## Modelling of heat and mass transfer processes in phase transformation cycle of sprayed water into gas: 5. Numerical modelling optimization of phase transformation cycle for droplets slipping in gas flow

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#### Nomenclature

*A* - droplet surface area, m<sup>2</sup>; *a* - thermal diffusivity, m<sup>2</sup>/s; *D* - mass diffusivity, m<sup>2</sup>/s; *Fo* - Fourier number; *k<sub>c</sub>* - effective conductivity parameter; *L* - latent heat of evaporation, J/kg; *m* - vapor mass flux, kg/(m<sup>2</sup>s); *M* - mass, kg; *Nu* - Nusselt number; *p* - pressure, Pa; *P* - symbol of free parameter in heat-mass transfer; *Pe* - Peclet number; *Pr* - Prandtl number; *Re* - Reynolds number; *q* - heat flux, W/m<sup>2</sup>; *r* - radial coordinate, m; *R<sub>v</sub>* - water vapor constant J/(kgl K); *T* - temperature, K; *V* - volume, m<sup>3</sup>; η - non-dimensional radial coordinate; λ - thermal conductivity, W/(m K); ρ - density, kg/m<sup>3</sup>; τ - time;

subscripts-

*C* - droplet centre; *co*- condensation; *e* - evaporation; *ee* - equilibrium evaporation; *f* - phase change; *g* - gas; *i* - time index in a digital scheme; *it* - number of iteration; *I* - index of control time; *j* - index of radial coordinate; *J* - index of droplet surface; *k* - conduction; *l* - liquid; *m* - mass average; *rt* - dew point; *R* - droplet surface; *v* - vapor; *vg* - gas-vapor mixture; 0 - initial state;  $\infty$  - far from droplet; *superscripts*-

+ - external side of a droplet surface; - - internal side of a droplet surface.

#### 1. Introduction

Liquid droplet spraying process is very important for thermal technologies. In two-phase flow heat transfer processes between droplets and its carrying gas flow takes place. Together, phase transformations observe on the surface of a droplet. Therefore, the combined heat and mass transfer is typical for sprayed liquid technologies. In order to define its intensity an interplay transfer interaction must be taken into account. It is influenced by many factors. It is important to distinguish essential factors that allows to define transfer format and transfer process intensity. Process interaction is closely related with liquid droplet thermal state. For last-mentioned determination the liquid droplet surface  $T_R$  and equilibrium evaporation  $T_{ee}$  temperatures, as well as dew point temperature  $T_{rt}$  are very important. Liquid droplets equilibrium evaporation conditions are influenced by gas parameters and droplets transfer peculiarities while a dew point temperature is defined by liquid vapour component in gas mixture. At the beginning droplet surface temperature  $T_{R,0}$  is equal to the sprayed liquid temperature  $T_{l,0}$ . These temperatures are determined by each thermal technology specifics.

A function of droplet surface temperature  $T_R(\tau)$ defines phase transformation process ongoing on the surface of the droplet. These function determination is directly related with analysis of droplet thermal state change. Phase transformation cycle is designed from transit and equilibrium evaporation regimes. In transit evaporation regime droplet heats up to equilibrium evaporation regime temperature  $T_{ee}/T_{l,0} > 1$ . Condensing and transit evaporation regimes composite transitional part to equilibrium evaporation phase transformation cycle. In transit stage a phase transformation regime is defined by parameter  $\overline{T}_{rl,0} = T_{rl}/T_{l,0}$ : when  $\overline{T}_{rl,0} < 1$  the transit evaporation regime begins right away, but when is condition  $\overline{T}_{n,0} > 1$ condensing evaporation regime takes place in the beginning. Droplet surface heats up to the dew point temperature and only then the transit evaporation regime starts. Droplet surface and central layers heats up at different rates, therefore non-isothermally observes in a droplet at transit phase transformation regime stage. Non-isothermal droplet thermal stage change is described by function of its average mass temperature  $T_m(\tau)$ . To define this function the describing droplet non-stationary temperature field  $T_l(r,\tau)$  and droplet volume  $V_l(\tau)$  functions must be known. It is clear that function  $T_l(r,\tau)$  defines a significant droplet surface temperature function  $T_R(\tau) \equiv T_l(r = R, \tau)$  for droplet heat transfer and phase transformations. A heat spread in droplet is analysed for function  $T_l(r,\tau)$  definition, while for function  $V_l(\tau)$  determination a droplet phase transformation cycle analysis is necessary. Heat and mass transfer problems are solved for these function definition in a droplet. Transfer processes inside a droplet and in its surrounding are closely related.

