation algorithm of the problem (39)–(45). Here stiffness matrix $\boldsymbol{K}$ is changed not only in each problem solution stage (like earlier), but also in every iteration of Rosen algorithm. It is important to mark that in spite of $\boldsymbol{K}$ changes influence matrix $\boldsymbol{\alpha}$ (herewith $\boldsymbol{H}$) remains constant up to the determination of problem (39)–(45) solution $\widetilde{A}_j$, $j \in J$, $\boldsymbol{\lambda}_{max}$, $\boldsymbol{\lambda}_{cr}$ at the end of calculation stage. Meanwhile only one part

$$\boldsymbol{G}_{const} = \boldsymbol{A}^T \boldsymbol{H} - \boldsymbol{I} \tag{46}$$

of matrix $\boldsymbol{G} = \boldsymbol{K}\left(\boldsymbol{A}^T \boldsymbol{H} - \boldsymbol{I}\right) = \boldsymbol{K}\boldsymbol{G}_{const}$ remains constant in the whole stage.

8. Numerical example

Minimum volume problem of nine-bar truss, shown in Fig. 2, is solved. Truss loading domain is also presented in Fig. 2. The elasticity modulus $E = 21000$ kN/cm^2 and the yield stress $\sigma_y = 20$ kN/cm^2 of the material are the same for all bars. The prescribed minimum values of cross-sectional areas of truss bars are $A_{1,min} = A_{4,min} = A_{5,min} = A_{6,min} = 8$ cm^2, $A_{2,min} = A_{3,min} = A_{9,min} = 5$ cm^2 and $A_{7,min} = A_{8,min} = 10$ cm^2, respectively. Stiffness constraints are realised via vertical residual displacement restriction of node 2 (Fig. 2), $|u_{r2}| \leq 0.04$ cm.

The main task is to solve minimum volume problem (30)–(36), i.e. determine cross-sectional areas A_j, $j = 1, 2, ..., 9$ corresponding optimality criterion, in three following cases:

C1) when stiffness (36) and stability constraints ($\varphi_j = 1.$, $j = 1, 2, ..., 9$) are neglected (the state close to cyclic-plastic collapse);

C2) when stiffness constraints (36) are taken into account;

C3) when both, stiffness and stability, constraints are evaluated.

The results are presented in Table. This research mathematical model of minimum volume problem (30)–(36) is general enough. When stiffness and stability constraints are neglected, minimum volume $V_{min} = 198959$ cm^3 is obtained just before cyclic-plastic collapse of the truss (Table, the first case of problem).

Minimum volume of the truss $V_{min} = 199865$ cm^3 was determined when stiffness constraints were evaluated (Table, the second case of problem). From the eighth stage more sensitive calculation algorithm was applied, i.e. constant part $\boldsymbol{G}_{const}$ (46) of influence matrix $\boldsymbol{G}$ is used while stiffness matrix $\boldsymbol{K}$ changes in iterations of Rosen algorithm. Additional analysis confirmed that $\boldsymbol{u}_{r,inf} = \boldsymbol{u}_{r,sup}$ i.e. unloading phenomenon of truss bars did not appear. Thus, it is possible to apply $\boldsymbol{u}_{r,min} \leq \boldsymbol{H}_{\lambda} \boldsymbol{\lambda} \leq \boldsymbol{u}_{r,max}$ instead of condition (36).

Maximum value of minimum volume $V_{min} = 210835$ cm^3 was obtained when both, stiffness and stability, constraints were taken into account (Table, the third case of problem).

