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# Improving theory for automatic balancing of rotating rotors with liquid self balancers

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## 1. Introduction

Creating and using technologies of new generation which give multiple reducing of production expenses would be impossible without high-speed rotor systems.

However rotors, whose operation in machines is very important, may be a source of harmful vibrations. And the problem focused in reducing vibration is always actual. Among the problems in the field of dynamics of rotor system the problem aimed at creating automatic balancing units (ABU) made as hollow rotor partially filled with liquid is paid special attention recently.

Analysis of a state of modern theory and practice for automatic balancing let us select the following implications:

- passive balancing is used in pendulum, ring, ball, and also liquid units;
- those units are direct controllers as their sensitive element produces force enough to balance rotor;
- such controllers are supplied by rotor's energy that is transfered to sensitive element;
- behavior of liquid in ABU has not been researched theoretically. In known works equations of motion of a system did not consider hydrodynamic properties of the liquid;
- study of motion stability of liquid balancer from the view point of dynamics considers basically the shell behavior, but the character of liquid flow is the second to consider. However, both motions interact that's why the equation of motion should consider both the shell and liquid;
- rotor - liquid auto-balancer system was frequently assumed as a plain model where gravity was not usually considered to evaluate the effectiveness of balancing vertical rotor;
- existing theory for automatic balancing describes the operation of automatic balancing units only at overcritical rotation of the rotor, but practice testifies that ABU can reduce vibration not only at overcritical frequencies of rotation but also at pre-resonance and resonance frequencies.

Experience of using automatic balancing units shows that existing theories for automatic balancing describe processes, particularly with liquids, during their operation with poor accuracy and need to be improved. Besides, there is a necessity to develop mathematical model, which would describe the work of liquid ABU and consider hydraulic properties of the liquid, what could help to explain why liquids move opposite to imbalance at over-resonance frequencies of the rotor rotation and pre-resonance and resonance frequencies as well.

## 2. Theoretical part

Developing mathematical model describing the behavior of liquid in ABU uses the properties of liquid listed below:

- the liquid volume can change its shape being forced by a small load;
- viscosity forces are considered only for high speed motion when shear in the liquid is significant; these forces are not considered for solving problems of liquid balance.

An equation describing the condition of stationary motion of solid body with liquid in its hollow [1] according to the principle of least action in the form of Hamilton-Ostrogradski is

$$\delta W = -\frac{1}{2}\frac{k_0^2}{I_0^2}\delta I + \delta \Pi = 0 \qquad (1)$$

where $W = \frac{1}{2}\frac{k_0^2}{I} + \Pi$ is potential energy of the system; $\Pi$ is potential energy of forces acting on the system: gravity and centrifugal forces, $k_0$ is the value of constant $k$ in case of uniform rotation of the whole system as one solid body about an immobile line with angular speed $\omega$, $I_0$ is moment of inertia of system for stabilized motion.

Thus, in case of stationary motion of the system equation $W$ has extreme value.

Let us assume a solid body being in uniform field of gravity and centrifugal forces and having one fixed point O, cylindrical hollow with radius $R$ and height $h$, and being partially filled with liquid. Axis $x_3''$ of the immobile system with coordinate origin in fixed point of the body is directed vertically. Mobile axes fixed with the body will be drawn so that the axis $x_3$ would go through gravity center of the system. Gravity center of the system is shifted $e$ relatively to geometric axis of the body.

To consider mechanical system without surface tension of the liquid potential energy of the system and inertia moment of the system relative to axis $x_3''$ are defined by the formulas

$$\left.\begin{aligned} &\Pi = Mg\left(x_{c1}\gamma_1 + x_{c2}\gamma_2 + x_{c3}\gamma_3\right) \\ &I = A\gamma_1^2 + B\gamma_2^2 + C\gamma_3^2 - 2D\gamma_2\gamma_3 - 2E\gamma_1\gamma_3 - 2F\gamma_1\gamma_2 \end{aligned}\right\} \qquad (2)$$

where $A$, $B$, $C$ are axial inertia moments, $D$, $E$, $F$ are centripetal inertia moments of the system; $M$ is system mass; $x_{c1}$, $x_{c2}$, $x_{c3}$ are coordinates of gravity center of the system; $\gamma_i$ is projections of ort $i_3''$ of immobile axis $x_3''$ to mobile axis are connected by the equation $\gamma_1^2 + \gamma_2^2 + \gamma_3^2 = 1$.