Therefore, a "droplet" problem is very miscellaneous and is solved by different methods that depends from applied assumptions. Transfer processes nonstationarity, radiation selectivity, Stefan hydrodynamic flow that accompanying phase transformations and complicated boundary conditions factor can be attributed to aggravating conditions of "droplet" task. Specific evaporation regime investigation is important for knowledge of droplets heat exchange and mass transfer specific cases [1]. Radiation input for droplet energetics assessment [2-8] is important for higher thermal potential technologies. In liquid spraying technologies attention is needed for temperature field at the droplet and its gradient peculiarities [5-11], combustion processes in the droplet surrounding should be taken into account [12-14], a possible turbulence in flow [15], a droplet slipping impact for condensing regime [16] and other factors [17-19].

When the liquid in droplet does not circulate, then heat spreads by conduction and radiation. In compound heat spread case a non-stationary temperature field function  $T_l(r,\tau)$  can be described according to integral-type models that are convenient for numerical analysis [5, 6]. Efficient heat conduction theory [11] creates assumptions for non-stationary temperature field integral type model application and definition of convective heat exchanger in droplet [16]. The abundance of factors influencing droplet heat transfer and phase transformation is determinate by a wide range of numerical methods applied in "droplet" studies. Often they are based by iterative numerical schemes, so it is important to define numerical schemes grids that are optimal for machine time costs. Therefore it is necessary to evaluate created assumptions and applied mathematical models impact for calculated droplet heat and mass transfer processes parameters.

In this work a heat transfer model influence for calculated water droplet thermal state and phase transition parameters is evaluated. Slipping droplets phase transformations cycle universality is based on by Fourier criteria time scale for different dispersity droplets combined into groups by Reynolds criteria.

#### 2. Research method

It is important to define conditions for optimal water droplets phase transformation. There are several important aspects. It is significant to take account into specifics of spraying technology. At the beginning water spraying application is evaluated for phase transformations in heat recovery from humid biofuel products technologies. This requires a special attention in condensing and transit evaporation phase regimes. An important factor is a variety of conditions that rises the demand of chosen boundary conditions for numerical experiment. A variety of boundary heat exchange conditions is defined by possible combinations of sprayed water and humid gas flow parameters. Important are gas temperature  $T_g$  and humidity  $\overline{p}_{y,\infty}$ , sprayed water temperature  $T_{l,0}$  and droplet dispersity R parameters. Dispersity parameter of droplet requires for special attention. Jet work characteristics define the droplets dispersity. In numerical experiment a droplet phase transformation cycle of particular diameter is defined. It would be useful to expand the modelled specific droplet phase transformation cycle for wider dispersity. Heat transfer between droplets and its carrying gas flow takes place in all heat transfer cases. Therefore it is necessary to take radiation influence into account and to assess droplet heat convection processes. Droplets that are heated up by conduction this problem is solved by phase transformation analysis in Fourier criteria time scale [16]. Therefore it is important to apply for droplets in phase transformation cycle, where droplets are heated by conduction, numerical modelling results, for droplets heated up by compound heating cases. This requires a deep knowledge of droplet heating influence for its volume and non-stationary temperature field variation in phase transformation cycle. To

$$\rho_{l}(\tau)\frac{dV_{l}(\tau)}{d\tau} = -V_{l}(\tau)\frac{d\rho_{l}(\tau)}{d\tau} - A_{l}(\tau)m_{v}^{+}(\tau).$$
(1)

When creating time coordinate grid a modelled duration  $\tau_f$  of droplet phase transformation cycle is selected in expression (1) numeric scheme. When defining time coordinate change step  $\Delta \tau_i$  recommendations in work [16] is followed. Providing an uniform step  $\Delta \tau_i$  in cycle  $0 \div \tau_f$ , control time moments  $\Delta \tau_i = \tau_{i-1} + \Delta \tau_i$  are defined, when  $1 < i \le I$  and  $\tau_{i=1} = 0$ , as well  $\tau_{i=l} \equiv \tau_f$ . An assumption of droplet volume sphericity  $V = 4\pi R^3/3$  is made and droplet surface area is described by expression  $A_l = 4\pi R^2$ .

