ISSN 1392–1207. MECHANIKA. 2016 Volume 22(3): 212–216

Kinematic and dynamic errors during the measurement of linear displacements using the angle transducers

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crossref http://dx.doi.org/10.5755/j01.mech.22.3.14180

1. Introduction

Simplification of measurement system can be made by using angle measurement transducers for measuring compounds' linear displacement by transforming the linear motion to rotational via adequate mechanisms. Most often as such mechanism can serve an accurate measurement drums which cover and turn a measurement tape connected with a movable compound. Rotation of a drum is then measured by an angle transducer and it is calibrated by the units of linear displacement. Such system is used also for measurement of linear displacements of tapes made without special steel marks or other material in a technological equipment of special purpose, e.g. in automation, production of metal raster scale by the laser method and other equipment [1-6].

Measuring the linear displacement by an angle transducer, e.g. of a tape or a connected with it movable component, the total error consist of errors of an angle measurement system, kinematic errors of a mechanism transforming the linear displacement of an elastic element tape to rotation. One of possible sources of these errors are the vibrations of tape and system. Therefore value and character of these vibrations must be researched, evaluated reasons of their occurrence and possible effect of the measurement error on the reliability.

2. Object and tasks of research

The research object are the errors of linear displacement measurement arising from the joint kinematic and geometric errors of a measurement system and dynamic processes happening while transforming the linear motion of a carriage to rotation of the shaft of an angle transducer. These errors can influence on the uncertainty of the displacement measurement. Influential dynamic factors can be seen while comparing the results of displacement measurement errors, defined by experimental method with the results of numerical modeling, which evaluate kinematical and geometrical measurement system [7] and angle transducer errors. Additionally, error components of high frequency can be noted.

Special stand was designed for the research (Fig. 1). It consists of the components of linear tape motion transformation into the shaft rotation of the angle measurement transducer, the tape displacement measurement system, composed of a carriage of linear displacement, to which the tape is connected, carriage displacement and its small angle oscillation in horizontal plain measurement laser interferometers.

Steel tape *7* is connecting with carriage *11*. The carriage moving on aerostatic supports on the guides of granite plate *12*. The tape is tightened by support roll and turns precision shaft *2*, which is connected through precise clutch *3* to a shaft of the angle measurement transducer *4*. Readings of the angle transducer are calibrated in units of linear displacement. Nominal resolution is equal to 0.0656 µm. Linear displacements of carriage *11* in accordance to plate *12* are measured by interferometer *8* of two frequencies. Environment and tape temperature gradients, sensors of environment humidity and air pressure monitoring are in a stand and in a scale of the real measurement time compensation of these factors uncertainty components is made. Because the requirement of Abbe principle in a horizontal plane is not kept, together by laser interferometer *13* angular oscillations of the carriage in this plane are measured. The measurement error of a researched system is defined as difference of readings of linear displacements, measured by the angle measurement system and by the laser interferometer, as a displacement function.

Fig. 2, a shows curves of measurement of three trials of the described system in limits of three revolutions of a measurement drum. Fig. 2, b shows the curves of mean values of carriage angular oscillations (φ) and their dispersion for confidence interval ± 2S.

According to models of the angle measurement transducer and rotor system kinematic and geometric errors, defined by experimental method, compensation values of these errors were calculated as functions of tape displacement. Dependence on the same argument Abbe error compensations were calculated. Curves of three trials mean values in limits of three revolutions of the drum, after fulfilment of compensation of mentioned errors, are presented in Fig. 3, a.

From the curve of mean errors can be seen that the harmonic component is present, when the frequency coincides with the measurement drum revolving frequency; the component of high frequency is added. Obviously that the first component is the total residual after compensation of the transducer, rotor system and Abbe errors. It may be decreased correcting the compensation function values. Frequency of the second error component does not coincide with the frequency of the angle transducer high frequency component, and its values are significantly larger than the values calculated by the model, if to evaluate experimentally defined values of these errors.