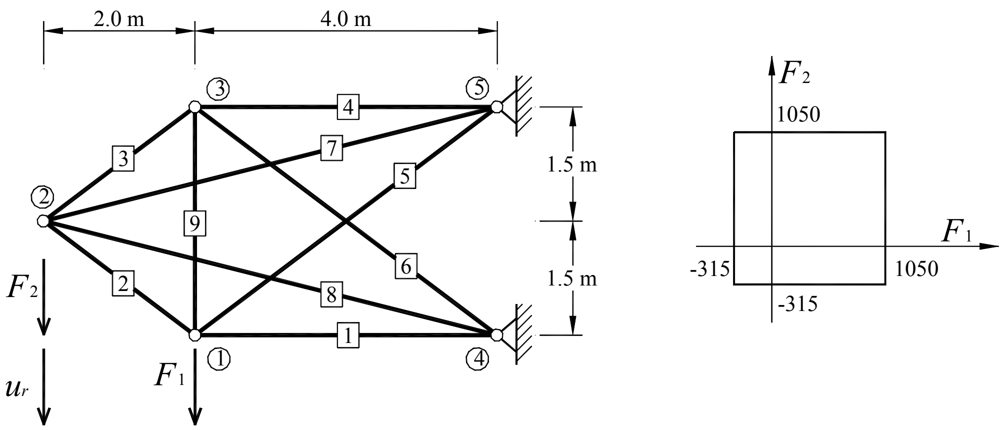

Fig. 2 Nine-bar truss geometry and loading (forces in kN)

Table

Solutions of minimum volume problem (30)–(36) for nine-bar truss

Problem (30)-(36) cases	Stage No	A_1, cm^2	A_2, cm^2	A_3, cm^2	A_4, cm^2	A_5, cm^2	A_6, cm^2	A_7, cm^2	A_8, cm^2	A_9, cm^2	Volume, cm^3
	Initial A	65.000	65.000	65.000	65.000	65.000	65.000	65.000	65.000	65.000	249405
C1	1	94.730	41.951	37.791	69.674	90.974	55.529	33.553	36.955	14.379	206871
	2	114.440	45.698	38.390	69.262	110.370	48.188	17.977	22.693	10.127	201974
	…	…	…	…	…	…	…	…	….	…	…
	8	133.440	43.888	39.708	63.624	127.100	39.821	10.000	10.000	11.319	198951
	9	133.470	43.878	39.708	63.600	127.130	39.791	10.000	10.000	11.345	198959
	10	133.480	43.874	39.708	63.589	127.140	39.778	10.000	10.000	11.357	198959
C2	1	134.670	44.616	39.592	63.400	127.890	39.657	10.000	10.000	11.362	199832
	2	134.360	44.319	39.895	63.766	127.760	39.813	10.000	10.000	11.358	199868
	3	134.780	44.639	39.589	63.350	127.980	39.598	10.000	10.000	11.354	199874
	4	134.360	44.320	39.893	63.759	127.770	39.806	10.000	10.000	11.359	199867
	5	134.780	44.639	39.589	63.347	127.980	39.595	10.000	10.000	11.354	199872
	6	134.940	44.682	39.532	63.181	128.130	39.444	10.000	10.000	11.359	199867
	7	135.010	44.687	39.518	63.107	128.210	39.366	10.000	10.000	11.368	199866
	8	135.040	44.687	39.513	63.073	128.250	39.328	10.000	10.000	11.376	199867
	9	135.060	44.687	39.511	63.057	128.260	39.310	10.000	10.000	11.381	199866
	10	135.070	44.686	39.510	63.050	128.270	39.302	10.000	10.000	11.384	199868
	11	135.070	44.686	39.510	63.047	128.270	39.298	10.000	10.000	11.385	199865
C3	1	144.960	56.236	38.505	63.368	128.180	49.262	10.000	10.000	12.665	211907
	2	143.410	55.163	38.295	64.581	126.360	51.151	10.000	10.000	12.717	211497
	3	142.730	51.018	38.442	65.349	125.610	52.091	10.000	10.000	12.659	210615
	4	142.380	51.009	38.618	65.827	125.280	52.579	10.000	10.000	12.653	210785
	5	142.200	50.993	38.674	66.047	125.090	52.833	10.000	10.000	12.628	210835

9. Conclusions

Not only strength, but also stiffness and stability constraints are included into mathematical models of structure optimization problems at shakedown. Usually stiffness conditions are ensured by restriction of structure deflections or nodal displacements (residual or total ones). During shakedown process residual displacements are varying non-monotonically, that is the result of unloading phenomenon of cross-sections. Complementary slackness conditions of mathematical programming do not allow the evaluation of this physical phenomenon. If variable repeated load is prescribed by its variation bounds, it is possible to prognosticate only variation bound of displacements. The method of fictitious structure for residual displacement variation bounds determination is developed in this paper. Using Kuhn-Tucker conditions new solution algorithm of minimum volume problem at shakedown, based on Rosen project gradient method, is proposed.