An influence of water expansion for droplet volume change is evaluated by the first member at expression (1) right side. Variation rate of water density inside droplet is defined by ratio of density change  $\Delta \rho_{l,i} = \rho_{l,i} - \rho_{l,i-1}$  and time  $\Delta \tau_i$  interval  $d\rho_l/d\tau \equiv \Delta \rho_{l,i}/\Delta \tau_i$ . Density of water  $\rho_{l,i}$  is selected according to momentary droplet mass average temperature  $T_{l,m,i}$ . An assumption of non-stationary temperature field  $T_l(r,\tau)$  symmetry is made  $\partial T_l(r = 0,\tau)/\partial r = 0$ . Droplet volume  $V_l$  is divided into J number spherical  $V_j = 4\pi r_j^3/3$  sizes, so that  $\sum_{j=2}^{j=J} (V_{j,i} - V_{j-1,i}) = V_{l,i}$ . By radial coordinate  $r_j$  defined droplet internal layers is  $\Delta r_j = r_j - r_{j-1}$ , where  $j < 1 \le J$ . Calculated average water density

where  $j < 1 \le J$ . Calculated average water density  $\hat{\rho}_{l,j} \equiv |\rho_{l,j}+\rho_{l,j-1}|/2$ . Droplet mass average instant temperature  $T_{l,m,i}$  is calculated according to scheme:

$$T_{l,m,i} = 0.5 \frac{\sum_{j=2}^{J} \hat{\rho}_{l,j,i} \times \left(V_{l,j,i} - V_{l,j-1,i}\right) \times \left(T_{j,i} + T_{j-1,i}\right)}{\sum_{j=2}^{J} \hat{\rho}_{l,j,i} \times \left(V_{l,j,i} - V_{l,j-1,i}\right)}.$$
 (2)

For time moments  $\tau_i$ , starting from i = 2, iterative cycle  $it \equiv 1 \div IT$  is carried out for instantaneous droplet surface temperature  $T_{J,i}$  definition. In iteration  $it \equiv IT$  for temperature  $T_{J,i}$  a selected temperature  $T_{J,i,IT}$  is attributed:  $T_{J,i} \equiv T_{J,i,IT}$ . For iterative cycle finishing iteration IT a requirement of flowing in and flowing down heat flow accuracy is raised in very high dependability. In this case requiring accuracy of the balance must be not less than  $P_{all} < 0.02\%$ . After defining a droplet surface temperature  $T_{J,i}$ , the droplet inner layers temperatures are calculated:

$$T_{j,i} = T_{J,i} + f_{j,i}(r,\tau).$$
(3)

In expression (3) by time  $\tau$  and radial coordinate r function  $f_{j,i}$  a droplet surface heating rate and heat spread inside the droplet peculiarities impact for non-stationary temperature field in the droplet is evaluated [6]. When heat spreads only by conduction in a droplet, then function  $f_{j,i} \equiv f_{j,i,"k"}$  is defined for case  $q_{r,i,j} \equiv 0$  by modified expression (19) presented in work [6]:

$$f_{j,i,"k"} = \frac{2}{r_j} \sum_{n=1}^{N_{\infty}} \sin\left(n\pi\eta_j\right) \sum_{i=2}^{I} \left(-1\right)^n \frac{R_i}{n\pi} \frac{T_{i,J} - T_{i-1,J}}{\tau_i - \tau_{i-1}} \times \\ \times \frac{1}{\hat{a}_i} \left(\frac{\hat{R}_i}{n\pi}\right) \left\{ exp\left[-\hat{a}_i\left(\frac{n\pi}{\hat{R}_i}\right)(\tau_I - \tau_i)\right] - \\ - exp\left[-\hat{a}_i\left(\frac{n\pi}{\hat{R}_i}\right)(\tau_I - \tau_{i-1})\right] \right\}.$$
(4)

In expression (4)  $\hat{R}_i = (R_i + R_{i-1} / 2)$  and  $\hat{a}_i = (a_i + a_{i-1} / 2)$ .