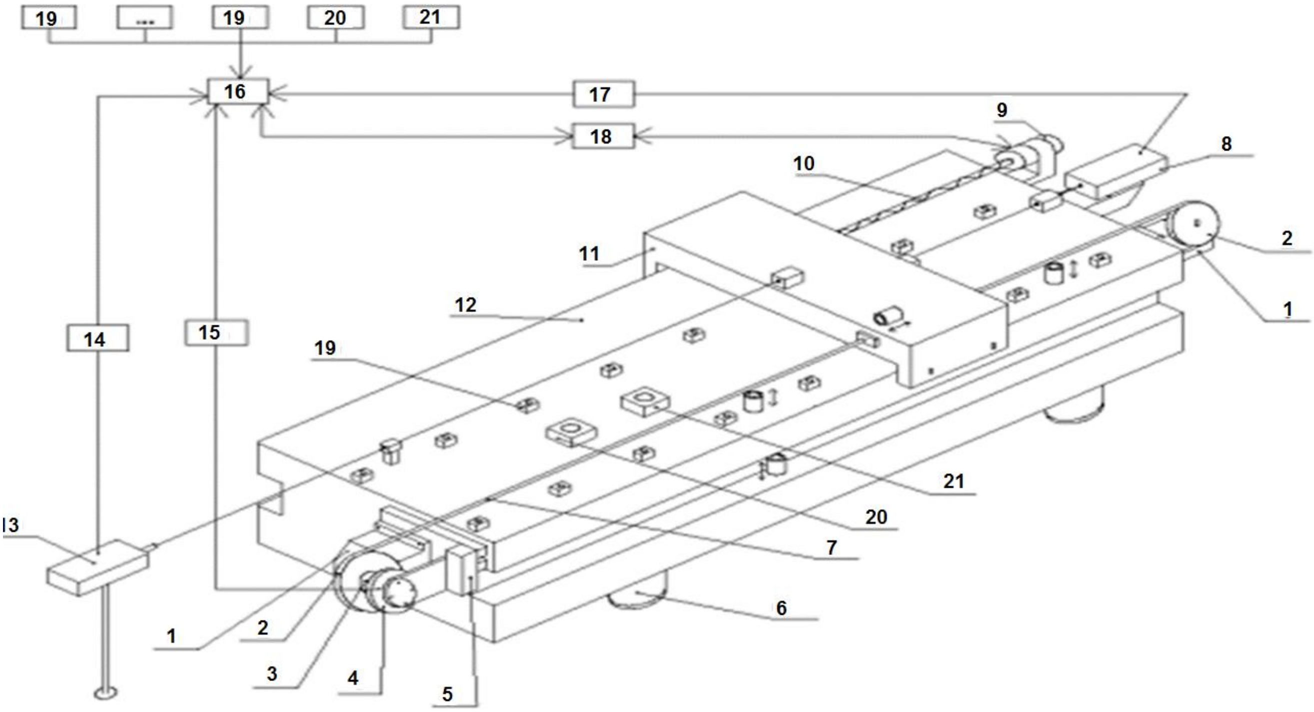
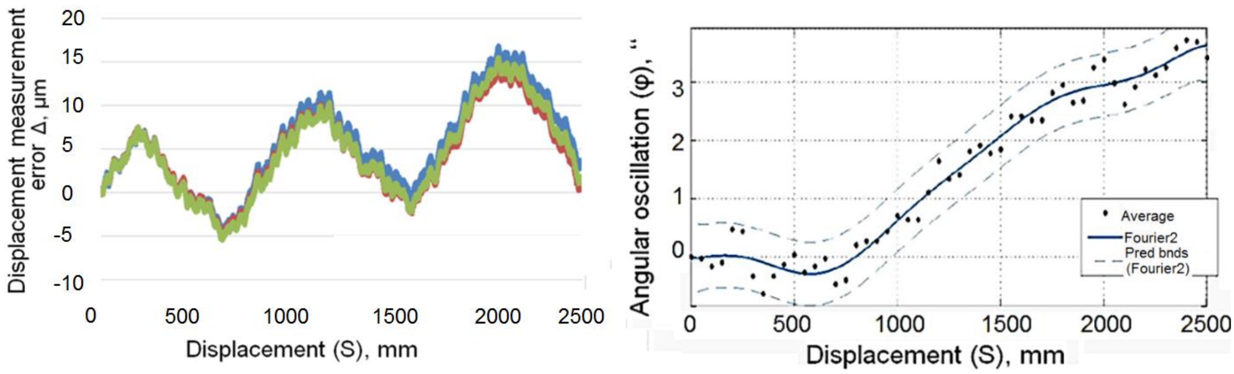
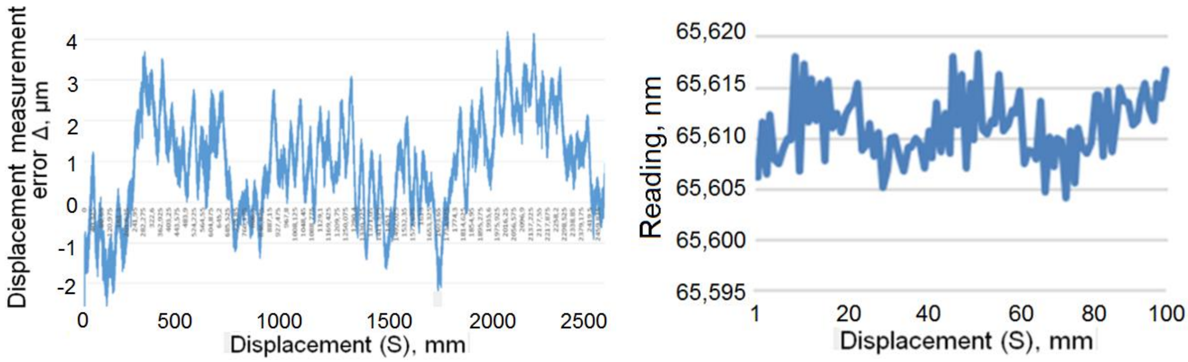


