

Dynamic characteristics identification of aircraft engine by method of trial parameters

V. Royzman, A. Bohorosh

*Khmelnitsky National University, 11, Institutskaya str., 29016 Khmelnitskiy, Ukraine,
E-mail: roizman@mailhub.tup.km.ua*

1. Introduction

Amplitude-frequency characteristics of an aircraft engine usually are based on continuous recordings of local vibrations of its body beginning from the moment of starting the engine up to the maximum possible rotational speed. Recordings not only characterize the level of machine vibrations but also carry latent or patent information on dynamic characteristics (masses, rigidity, inertia moments, damping) of many parts and units of the engine. For a complex machine these characteristics are of multi-peaked type which means that the engine comes through some resonances while operated but their sources might be unknown. At the same time it is very important to identify each of resonances to know how to prevent them.

Usual researches on unit dynamics and machine dynamics as a whole do not always lead to the success if tests are far from real operation conditions and do not take into account all the variety of effect factors in dynamic model of the system, but at the same time it is very important to find exact values of dynamic characteristics applied in mathematical model. All this is possible to get by solving converse problem of dynamics with amplitude-frequency characteristics obtained by an experiment.

This paper shows a compressor of aircraft engine AE-20 as an example how with the help of mathematical model of rotor vibrations of multi staged compressor on elastic supports connected to the masses of crankcase frames and combustion chamber it is possible to identify unknown parameters of the system and to calculate origins of all five resonances. For that the method of trial parameters is suggested.

2. Trial parameters method

Usually the information obtained by simple observation of characteristics calculated more or less precisely is not enough for some reasons to identify parameters of mathematical model.

Therefor active, controlled effect on an object might be the base of algorithmic presentation of operative mathematical modeling. Let the rated model of an investigated object is represented in the form

$$Y_i = f(q_1, q_2, \dots, q_n; \beta_1, \beta_2, \dots, \beta_n), i=1,2,\dots, \ell \quad (1)$$

If the values of output characteristics Y_i , obtained under real functioning conditions of an object are substituted into (1) as well as some reliable initial factors (q, β), then the system (1) should be fulfilled also by some number k of unknown values of initial factor and by j coeffi-

cients of a model.

More often the system (1) when $k+j > \ell$ has infinite set of solutions. When an object is real and functions in reality the solution should be chosen that it. For this the problem should be redefined and the practical method such of a definition extension is the method of trial parameters that requires some replicate experiments.

To provide it in practice some additional or changed $k+j - \ell$ elements should be substituted in and then the subject should be tested on trial mode that is the functioning of the subject should be activity controlled.

Input of trial elements (modes) are identified the deficit number of values of output parameters to complete the system (1) are known and the desired factors and coefficients can be identified that is inverse point problem solved.

3. Subject description and problem statement

Rotor of GTE-AE-20 compressor is of disc and drum type and consist of 10 separate discs rimmed by the blades of rear rotor shaft and seal labyrinth of front and rear bearings units (Fig. 1).

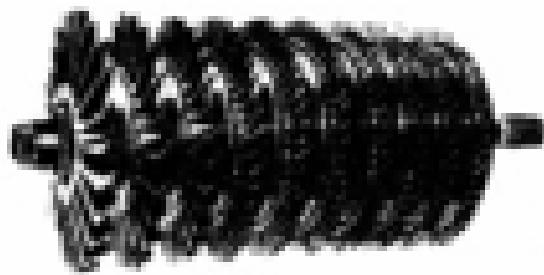


Fig. 1 Rotor of compressor GTE-AE-20

During machine assembly the front bearing is installed in frontal case and the rear joint in the body of combustion chamber. Stiffness of rotor supports is determined by these units.

Long time operation of the engine AE-20 and the number of bed tests showed such defects as following - bending of the rear shaft, breakdown of pin joints of the rotor mainly of the joint of rear shaft with 10th stage of rotor and some other defects of combustion chamber body.

It was assumed that mentioned defects arise in connection with the rotor of compressor running through critical revolution frequencies in engine system.

This assumption is increased when the level of vibrations of some engines AE-20 in idling conditions where oscillations are of resonance character are inadmis-

sible high. Calculations of the first critical rotation frequency of the rotor is carried out by Stodola method with traverse forces taken into consideration but ignoring supports compliance and the compliance of stages joints. Having no reliable data it was obtained equal to 22150 rpm and critical frequency of rotation found by integral method ignoring traverse forces turned to be 25400 rpm. Critical rotation frequency found with taking in account local compliance was equal to 19400 rpm and the account of local compliance was done with the help of experimentally found compliance of the rotor in midsection.