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MINIMALAUS TŪRIO PRISITAIKANTI SANTVARA: UŽDAVINIO MATEMATINIAI MODELIAI IR NAUJI SPRENDIMO ALGORITMAI

R e z i u m ė

Nagrinėjama idealiai tampriai plastinė žinomos geometrijos santvara, prisitaikanti prie kintamos kartotinės apkrovos. Nurodytos tik viršutinės ir apatinės nuo laiko nepriklausančios apkrovos kitimo ribos (konkreti apkrovimo istorija nežinoma). Sudaryti nauji netiesinių minimalaus tūrio santvaros skaičiavimo uždavinių matematiniai modeliai prisitaikomumo sąlygomis. Juose įvertinamos ne tik konstrukcijos prisitaikomumo ir standumo sąlygos, bet ir stabilumo netekimo galimybė plastinėje jos darbo stadijoje. Netiesinis uždavinys sprendžiamas etapais, kiekviename iš jų perskaičiuojant santvaros strypų besikeičiančius standžius. Pasiūlytas naujas algoritmas santvaros strypų nusikrovimo įvertinimui standumo sąlygomis. Jis iliustruojamas strypinės šarnyrinės santvaros minimalaus tūrio uždavinio sprendimu. Rezultatai gauti, darant mažų poslinkių prielaidą.

D. Merkevičiūtė, J. Atkočiūnas

MINIMUM VOLUME OF TRUSSES AT SHAKEDOWN - MATHEMATICAL MODELS AND NEW SOLUTION ALGORITHMS

S u m m a r y

Perfectly elastic-plastic truss of known geometry adapted to variable repeated load (only its lower and upper variation bounds are prescribed) is considered. Non-linear mathematical models of truss minimum volume problem are formulated. Not only strength and stiffness constraints are included in problem formulations, but also possible bar buckling is taken into account. As during volume minimization stiffness of truss elements is changing, non-linear optimization problem is solved step-by-step. Solution algorithm allows the evaluation of bar unloading phenomenon, which often occurs during shakedown process. The technique is illustrated by numerical example of pin-joined bar system calculation. The results are valid for the small displacement assumptions.

Д. Меркявичюте, Ю. Аткочюнас

МИНИМАЛЬНЫЙ ОБЪЁМ ФЕРМЫ В УСЛОВИЯХ ПРИСПОСОБЛЯЕМОСТИ – МАТЕМАТИЧЕСКИЕ МОДЕЛИ ЗАДАЧ И И НОВЫЕ АЛГОРИТМЫ РЕШЕНИЯ

Р е з ю м е

Рассматривается идеально упругопластическая ферма заданной геометрии, приспособившаяся к повторно-переменному нагружению. Нагрузка характеризуется только независящими от времени верхними и нижними пределами изменения (конкретная история нагружения при этом неизвестна). Построены новые математические модели нелинейных задач определения ферм минимального объема в условиях приспособляемости. В математических моделях учитываются не только условия прочности и ограничения на жесткость конструкции, но и возможность учета потери ее устойчивости в условиях пластической работы. Решение нелинейных задач осуществляется этапами, в пределах которых пересчитываются жесткости претерпевших изменения элементов фермы. Разработан новый алгоритм учета явления разгрузки элементов фермы, столь характерного для процесса приспособляемости конструкций. Предлагаемый алгоритм иллюстрируется примером минимизации объема шарнирно-стержневой системы. Полученные результаты действительны при малых перемещениях.

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