Condition (1) considering $\omega I = k$ gives the equations in generalized coordinates of solid body in stationary motion

$$\left.\begin{aligned} \frac{\partial W}{\partial \gamma_1} &= -\omega^2\left[(A-C)\gamma_1 + D\gamma_1\gamma_2\gamma_3^{-1} - E\left(\gamma_3-\gamma_1^2\gamma_3^{-1}\right) - F\gamma_2\right] + M\,g\left(x_{c1} - x_{c3}\gamma_1\gamma_3^{-1}\right) = 0 \\ \frac{\partial W}{\partial \gamma_2} &= -\omega^2\left[(B-C)\gamma_2 - E\left(\gamma_3-\gamma_2^2\gamma_3^{-1}\right) + E\gamma_1\gamma_2\gamma_3^{-1} - F\gamma_1\right] + M\,g\left(x_{c2} - x_{c3}\gamma_2\gamma_3^{-1}\right) = 0 \end{aligned}\right\} \qquad (3)$$

Eq. (3) let the solution

$$\gamma_1 = \gamma_2 = 0,\ \gamma_3 = 1 \qquad (4)$$

which will be true at any value of ω, if gravity center of the system is located on the axis $x_3$, which is in turn the general symmetry axis, under the condition that

$$x_{c1} = x_{c2} = 0,\ D = E = 0 \qquad (5)$$

The solution of Eq. (4) corresponds to uniform rotation of the whole system around axis $x'_3$ aligned with axis $x_3''$, with angular speed $\omega$, and axis $x'_3$ being general inertia axis of the system, what is the condition of self balancing. In this condition free surface of the liquid will have the shape of revolution paraboloid

$$\frac{1}{2}\frac{\gamma\omega^2}{g}(x_1{}^2 + (x_2+e)^2) - \gamma x_3 - p_0 = C_* \qquad (6)$$

where $\gamma = g\rho$ is volume weight of the liquid (Fig. 1).

Let the point $O'$ be the coordinate origin, axis $x_3'$ be directed as revolution axis, and axis $x_2'$ go through points $O$ and $O'$ (Fig. 1). Although another coordinate system $Ox_1x_2x_3$ will be considered, in which axis $x_3$ is geometric axis, axis $x_2$ coincides with axis $x_2'$.

Free surface has the shape of revolution paraboloid with parameter $\frac{1}{\beta} = \frac{2g}{\gamma\omega^2}$, what depends on angular rotation speed; growing angular rotation speed makes the parameter decrease and the paraboloid be similar to the surface of circular cylinder: $r^2 = x_1{}^2 + (x_2+e)^2 = b^2$ and in the limit when $\frac{2g}{\gamma\omega^2} \to 0$ it becomes a circular cylinder. Correspondingly, in horizontal section with axis $x_3 = 0$ that coincides with lower base of the hollow, free surface of the liquid has a circle shape with center O and radius $R_1$. Let us find a relation between radius $R_1$ and quantity of the liquid in cylindrical hollow $V$. Therefore we'll find constant $C_*$ in (6) condition that volume of the liquid retains in rest (when $\omega = 0$) and in revolution.

There are three cases of liquid position in the hollow which depend on relation between the liquid and imbalance:

1.quantity of the liquid is greater than it is needed for balancing imbalance (Fig. 1);

2.quantity of the liquid is that it can balance the imbalance in considered frequencies (Fig. 2);

3.quantity of the liquid is not enough to balance the imbalance (Fig. 3).