At last allowing supports compliance the critical rotation frequency is 15800 and 16600 rpm allowing gyroscopic effect.

Calculated with position coefficients of influence the critical rotation frequency of compressor rotor of AE-20 motor on rigid supports is in the limits of $n=11200$ and $n=12200$ rpm that is it might vary 9-10% depending on the values of stresses at interference fits of discs and pins and depending on rigidity of sections in between rotor stages.

In Fig. 2 amplitude-frequency characteristics of an engine AE-20 is shown obtained during the test on a vacuum test bed. 4-5 resonances are clearly seen in it as well as the mentioned defects connected with rotor overcoming the resonances or even with the operation at or near them. It was necessary to know the origin of all resonances in order to have the possibility of affecting amplitude and frequency values of the resonances.

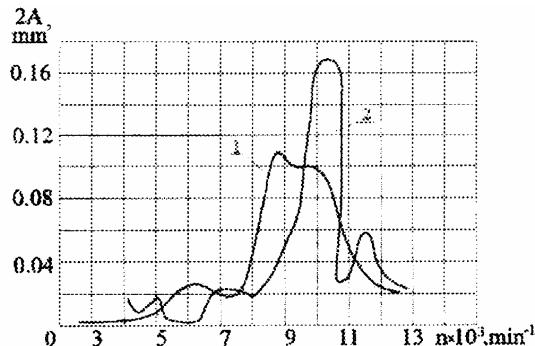


Fig. 2 Amplitude – frequency characteristics of compressor: 1 - on front support; 2 - on rear support

4. Calculation scheme and dynamic model

Dynamic model of a system rotor-supports vibrations is represented in Fig. 3. The compressor rotor of mass m and compliance under mass center α_{11} lies on two rigid supports of compliances $\alpha_{AA} = \alpha_{22}$ and $\alpha_{BB} = \alpha_{33}$ jointed to masses of front crankcase m_2 and of combustion chamber m_3 .

It should be mentioned that the rotor itself is outlined by point mass which is equal to the mass of all rotor applied to mass center and has the rigidity at the joint point that is equal to the rigidity of the rotor in the section that contains mass center.

Let us show that such simplification does not affect the values of critical rotation frequency to compare with the one calculated for multi-disk rotor when real values of rigidities and masses are used into both schemes.

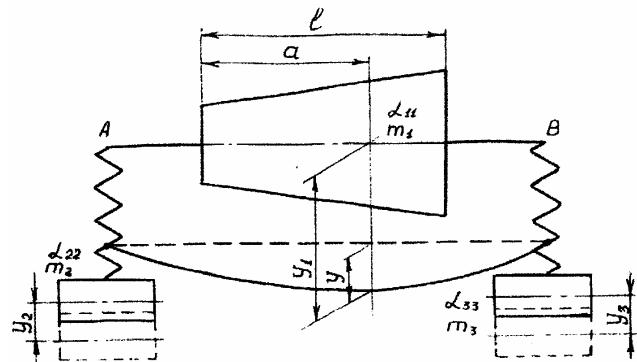


Fig. 3 Calculating scheme

To find ω_{kp} it is necessary to solve the equation

$$1 - m_1 \alpha_{11} \omega^2 = 0$$

or

$$n_{kp} = \frac{300}{\sqrt{\delta_{ct}}} \text{ rpm} \quad (2)$$

where δ_{ct} is rotor weight deflection. The center of gravity of compressor rotor for engine AE-20 is in between 5 th and 6 th stages $a=l_{ct}=43.74$ cm.

According to static stresses of rotor we may choose the affect coefficient for rotor section along the center of gravity

$$\alpha_{11} = \alpha_{ct} = 6.35 \cdot 10^{-6} \text{ cm}$$

$$\delta_{ct} = 6.35 \times 115.4 \times 10^{-6} = 770 \times 10^{-6} \text{ cm}$$

$$n_{kp} = \frac{300 \times 10^3}{27} = 11200 \text{ r/m}$$

where 115.4 N is weight of rotor.

More precise value calculated by Reley method the rotor allowing the effect coefficients of all stages was equal to 11220 rpm.

So the simplification error of 10 masses rotor with one mass is not more then 0.4% that is why for the calculations of ω_{kp1} a rotor may be considered to be one mass system and exact values of rigidity and masses may be used.