If the liquid circulates inside the droplet, convection influence for heat spread in droplet should be taken into account. Then in expression (3) function is  $f_{j,i} \equiv f_{j,i,"c"}$ . Heat spread in droplet intensification due liquid circulation [11] is evaluated by effective heat conductivity parameter  $k_c^-$ :

$$Nu^{-}(Pe_{l}) = k_{c}^{-} \times Nu^{-}(Pe_{l} = 0).$$
(5)

Discussed methodology [11] is applied for parameter  $k_c^-$  definition, while function  $f_{j,i}$  for case of circulating liquid inside the droplet is described by [16] work recommendations:

$$f_{j,i,"c"} = -\frac{R_i}{k_c^-} \sum_{j}^{J-1} \frac{gradT_{j,i,"k"} + gradT_{j+1,i,"k"}}{2} \Big(\eta_{j+1} - \eta_j\Big).$$
(6)

In expression (6) temperature field local gradient  $gradT_{j,l,"k"}$  inside a droplet is calculated according to heat spread model by conduction and radiation [6].

In Eq. (1) steam flow density  $m_{\nu}^{+}$  is considered to be negative at condensing phase transformation regime. Then  $m_{\nu}^{+}$  define a droplet mass increase due to condensing liquid vapour near the droplet surface. In evaporation mode  $m_{\nu}^{+}$  considered to be positive sign size. The size  $m_{\nu}^{+}$  defines a droplet mass decrease due to vaporing liquid. A numerical scheme  $m_{\nu}^{+}$  created according to mathematical model in work [8] allows taking in account these approach peculiarities of vapour flow density on the surface:

$$m_{\nu,i,it}^{+} = \frac{D_{\nu g,i,it}}{T_{R,i,it} R_{\nu} R_{i,it-1}} \left[ p_{\nu,R,i,it} - p_{\nu,\infty} + \frac{M_{\nu}}{M_{g}} \times \left( p \ln \frac{p - p_{\nu,\infty}}{p - p_{\nu,R,i,it}} - p_{\nu,R,i,it} + p_{\nu,\infty} \right) \right].$$
(7)

Calculation scheme (7) of liquid vapour flow density  $m_v^+$  provides automatic change from condensing to evaporation phase transformation regime. In numerical research process, this requires special attention for iterative cycle  $it_{i=i_{co}} \equiv 1 \div IT_{i=i_{co}}$  that defines condensing phase transformation regime duration  $\tau_{co} \equiv \tau_{i=i_{co}}$ . First of all it is necessary to achieve, that in this iterative cycle a defined droplet surface temperature  $T_{R,i=i_{co}} \equiv T_{J,i=i_{co},i!\in IT}$ , would be closer to dew point temperature defined according to gas humidity  $T_{rt} \equiv T_{sol}(p_{v,\infty})$ . It should be noted, that indetermination is possible in this cycle. Predictions for that is made by possible vapour flow density sign change at the iterations  $it_{i=i_{co}}$  when for droplet surface temperature  $T_{J,i=i_{co},it}$  a value of lower or higher than dew point temperature is attributed. This can happen, when iterations are carried out to ensure independently for permissible flow imbalance in fastest descent method.

Numerical experiment is carried oud in two stages, in case to avoid a discussed indetermination. Primarily, a condensing phase transformation regime is analysed providing for it twenty time steps  $\Delta \tau_{i,co} = \tau_{co}/(i_{co}-1)$ , when  $i_{co} = 21$ . While selecting different condensing regime duration  $\tau_{co}$  and consistently correcting it, a time step change  $\Delta \tau_{i,co}$  that satisfies requirement  $\left|T_{J,i_{co},T} - T_{rt}\right| \cong 0$  is defined. This step also ensures additional condition  $T_{J_{i,r_0},IT} < T_{rt}$ . In the second stage cycle is expanded to 225 time steps when numerical research starts again from condensing regime when  $\Delta \tau_i \equiv \Delta \tau_{i,co}$ . So in the second stage the modelled phase transformation  $0 \div \tau_{I=225}$  cycle includes condensing and transit evaporation regimes as well the initial stage of equilibrium evaporation. For full phase transformation cycle modelling  $0 \div \tau_f$  a time step change duration in equilibrium evaporation regime should be lengthen.

Numerical research is performed in Fourier criterion time scale  $Fo \equiv a_0\tau / R_0^2$ . It was mentioned, that phase transformation cycle  $0 \div Fo_{f,"k"}$  makes preconditions to reduce numerical experiment sizes for droplets heated up by conduction ("*k*" heating case): enough to create a phase transformation cycle  $0 \div Fo_{co,"k"} \div Fo_{nf,"k"} \div Fo_{f,."k"}$  for freely chosen diameter droplet, to define its heat and mass transfer parameters *P* relatively dimensionless forms  $\overline{P}(Fo) \equiv P(Fo) / P_0$  and according to them define  $P(\tau)$  functions for wanted droplets dispersity [16].