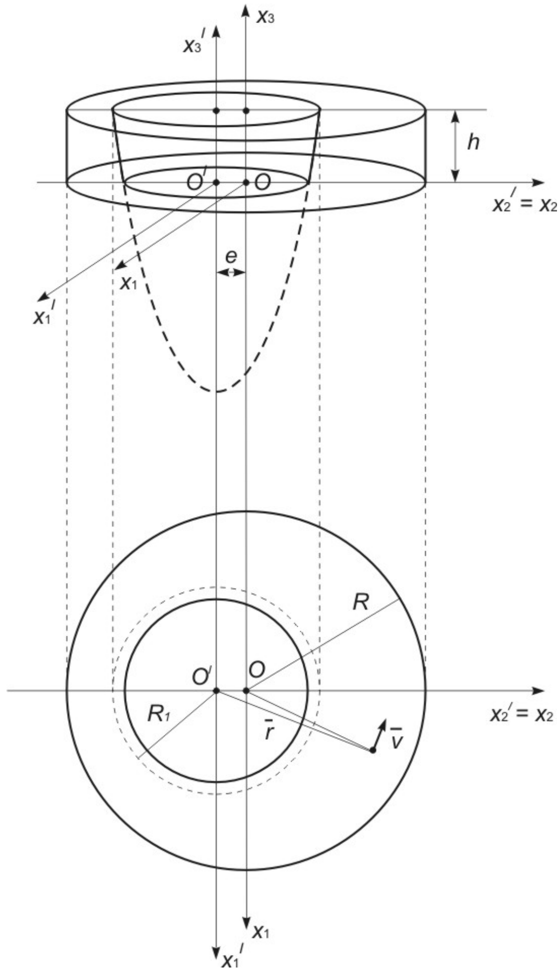



Fig. 1 Liquid position in the hollow (case 1)

Thus, the free surface of heavy liquid revolving about vertical eccentrically axis will be described by the equation

$$x_3 = \frac{\omega^2}{2g}\left(x_1{}^2 + (x_2+e)^2\right) - \left(\frac{\omega^2}{2g}R^2 - \frac{\omega^2 V}{2\pi g h} - \frac{h}{2}\right) \qquad (7)$$

Considering horizontal section (Fig. 1), it is possible to make a conclusion that the liquid displaces towards rotor axis, opposite to imbalance. Its motion stops when total imbalance of the liquid and rotor becomes equal zero, that is the balance where principal inertia axis of the rotor system coincides with revolution axis. It's automatic balancing when liquid has such shape.

If the Eq. 6 of free surface of the liquid is known one can define equation for liquid optimal volume what is necessary for accurate imbalance balancing in considered frequencies of system rotation with given physical and geometrical parameters

$$V=\pi\left(R^2h-\frac{h+\beta\left(e-R\right)^2}{4}\left(\sqrt{\frac{h}{\beta}+\left(e-R\right)^2}-e+R\right)^2+\frac{3\beta}{32}\left(\sqrt{\frac{h}{\beta}+\left(e-R\right)^2}-e+R\right)^4+\frac{\left(R-e\right)^4\beta}{2}\right) \quad (8)$$

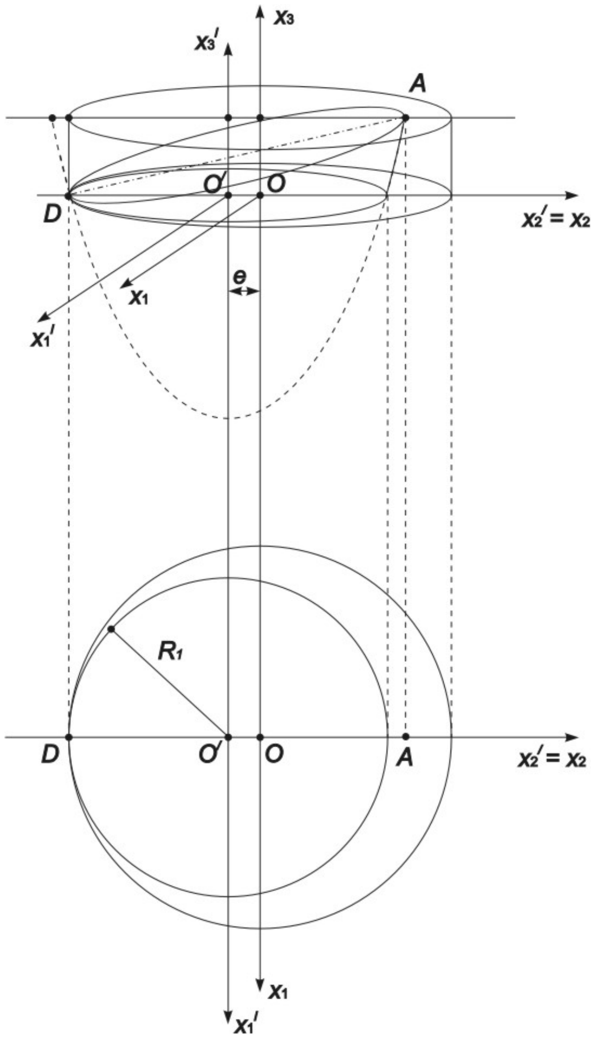



Fig. 2 Position of liquid in the hollow (case 2)