Values of critical rotation frequencies and resonances of such three mass system may be found with a help of solution of "ancient" determinant of 3rd order that describes its oscillations

$$\begin{vmatrix} 1 - m_1 \alpha_{11} \omega^2 & -m_2 \alpha_{12} \omega^2 & -m_3 \alpha_{13} \omega^2 \\ -m_1 \alpha_{21} \omega^2 & 1 - m_2 \alpha_{22} \omega^2 & -m_3 \alpha_{23} \omega^2 \\ -m_1 \alpha_{31} \omega^2 & -m_2 \alpha_{32} \omega^2 & 1 - m_3 \alpha_{33} \omega^2 \end{vmatrix} = 0 \quad (3)$$

In this case all determinant compliances but α_{11} are unknown.

To find unknown compliances the method of Trial parameters was used. As a test sample rigid ring shown in Fig. 4 was used.

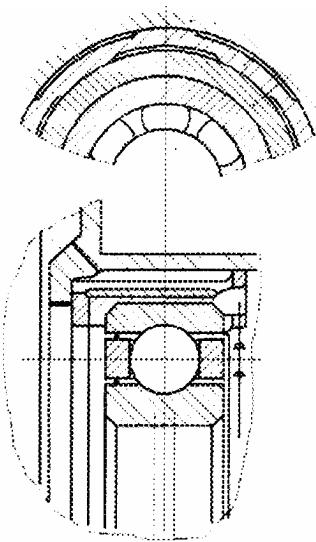


Fig. 4 Rotor support with elastic ring

Rigidity which was found with the help of special device turned out to be considerably greater than the rotor compliance

$$\alpha_{ring} = 50 \times 10^6 \text{ 1/m}$$

The compliance of rotor supports that are in a front crankcase in combustion chamber where identified step by step:

1. knowing the ring rigidity and assuming the front support rigidity of a compressor with a help of engine resonance value (Fig. 5) the compliance of back support was identified by the study of oscillations of 1-mass rotor on 2 compliant supports;

2. by means of compliance value of back support and resonance values of the engine without rigid support (Fig. 5) the compliance of front support was identified.

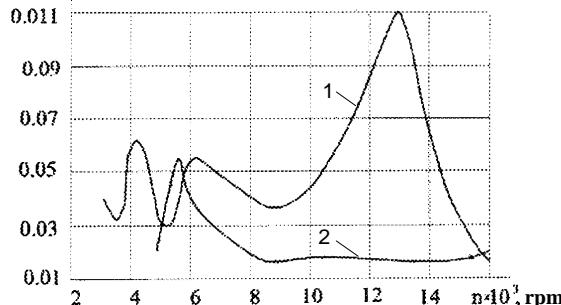


Fig. 5 Smoothed amplitude - frequency characteristics of AE-20 engine: 1 - without of rigid ring; 2 - with rigid ring

First the displacements of supports that were initiated by unit force applied of the center of masses of the rotor (or overoutlined system) were found that is effect coefficients α_{21} and α_{31} and then displacements of gravity, center to support rigidities and new effect coefficient at this point (Fig. 6) were found

$$\alpha_{21} = \alpha^k_{22} \frac{b}{l}; \alpha_{31} = \alpha_{33} \frac{a}{l}$$

$$AE = \frac{b}{l} \alpha^k_{22}; \quad BD = \frac{a}{l} \alpha_{33}; \quad CE = \frac{b}{l} \alpha^k_{22} - \frac{a}{l} \alpha_{33}$$

$$\delta_{11} = FK + SF = \frac{b^2}{l^2} \alpha^k_{22} - \frac{ab}{l^2} \alpha_{33} + \frac{a}{l} \alpha_{33}$$

$$\delta_{11} = \frac{b^2}{l^2} \alpha^k_{22} + \frac{a^2}{l^2} \alpha_{33}$$

Therefore the sum coefficient of the effect is equal to

$$\alpha *_{11} = \alpha_{11} + \frac{b^2}{l^2} \alpha^k_{22} + \frac{a^2}{l^2} \alpha_{33} \quad (4)$$

From equation

$$\omega^2_{kp} = \frac{1}{m_1 \alpha *_{11}} = \frac{q}{G_1 \alpha *_{11}}$$

it was found α_{11} and from Eq. (4) α_{33} supposing that for the first stage of calculation α_{22} is compliance of the rigid ring. In this case α_{33} was identified by considering symmetrical oscillations of the rotor as of a body on pliable supports. In case of anti-symmetrical oscillations that correspond to more high frequency resonance (Fig. 6) and by means of similarity of triangles CDD and FKC it was found