Traditionally, the theoretical heat conduction case is defined by Nusselt criteria Nu and for non-evaporating droplet  $Nu \equiv 2$ . However the "k" heat transfer case can be defined by requirement, that in all droplet phase transformation cycle the gas flow carry it without slipping. Then  $\Delta w_l \equiv |w_l - w_g| = 0$ . Droplet slipping intensity in gas flow is defined by Reynolds criteria  $Re = 2\Delta w_l R / v_g$ . Universality of modified criterion  $Re = 2\rho_g \Delta w_l R / \mu_{vg}$  is based on at the droplet research [11], when  $\rho_g = f(T_g)$ ,  $\mu_{gd} = f(T_{gd})$  and  $T_{gd} \equiv T_R + (T_g - T_R)/3$ . Non-evaporating droplet convective heating intensity is defined by expression [11]:

$$Nu = 2 + 0.552 \, Re^{1/2} Pr^{1/3} \,. \tag{8}$$

It becomes clear, that "k" heat transfer case definition for conditions  $Nu \equiv 2$  or Re = 0 are identical.

#### 3. Results and its analysis

Slipping droplet in non-radiating gas flow is heated by convection ("c" heat transfer case). Droplet slipping intensity suffocates due to resistance force. Therefore "c" heat transfer consistently comes up to "k" heat transfer conditions. Easements of phase transformations cycle are valid for last-mentioned conditions. Sprayed liquid droplets according their dispersity and initial velocity can be combined into clouds that have the same criteria  $Re_0$ . Because for all droplets in a cloud  $Re_0 \cong 0$ , where condition  $Nu \cong 2$  is valid, a phase transformation cycle in Fourier criteria time scale is equal [16], therefore it can be guessed that in other droplet clouds, defined by criteria  $Re_{l,0}$ , the phase transformation cycles  $0 \div Fo_{f,R}$  should be identical. If this assumption confirms then numerical experiment of "droplet" can be optimized and volume reduced.

# 3.1. Consistent patterns of transfer parameters change in droplet clouds defined by criteria $Re_0$

Droplets phase transformations for  $2Re_0$  diameter (50, 100, 150 and 200 micrometres) of initial temperature  $T_0 = 278$  K were modelled, when they are carried out by dry air flow whose temperature is  $T_g = 450$  K, when humidity is  $\bar{p}_{y,\infty} \equiv p_{y,\infty} / p = 0.3$  and velocity  $w_g = 9$  m/s.



Fig. 1 Droplet thermal parameters functions  $T_R(\tau)$ ,  $T_C(\tau)$ and  $T_m(\tau)$  graphics in real (a) and Fourier criteria (b) time scales. Initial diameter of droplet is  $2R_0 \times 10^6$ , m: (1) 50, (2) 100, (3) 150, (4) 200.  $Re_0 = 15$ 

Selected individual droplet velocity  $w_l$  ensures for their cloud the provided initial Reynolds number  $Re_0 = 0$ ,  $Re_0 = 15$ ,  $Re_0 = 30$  or  $Re_0 = 50$  (Table 1). Single-droplet model was applied, therefore droplet heat transfer and phase transformation impact for air flow parameter is denied.

In droplet groups defined by Reynolds number a calculated droplet heat and mass transfer parameters P was analysed in thermal  $P_T$ , dynamic  $P_D$ , energetic  $P_q$  and phase transitions  $P_f$  groups.

Table 1 Conditions defining droplet initial slipping



Fig. 2 Dynamics of droplet slipping in air flow at real time (a) and in time scale expressed by Fourier number (b).  $Re_0 = 15$ 

Droplet thermal parameters are related with nonstationary temperature field function  $T(r,\tau)$  that describes temperature division change inside the droplet. According to this function described functions of the droplet surface  $T_R(\tau) \equiv T(r = R, \tau)$  and centre  $T_C(\tau) \equiv T(r = 0, \tau)$  are very important for heat transfer and phase transformations. Also droplet mass mean temperature function  $T_m(\tau)$  is defined according to (2) expression. Functions graphs  $P_T(\tau)$  describing these temperatures are sensitive for droplets dispersity in real time (Fig. 1, a). However, as example  $Re_0 = 15$  shows, that for droplet cloud combined by Reynolds criteria temperature functions can be reflected by one graph in Fourier time scale (Fig. 1, b). Points in Figs. 1-3 indicates droplet thermal parameters at the moment of phase transition regime change from condensing to transit evaporation regime. Droplet surface temperature then matches the dew point temperature  $T_R(\tau \equiv \tau_{co}) = 342.28$  K, water inside the droplet centre is heated up to  $T_C(\tau \equiv \tau_{co}) = 339.63$  K, while water mass mean temperature is  $T_m(\tau \equiv \tau_{co}) = 341.27$  K.