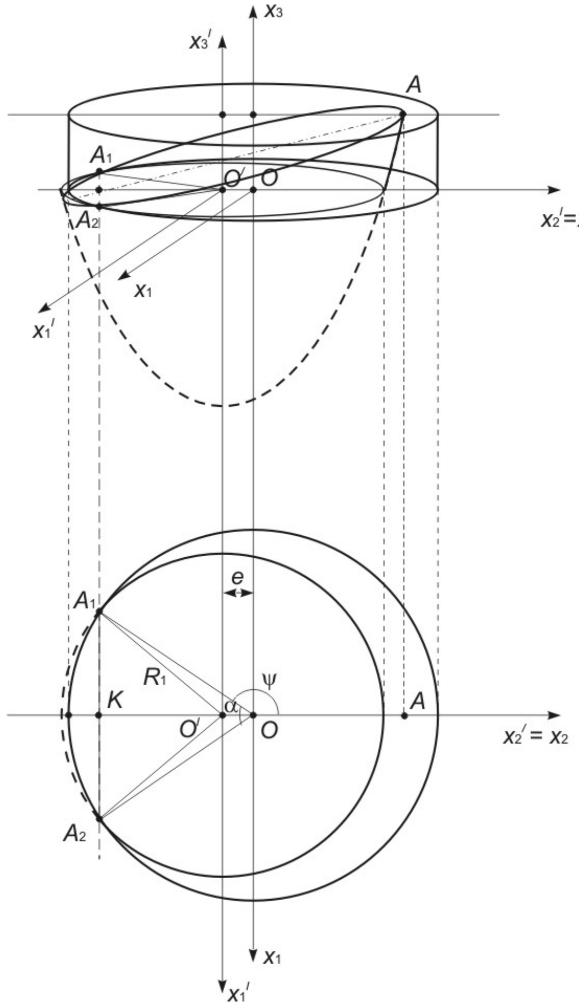



Fig. 3 Liquid position of in the hollow (case 3)

In this case radius $\alpha$ of free liquid surface in horizontal section made by the plain $x_3=0$ equals

$$R_1=R-e \quad (9)$$

It explains that thickness of the layer in horizontal section made by plain $x_3=0$ opposite to imbalance shouldn't exceed 2*e*. Thus, only thin layer of liquid takes part in changing imbalanced state of rotating system.

In the third case (Fig. 3) constant $C_*$ will be found from Eq. (10).

In Fig. 3 thickness of the layer in horizontal section having the area $x_3=0$ opposite to imbalance is much less than 2*e*. Thus, the liquid volume is not enough to reduce the deviation of longitudinal axis of the rotor from revolution axis. Research of liquid's free surface helps to make a conclusion that not the whole volume of the liquid is involved into imbalance balancing. Optimal volume of the liquid balance the imbalance in the given revolution frequencies is estimated by Eq. (8).

$$V=\pi R^2h-\left[\frac{R^2h}{2}\,ar\cos\left(\frac{C-\beta\left(R^2+e^2\right)}{2eR\beta}\right)-\frac{h\left(-C^2+2C\beta\left(R^2+e^2\right)-\beta^2\left(R^2-e^2\right)^2\right)}{4e\beta^2}+\right.$$
$$\left.+\frac{h\left(h+2C\right)}{2\beta}\left(\pi-tg\left(\frac{-C^2+2C\beta\left(R^2+e^2\right)-\beta^2\left(R^2-e^2\right)^2}{2e\beta\left(C-\beta\left(R^2+e^2\right)\right)}\right)\right)\right] \quad (10)$$

This value may be treated as maximal volume of the liquid necessary for accurate balancing at the given geometric dimensions of cylindrical surface. Eq. 9 delivers that thickness of the layer in horizontal section having an area $x_3=0$ opposite to imbalance should not exceed 2*e*. Thus, only thin layer of the liquid takes part in changing imbalance of revolving system. This fact confirms experimental conclusion about the highest effectiveness of using multichamber ABU.

Hence if to set liquid ABU on the rotor, then at any angular speed $\omega$ of stationary motion of the system the liquid, which moves about absolute rotation axis under the action of centripetal forces, flows towards the side opposite to imbalance and tends to bring the common gravity center to rotation axis.

The value of constant $C_*$ in the right part of the equation (6) is estimated by the liquid volume in the hollow of solid body (ABU).

Motion stability of a heavy body with hollow partially filled with liquid demands minimum $W$ [1].