$$\delta_{11} = \alpha^k_{22} \frac{b^2}{l^2} + \alpha_{33} \frac{a^2}{l^2}$$

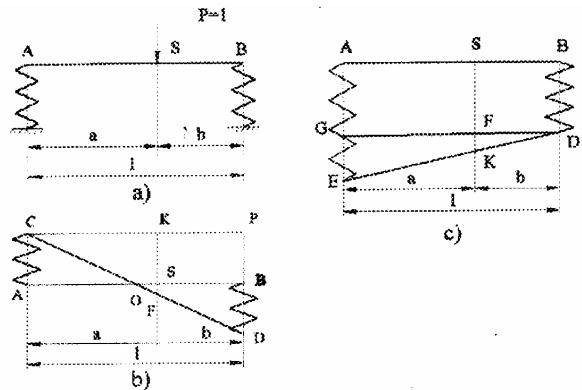


Fig. 6 Calculation of effect coefficients

According to vibrogramm of AE-20 engine with rigid support the critical rotation frequency was taken 7500 r/m and with the help of mentioned formulas

$$\delta_{11} = \frac{40.26}{84^2} \times 50 \times 10^{-6} + \frac{43.74^2}{84^2} \alpha_{33} =$$

$$= 11.5 \times 10^{-6} + 0.271 \alpha_{33}$$

$$\alpha *_{11} = \alpha_{11} + \delta_{11} = 17.8 \times 10^{-6} + 0.271 \alpha_{33}$$

From Eq. (4)

$$\alpha *_{11} = \frac{780}{n_{kp}^2}; 17.8 \times 10^{-6} + 0.271 \alpha_{33} = \frac{780}{7500^2} = 138 \times 10^{-6}$$

$$\alpha_{33} = 14.7 \times 10^{-6} \text{ cm}$$

During the second stage an effect coefficient of the front rotor support was identified. Trial output of the test of compressor rotor without rigid ring with operated engine under the conditions of vacuum chamber $n_{kp1} = 7800$ rpm and $n_{kp2} = 1000$ rpm.

Starting with these data and similar to the mentioned procedure we found the pliability of front support

$$1. \delta_{11} = \frac{40.26}{84^2} \alpha_{22} + \frac{43.74^2}{84^2} x 14.7 x 10^{-6} = 0.23 \alpha_{22} + 4 x 10^{-6};$$

$$2. \alpha *_{11} = \alpha_{11} + \delta_{11} = 6.3 x 10^{-6} + 0.23 \alpha_{22} +$$

$$+ 4 x 10^{-6} = 10.28 x 10^{-6} + 0.23 \alpha_{22}$$

From Eq. (4) we have

$$\alpha *_{11} = \frac{780}{10000^2}$$

For the second form of oscillations of rigid rotor on pliable support

$$10.28 x 10^{-6} + 0.23 \alpha_{22} = 7.8 x 10^{-6}$$

whence $\alpha_{22} = -11.75 x 10^{-6}$ cm.

Calculated by first form of oscillations

$$10.28 x 10^{-6} + 0.23 \alpha_{22} = \frac{780}{7800^2} = 12.85 x 10^{-6}$$

$$\alpha_{22} = +11.5 x 10^{-6}$$
 cm

Thus when the rotation frequency is 7800 rpm both have the same sign that is the motions of both support are to the same direction and when it is 10000 rpm α_{22} and α_{33} have different signs and different directions.

Then we calculated natural frequency of they the system oscillations that is natural frequency of the rotor oscillations when it is on rigid supports, front case oscillations at pliability α_{22} and of combustion chamber body at pliability α_{33} , taking its dimensions $\omega(1/\text{sec})$ and $n(\text{rpm})$ than correspond to resonances of these units.

1. Vibration frequency of compressor rotor system on rigid supports as calculated before and was equal to 11200 rpm.

2. Resonance frequency of the body of combustion chamber at α_{22} was found by Eq. (2)

$$n_{Res}^k = \frac{300}{\sqrt{G_s \alpha_{11}}} = \frac{300}{\sqrt{300 x 11.5 x 10^{-6}}} = 8200 \text{ rpm}$$

Also by Eq. (2) was found the resonance of combustion chamber body at pliability α_{33}

$$n_{Res}^k = \frac{300}{\sqrt{60 x 14.7 x 10^{-6}}} = 10000 \text{ rpm}$$

At last the calculation of combined oscillations of the system where the rotor was defined by concentrated

mass was performed.

Oscillations of the masses of front case and the body of combustion chamber were taken into account (Fig. 3).

$$\alpha_{12} = \alpha_{21} = \frac{b}{l} \alpha_{22} = 0.521 x 11.6 x 10^{-6} = 6 x 10^{-6} \text{ cm}$$

$$\alpha_{13} = \alpha_{31} = \frac{a}{l} \alpha_{33} = 0.43 x 14.7 x 10^{-6} = 7.05 x 10^{-6} \text{ cm}$$

$$\alpha_{23} = \alpha_{32} = 0$$

Natural frequencies of this system were found from the solution of the determinant (3).