Fig. 3 Water flow maximum velocity variation on the droplet surface in real (a) and Fourier criteria time scale (b).  $Re_0 = 15$ 

Dynamic  $P_D$  parameters describe a droplet movement in gas flow and forces operating droplet. The main dynamic parameter is velocity  $\Delta w_l$  that defines droplet convective heating intensity in aspect of its slipping in gas. Maximum velocity  $w_R$  of liquid flow on the slipping droplet surface is important to define liquid circulation intensity inside the droplet [11]. The flow of liquid on slipping droplet surface is caused by rising friction forces in the surface. Due to their impact forming local whirpools make preconditions for heat convection transfer to take place in droplet. Velocity  $w_R$  is defined by friction drag coefficient  $C_F$ , droplet slipping velocity  $\Delta w_l$ , liquid and gas physical properties as well droplet flow regime function [11]. Droplet slipping velocity (Fig. 2) and water flow velocity on the surface of a droplet (Fig. 3.) function graphs depends from droplet dispersity in real and Fourier criteria time scales. It is clear that universality occurred in thermal parameters  $P_T(Fo)$  graphs is invalid in dynamic parameters  $P_D(Fo)$  functions graphs. A qualitative difference of function graphs  $P_D(\tau)$  and  $P_D(Fo)$  can be seen: bright curves distinctive in functions graphs  $\Delta w_l(\tau)$ (Fig. 2, a) and  $w_R(\tau)$  (Fig. 3, a) at Fourier criteria time scale becomes regular (Fig. 2, b) and (Fig. 3, b), respectively. In addition, points located in vertical line at (Fig. 2, b) and (Fig. 3, b) justifies that duration of condensing phase transformation regime  $0 \div Fo_{co}$  is insensitive for droplets dispersity at Fourier criteria time scale. According to that a dimensionless  $\overline{P}(Fo) \equiv P(Fo) / P_{norm}$  function form application for droplet parameters dynamics is described in phase transformation cycle.





The parameter  $P_{norm}$  can be used for characterizing moments in droplet phase transformation cycle. These moments can be related with regime variation in phase transformation cycle or with parameters at the beginning of the cycle. Initial droplet cycle parameters  $P_0$  are ordinary and clear, because they are defined by liquid spraying conditions. Then functions  $\overline{P}(Fo) \equiv P(Fo)/P_0$  graphs should be universal. Droplets dynamic parameters  $\Delta w_l$  and  $w_R$  are combined by Reynolds criteria  $Re_0 = 15$  and functions  $\Delta \overline{w}_l(Fo) = \Delta w_l(Fo)/\Delta w_{l,0}$  as well  $\overline{w}_R(Fo) = \Delta w_R(Fo)/\Delta w_{R,0}$ graphs proves this assumption (Fig. 4)

Droplet energy state change in phase transformation cycle is defined by droplet energy parameters  $P_q$ . The most important among them is heat flows on the droplet surface (Fig. 5) which variation in phase transformation cycle is specific and sensitive for droplets dispersity in real time (Fig. 5, a).



Fig. 5 Droplet dispersity influence for heat flow variation in phase transition cycle in real (a) and Fourier criteria (b) time scales.  $Re_0 = 15$ 