Fig. 1 Scheme of a research stand



a b

Fig. 2 Three trials of system errors (a) and carriage angular oscillation mean value and dispersion (b)



a b

Fig. 3 System three realisation mean (a) and reading changing (b) graphics

According to readings of the angle measurement system and laser interferometer the resolution of the researched system was calculated. Fig. 3, b shows its change graphic in limits of 100 mm of tape displacement.

Evaluating the character of residuals after system errors compensation, these errors are associated with dynamic processes in a pulled tape at action of the system and environment dynamic excitation. Such presumption it is possible to make while evaluating the resolution change. Purpose of dynamic research is to define vibration of the tape and moving components, which can influence the linear displacement transformation into the measurement drum rotation change function, its connection with displacement measurement uncertainty.

3. Modelling of steel tape

Modal analysis of the steel tape was done using SolidWorks software. Modelling of the tape used in the stand of experimental research was made with evaluation of mechanical characteristics of steel AISI 420. Mesh information is shown in Table 1.

Mathematical model of the steel tape. The behaviour of any elastic object interacting with dynamic forces can be specified by the dynamic equilibrium equation:

$$[M]\{\ddot{\delta}\}+[C]\{\dot{\delta}\}+[K]\{\delta\}=\{F\}, \tag{1}$$

where $[M]$, $[C]$, $[K]$ are the mass, damping and stiffness matrices, respectively; $\{F\}$ is the vector of external mechanical forces; $\{\ddot{\delta}\}=\{d^2\delta/d^2t\}$, $\{\dot{\delta}\}=\{d\delta/dt\}$, $\{\delta\}$ are the vectors of accelerations, velocities and displacements, respectively.

The proportional damping model expresses the damping matrix as a linear combination of the mass and stiffness matrices, that is:

$$[C]=\alpha_1[M]+\alpha_2[K], \tag{2}$$

where α_1, α_2 are proportional damping constants (α_1, α_2 are real scalars).

Modal equations have the form:

$$\ddot{z}_i+2c_i\omega_i\dot{z}_i+\omega_i^2 z_i=\{\delta_i\}^T\{F\}, \tag{3}$$

where ω_i are natural frequencies; $\{\delta_i\}$ are eigenmodes, where $i = 1, 2, \ldots, n$ and n is the number of degrees of freedom of the structure; the modal damping c_i is expressed as:

$$c_i=1/2(\alpha_1/\omega_i+\alpha_2\omega_i). \tag{4}$$

Results of numerical study by using FEM (finite element model), by which deformations of steel tape dangerous frequencies are defined, are presented in Table 2.