The condition of minimum $W$ in our case is narrowed to the inequality

$$\left(C_0 - A_0\right)\omega^2 - Mgx_{c3}{}^0 - a > 0 \qquad (11)$$

where $A_0 = B_0$ (because the hollow is cylindrical), $A_0$, $B_0$, $C_0$ are general inertia moments of a body having unperturbed motion

$$a = \rho g \int_0^{2\pi} \int_{R_1 - e\cos\varphi}^{R} \left[ \frac{\omega^2}{g^2}\left( \frac{\omega^2}{2} r^2 - C \right) + 1 \right]^2 r^3 dr \qquad (12)$$

Matching condition (11) with condition $\left(C_0 - A_0\right)\omega^2 - Mgx_{c3}{}^0 > 0$ of stability of uniform vertical rotation of a solid body with the liquid in case when its hollow is completely filled up with liquid shows that having free surface of the liquid destabilizes stationary rotation of the system.

Stability condition (11) can still be tolerated at high-speed rotation if the hollow with liquid is oblate enough ($A_0 < C_0$) when the height of cylindrical hollow $h$ is much greater than its radius $R$.

Condition (11) is valid if the liquid that fills the hollow has ultimate viscosity. In case considered here when the liquid rotates together with the body as one, liquid's viscosity has no effect on its motion.

In case of nonstationnary motion of the hollow, which is partially filled with liquid, results look different: one should consider oscillations of free surface of the liquid [2]. Some simplifying assumptions [3] show that there is always a risk of losing stability when natural frequency of any form of liquid oscillations is close to nutation frequency of the body carrying liquid. Infinite range of forms of natural oscillations of the liquid corresponds to infinite range of instability regions, however practice testifies that only a few first regions can be useful. One may assume that internal friction damping natural oscillations of the liquid neutralizes the regions of instability of higher orders. Natural frequencies and hence the regions of instability depend on how much the hollow is filled up.

## 3. Experimental part

In order to demonstrate the process of self balancing as a whole and analyse contradictions between the theory and the practice, a high-speed video shooting of working bodies behaviour is suggested to be used. For this purpose, we have elaborated a testing stand to investigate self balancing process with video and computer techniques applied. Block scheme of the stand is shown in Fig. 4.

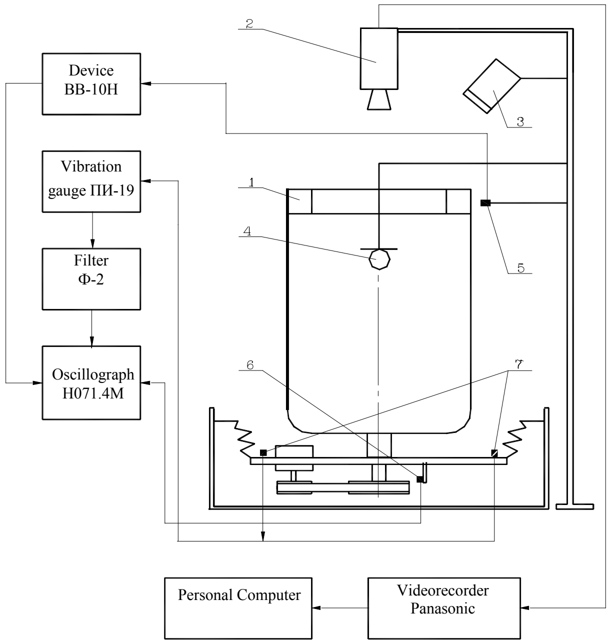



Fig. *4* Block-scheme of the equipment connecting: *1* – the ABD breadboard model; *2* – video camera; *3* – stroboscope; *4* – incandescent lamp; *5* – induction data unit; *6* – revolutions data unit; *7* – accelerometers

The stand includes measuring, registering, video techniques as well as a research installation which consists of rigid cantilever vertical rotor connected with rigid platform by means of a pair of roller bearings. The platform is fastened to the stationary frame with four spring suspension brackets.

In the upper part of the rotor there is a breadboard model of ABD made of optically transparent material (Fig. 5). It is a ring ∅400 mm with two concentred partitions ∅300 and ∅200 mm respectively which form concentred chambers for working bodies to be placed into. All the three chambers are hermetically insulated from each other. It enables using simultaneously both liquid (finctured water) and solid working bodies (balls made of various materials).

The start of the rotor is synchronized with the start of loop oscillograph and video camera. The use of current engine allows to chage operation frequencies gradually (stop by step) and to determine the positions of working bodies at fixed values of rotation frequency.