External convective warming heat flow  $q_c^+$  density is consistently weakening in transit phase transformation regime. This is affected by air temperature and droplet surface  $T_g - T_R$  difference that defines process driving force reduction and weakening Stefan hydrodynamic flow impact in condensing regime. In evaporation regime decreasing droplet diameter and growing Stefan hydrodynamic flow impact becomes the main factors affecting droplet convective heating intensity and having the opposite influence: the first factor intensifies convectional heat flow intensity, while the second represses. In equilibrium evaporation regime a droplet dimension decreasing effect becomes stronger, therefore convectional heat flow density begin to grow (Fig. 5, a). Phase transformations heat flow density  $q_f^+ = m_v^+ L$  in condensing regime is consistently suffocating and becomes zero in phase transformation regime change from condensing to transit evaporation regime (Fig. 5). After that consistently grows up to external convectional heating intensity at beginning moment of equilibrium evaporation (Fig. 5, a). Inside the water droplet at condensing regime a convectional heating flow density  $q_c^$ that defines heating intensity from initial value  $q_{c,0}^- \equiv q_{c,0}^+ + q_{f,0}^+$  decreases to  $q_{c,co}^- \equiv q_{c,co}^+$  and becomes zero at the end of transit evaporation regime (Fig. 5). It is important that in normed  $\overline{P_q} \equiv P_q / P_{q,0}$  form energy parameters  $\overline{P}(Fo)$  function graphs are insensitive for droplet dispersity in the cloud defined by Reynolds criteria (Fig. 5, b). In phase transformation cycle energy parameters dimensionless form  $\overline{P} \equiv P_q / q_{c,0}^+$  is more convenient to assess the droplet energy state. This form clearly defines heat flows interplay ratio dynamics in phase transformation cycle (Fig. 6). In condensation process at the beginning of cycle the separating phase transformation heat flow almost two times exceeds external convection heat flow, and later on Fig. 6 clearly reflects other regularities that was previously discussed.



Fig. 6 Normed energy parameters function graphs in Fourier criteria time scale, in aspect of external convection initial heat flow.  $Re_0 = 15$ 

In droplets dispersity groups combined by Reynolds  $Re_0 = 15$  criteria observed regularities of thermal, energetic and dynamics droplet parameters ensures universality of phase transformation parameters  $\overline{P}_f$  (Fo) function graphs. This is confirmed by main parameters of droplet phase transformation – droplet mass  $\overline{M}_l \equiv M_l/M_{l,0}$  volume  $\overline{V}_l \equiv V_l/V_{l,0}$  and vapour flow on the surface



Fig. 7 Droplet phase transformation  $P_F$  parameters functions graphs universality  $\overline{P_f}(Fo) \equiv P_f(Fo) / P_{f,0}$  in droplet dispersity cloud when  $Re_0 = 15$ 

3.2. Droplet volume model influence for calculated parameters of phase transformation cycle

Intensity of heat and mass transfer processes is sensitive for sprayed liquid dispersity in two-phase droplet and gas flows: for smaller droplets heat and mass transfer flow densities are higher. Therefore it is important to define correctly function  $A_l(Fo) \equiv 4\pi R^2(Fo)$  that describes droplet surface area variation in phase transformation cycle. The area function is related with functions of droplet volume  $V_l(Fo)$  and droplet average mass temperature  $T_m(Fo)$  defined by (1-8) equations. Warming droplet volume variation is influenced by water expansion effect that is defined in expression (1) according to selected water density for droplet mass average temperature. Therefore functions  $V_l(Fo)$  and  $T_m(Fo)$  are related. When defining droplet dimension variation in transit phase transformation regime, the model of droplet non-stationary field  $T(\eta, Fo)$ can be an important factor. Equation system (1-8) allows to define temperature field  $T_{"k"}(\eta, Fo)$  function unambiguously for droplets heated up by conduction. When droplet is heated up by radiation and conduction in semi-transparent droplet an absorbable radiation flow is evaluated by taken into account work [6] recommendations. Due to possible water circulation inside droplet a problem of analytical description for non-stationary temperature field  $T_{"c"}(\eta, Fo)$ function rises. In expression (6) function  $T_{"c"}(\eta, Fo)$  definition is based on by efficient heat conduction theory that is developed to describe a local convectional heat transfer in slipping droplet:

$$q_{c,l}(\eta,Fo) \equiv -f_{r_{c''}}(\eta,Fo)\lambda_{l}(\eta,Fo)\frac{\partial T(\eta,Fo)}{\partial \eta},\qquad(9)$$



Fig. 8 Non-stationary temperature field model influence for calculated droplet mass mean temperature.  $Re_0 = 50$ 



Fig. 9 Non-stationary temperature field model influence for calculated temperature difference  $\Delta T_{\rm m}(Fo) \equiv T_{m,"c"}(Fo) - T_{m,"k"}(Fo)$ .  $Re_0 = 50$ 

providing validity of assumption  $f_{c}^{*}(\eta, Fo) \equiv k_{c}^{-}(Fo)$  [16].