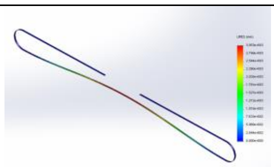
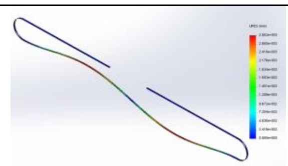
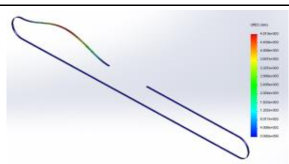
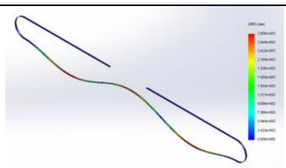
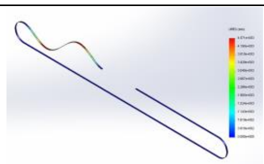
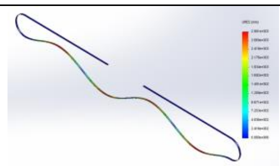
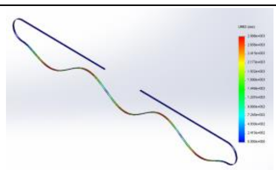
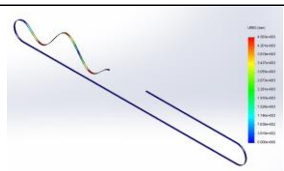
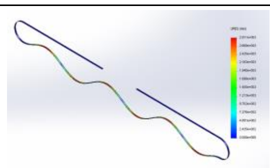
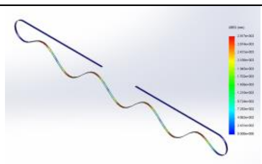
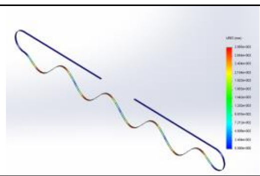
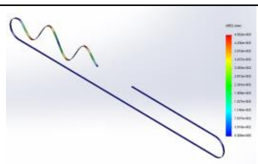
Table 1

Mesh information

Mesh type	Solid Mesh
Mesher Used:	Standard mesh
Automatic Transition:	Off
Include Mesh Auto Loops:	Off
Jacobian points	4 Points
Element Size	16.418 mm
Tolerance	0.821 mm
Mesh Quality	High
Mesh Information - Details	
Total Nodes	35009
Total Elements	15023
Maximum Aspect Ratio	34.09
% of elements with Aspect Ratio < 3	0
% of elements with Aspect Ratio > 10	98.9
% of distorted elements(Jacobian)	0

Table 2

Results of the modal analysis

Mode No.	1	2	3
Frequency, Hz	7.43	14.27	15.323
Mode shape			
Mode No.	4	5	6
Frequency, Hz	21.054	27.113	28.177
Mode shape			
Mode No.	7	8	9
Frequency, Hz	37.220	42.522	44.710
Mode shape			
Mode No.	10	11	12
Frequency, Hz	52.220	55.122	59.710
Mode shape			

Results of the modeling show that resonant frequencies of the first 12 modes of the researched tape range from 7.43 to 57.7 Hz, and from 12 modes four modes are of upper part and eight of bottom part. Obtained modelling results verify the errors of high frequency, presented in Fig. 3.

4. Experimental research

For measurement of vibration parameters the Denmark firm „Bruel & Kjær" measurement tools were used: the data storage and processing installation 3660D with the computer DELL, displacement transducers, accelerometers 8306 and 8344. Measurements of tape vibrations were performed, on the ground of which the experimental modal analysis was made.

Experimental modal analysis (OMA operation modal analysis) of the tape was done. At the time of experimental modal analysis modal parameters (natural frequency), damping coefficient and a mode shape) were defined at use of experimental data. Modal parameters are defined in selected interval of frequencies. The mode form is deflection of the object, which shows relational displacement of the specific mode between all parts of object. Results of experimental modal analysis – the deformations at dangerous frequencies of the steel tape are presented in Table 3. The first examination shows resonant frequencies of an upper part (length of the tape between supports is 2.35 m), the second – of the bottom part (length of the tape between supports 4.7 m). Values of damping coefficients are presented in Table 4.