Ignoring the expression that contains ω^6

$$\omega^4 / 8.52 x 10^{-3} + 5.47 x 10^{-13} + 8.8 x 10^{-13} -$$

$$- 4.22 x 10^{-13} - 2.83 x 10^{-13} / - \omega^2 / 1.17 x 10^{-6} +$$

$$+ 0.724 x 10^{-6} + 0.751 x 10^{-6} / + 1 = 0$$

$$\omega_1 = 7811 / \text{sec.}; n_1 = 7450 \text{ rpm}$$

$$\omega_2 = 990 / \text{sec.}; n_2 = 9200 \text{ rpm}$$

The outcome of a trial with calculation results let us know full pattern of oscillations and the causes of resonances.

5. Conclusions

1. First resonance of the system near 7500 rpm is caused by oscillations of rigid shaft on pliable supports.

2. The second resonance at 8300-8500 rpm is of the system rotor-supports. The resonance of front support when rotation-frequency of the rotor coincides with natural oscillation frequency of front case is obtained.

3. The next resonance at 9600-10000 rpm is characterized by rotor oscillations as of a solid body upon pliable supports according to the second form of oscillations. Besides the rotor turns comparatively to the center of rotation.

4. The next resonance, near 10600 rpm appears when two frequencies coincide the frequency of natural oscillations of combustion chamber and the frequency of rotor rotation.

5. One more resonance near 11200 rpm corresponds to the critical frequency of rotor rotation on rigid supports.

Comparison of the trial outcome and analysis results shows that dynamic model and accepted algorithm of identification of support rigidity, reflects real system behavior to a considerable degree.

Except of the mentioned resonances there is one small increasing of supports vibration near 5000-6000 rpm that might be explained by the rotor weight influence.

Resonances of the first and second oscillation forms of the rotor as a solid body on elastic supports have the values not much different from the values of natural frequencies of the left and right supports and some times in practice they were considered to be only one and it was near 8000 and 10400 rpm.

Under these forms of oscillations the rotor bends insignificantly and shifting of the stages comes generally

together with that of the supports. Besides the correlation between rotor deflection and support oscillations depends on the proximity of given resonance from natural frequency of the supports.

The far it gets from natural frequency of the support the more deflections of the rotor itself prevail in general shift of stages and they reach maximum at 11200 rpm.

V. Royzman, A. Bohorosh

**AVIACINIO VARIKLO DINAMINIŲ
CHARAKTERISTIKŲ NUSTATYMAS BANDOMUJŲ
PARAMETRŲ METODU**

R e z i u m ē

Straipsnyje aviacinio variklio AV-20 pavyzdžiu parodoma, kaip, turint daugiapakopio kompresoriaus rotoriaus virpesių matematinį modelį, sudarytą iš tampių atramų, prie kurių prijungtos frontalinių karterio ir degimo kamerų korpusų masės ir užrašytas besisukančio rotoriaus dažnumines amplitudės charakteristikas, galima nustatyti nežinomus visos virpančios sistemos parametrus ir kaip susidaro visi penki rezonansai. Tam tikslui darbe pasiūlytas bandomųjų parametru metodas.

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**DYNAMIC CHARACTERISTICS IDENTIFICATION
OF AIRCRAFT ENGINE BY METHOD OF TRIAL
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S u m m a r y

This paper shows a compressor of aircraft engine AE-20 as an example how with the help of mathematical model of vibrations of multi staged compressor rotor on elastic supports connected to the masses of frames of crankcase and combustion chamber and amplitude-frequency characteristics of all the system being recorded during the time of rotor rotation it is possible to identify unknown parameters of the system and to calculate origins of all resonances.

B. Ройzman, А. Богорош

**ИДЕНТИФИКАЦИЯ ДИНАМИЧЕСКИХ
ХАРАКТЕРИСТИК АВИАЦИОННОГО ДВИГАТЕЛЯ
МЕТОДОМ ПРОБНЫХ ПАРАМЕТРОВ**

P e z y o m e

В данной статье на примере компрессора авиадвигателя АИ-20 показывается, как по математической модели колебаний ротора многоступенчатого компрессора, покоящегося на упругих опорах с присоединенными к ним массами корпусов лобового картера и камеры сгорания и записанной при вращении ротора компрессора амплитудно-частотной характеристике всей колеблющейся системы, удается идентифицировать неизвестные параметры этой системы, а затем рассчитать происхождение всех пяти резонансов. Для этого в работе предлагается метод пробных параметров.

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