Warming droplet temperature field model influence for water density  $\rho_l \equiv \rho_{H_2O}(T_m)$  function is evaluated by numerical experiment when defining droplet volume variation at transit phase transformation regime in cases  $T_m \equiv T_{m,"k"}$  and  $T_m \equiv T_{m,"c"}$  Fig. 8.

Calculated temperatures  $T_m \equiv T_{m,"c"}$  and  $T_m \equiv T_{m,"k"}$ imbalance  $\Delta T_m(Fo) \equiv T_{m,"c"}(Fo) - T_{m,"k"}(Fo)$  is brightest in condensing phase transformation regime (Fig. 9).

In case of  $Re_0 = 50$  the calculated temperature difference  $\Delta T_m(Fo)$  according to various heat transfer models in the droplet reached up to  $\Delta T_{m,\max} \approx 2.4$  K. In equilibrium evaporation regime an applied non-stationary temperature field model influence for temperature difference  $\Delta T_m$  becomes insignificant Fig. 9.



Fig. 10 Non-stationary temperature field model influence for calculated droplet volume.  $Re_0 = 50$ 



Fig. 11 Non-stationary temperature field model influence for calculated droplet surface area.  $Re_0 = 50$ 

Warming droplet nonstationary temperature field model influence for calculated droplet volume is brightest at the end of transit phase transformation regime Fig. 10. Calculated droplet volume up to 0.05% influences relational  $\overline{S} = (1 - S_{"c"} / S_{"k"}) \cdot 100 \%$  imbalance of calculated droplet surface areas  $S_{"c"}$  and  $S_{"k"}$  Fig. 11.

#### 4. Conclusions

For initial slipping according defined Reynolds criteria  $Re_0$  grouped droplet cloud universality of droplet transfer dimensionless parameters  $\overline{P}(F_0)$  function graphs in the aspect of droplet dispersity water temperature is  $T_0$ , fume temperature  $T_d$  and humidity  $\overline{p}_{g,\infty}$ , in "k" heat transfer case. Therefore water droplets phase transformation cycle in Fourier time scale  $0 \div Fo_{co} \div Fo_{nf} \div Fo_f$ , for selected initial parameters case  $Re_0$ ,  $T_0$ ,  $T_d$ , and  $\overline{p}_{g,\infty}$ , can be defined according to droplet heat and mass transfer modelling results for freely chosen droplet dispersity. This makes preconditions to optimize droplet research numerical experiment that is very important to define optimal water spraying in wide range boundary conditions of heat transfer.

According modelling results that it was get by conditions of intensively  $Re_0 = 50$  slipping droplet in phase transformation cycle where a calculated droplet mass moment temperature can exceed temperature  $T_{m,"k"}$  up to 2.5 K in "*c*" heat transfer based model, calculated according to "*k*" heat transfer model. Applied droplet mass mean temperature model may determine up to  $\pm 0.05$  % imbalance of calculated droplet surface area.

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MODELLING OF HEAT AND MASS TRANSFER PROCESSES IN PHASE TRANSFORMATION CYCLE OF SPRAYED WATER INTO GAS: 5. NUMERICAL MODELLING OPTIMIZATION OF PHASE TRANSFORMATION CYCLE FOR DROPLETS SLIPPING IN GAS FLOW

Summary

Condensing and transit evaporation regimes for phase transformation cycle were modelled for slipping droplets in humid gas. By concluded results of droplet thermal, energetic, dynamics and phase transformation parameters P it is based on that according to unit initial Reynolds number grouped droplet cloud, universality of dimensionless transfer parameters  $\overline{P}(F_o)$  functions graphs is valid in aspect of droplet dispersity for cases defined by water and gas initial temperature and humidity.

According to phase transformation cycle modelling results for intensively slipping droplet by conditions  $Re_0 = 50$  it was received that by "*c*" heat transfer model calculated droplet moment mass mean temperature  $T_{m,"c"}$  can exceed up to 2.5 K temperature  $T_{m,"k"}$  calculated according to "*k*" heat transfer model. An applied droplet mass mean temperature model can lead to  $\pm 0.05$  percent imbalance for calculated droplet surface area.

**Keywords:** humid gas, slipping water droplets, phase transformation cycle, numerical research optimization.

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