Table 3

Results of experimental modal analysis

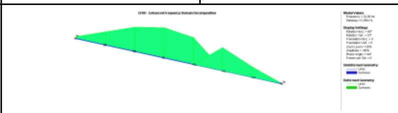
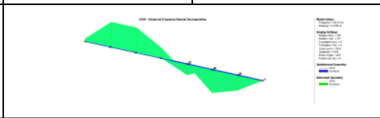
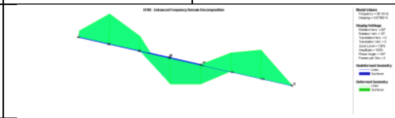
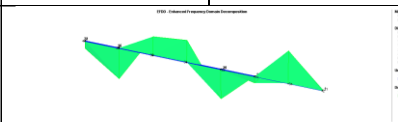
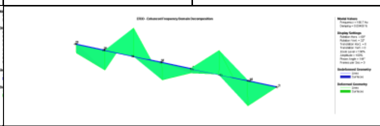
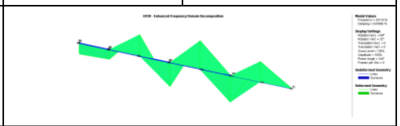
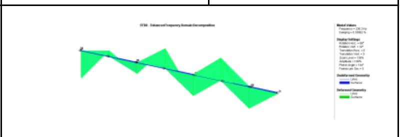
Mode No.	1		2		3	
	Upper	Bottom	Upper	Bottom	Upper	Bottom
Frequency, Hz	14.4	7.303	28.67	14.44	43.05	21.68
Mode shape						
Mode No.	4		5		6	
	Upper	Bottom	Upper	Bottom	Upper	Bottom
Frequency, Hz	57.41	28.85	71.87	36.13	86.39	43.35
Mode shape						
Mode No.	7					
	Upper	Bottom				
Frequency, Hz	100.9	50.64				
Mode shape						

Table 4

Damping coefficients received by experimental modal analysis

Mode	Frequency, Hz		Damping ratio, %	
	Upper	Bottom	Upper	Bottom
Mode 1	14.4	7.303	1.946	0.989
Mode 2	28.67	14.44	0.977	0.535
Mode 3	43.05	21.68	0.650	0.320
Mode 4	57.41	28.85	0.494	0.241
Mode 5	71.87	36.13	0.431	0.228
Mode 6	86.39	43.35	0.353	0.179
Mode 7	100.9	50.64	0.304	0.148

After evaluation of experimental results it can be seen that modes presented in Table 2 describe the modal forms of a steel tape, situated on the upper and on the bottom parts of comparator. The modes presented respectively in Table 3 separately describe steel tape mode forms existent on upper and bottom comparator parts. Evaluating the theoretical and experimental results it is necessary to compare 3, 5, 8 and 12 theoretical modes presented in Table 2 with the modes 1-4 presented in Table 3, which manifest on the upper comparator part situated steel tape. It would be necessary respectively to compare forms of the steel tape shown in Table 2, situated on the bottom part of the comparator: 1, 2, 4, 6, 7, 9 and 10 with the modes 1-7 shown in Table 3. So, the obtained results of experimental modal analysis supplement the results of numerical study by using FEM and show good validation of results (not coincidence from 1.2 to

6.4%). Results show, that resonances of bottom part alternate every 7.23 Hz, while resonance of upper part – every 14.41 Hz. In accordance with the results of modal analysis it is possible to conclude, that resonant frequencies of examined tape assert in range from 7.303 to 100.9 Hz at research of the first 7 modes. And it would be necessary to evaluate these results at exploitation of equipment of similar design.

5. Conclusions

1. Components of high frequency errors and resolution changes during the measurement of linear displacement by the angle transducers are present because of dynamic processes, acting in the elastic element (in a tape, cable).

2. Results, received at research of dynamic processes, acting in elastic element (tape, cable), can be used at analyzing speed stability of movable compounds passed by such elements.

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KINEMATIC AND DYNAMIC ERRORS AT MEASUREMENT OF LINEAR DISPLACEMENTS BY THE ANGLE TRANSDUCERS

S u m m a r y

Dependence of linear displacement error, while measuring with angle transducers, on angle measurement system errors, kinematic and geometric errors of mechanism of linear motion transformation to rotation is shown. Dynamic processes, acting in an elastic element, connecting a movable compound, which linear displacement is measured, with the shaft of the angle measurement transducer, are modeled. Vibration modes, influencing the uncertainty of linear displacement measurement by an angle transducer, are defined. Results of numerical study by using FEM are verified and complemented by experimental research. Research stands and equipment are described.

Keywords: linear displacements, measurement, an angle transducer, kinematic errors, dynamic characteristics, operational modal analysis.

Received February 17, 2016
Accepted May 11, 2016