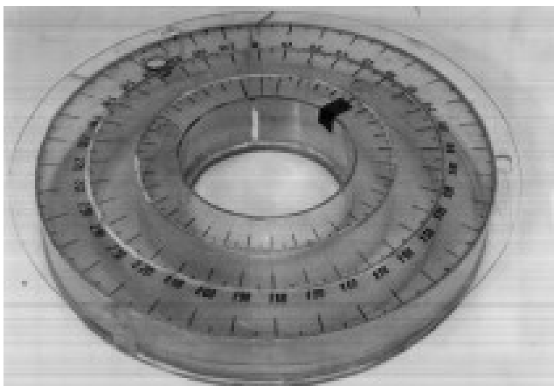
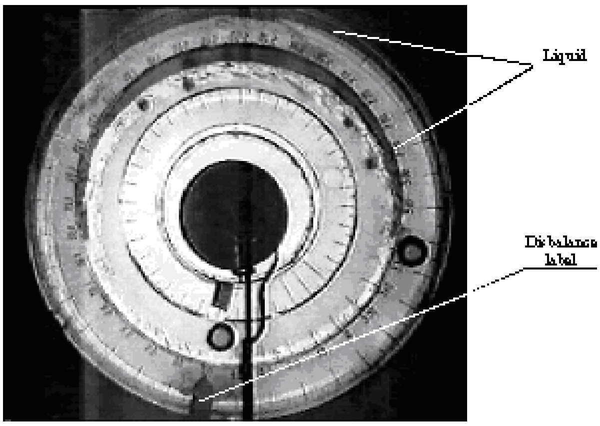


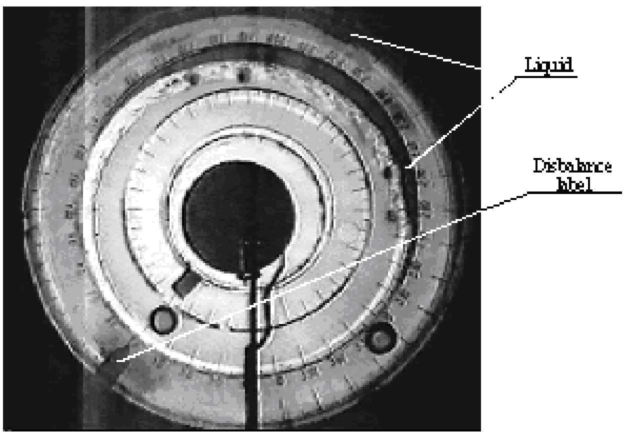
Fig. 5 General appearance of the ABD breadboard model

Combined development of video shots and oscillograph tapes shows that when the rotor reaches the assigned frequency of rotation the working bodies take the position against (in front of) disbalance, i.e. at the frequencies as low as 1.2-1.6 Hz which correspond to preresonance rotation frequency, without angular acceleration, the working bodies become balanced with the rotor disbalance. Examples of video shots are shown in Fig. 6.

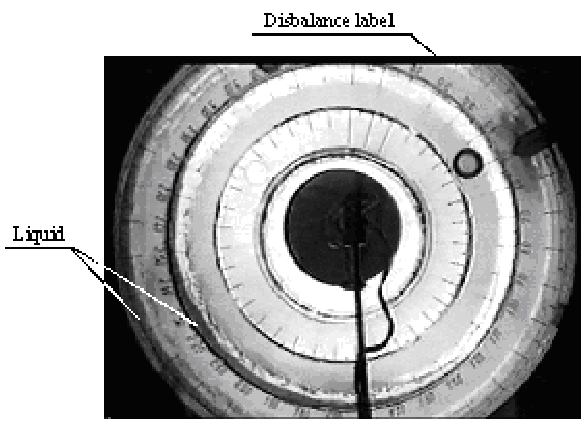
One can see the difference in time of working bodies being engaged into operation in case the rotor reaches the frequency of operation instantly. The liquid ceases its movement relative to frame walls of ABD much earlier than balls. As a result, at the first stage of acceleration ABD looks like a wide-range dynamic damper.




a




b




c

Fig. 6 Video shots: a – before resonance; b – at resonance; c – beyond resonance

Also, in the course of experiments it was noticed that the alteration of disbalance position during the work resulted in the changes of balls position and additional balance of the rotor.

Fig. 6 shows the most typical and representative computer slides of video recording of the liquid position in the ABD at under-resonance (Fig. 6, a, $\omega$=8$s^{-1}$), resonance (Fig. 6, b, $\omega$=12.5$s^{-1}$) and over-resonance (Fig. 6, c, $\omega$ = =90$s^{-1}$) frequencies of the rotor rotation with the ABD.

Analysis of these and other slides shows that the liquid compensates rotor disbalance at under-resonance, resonance and over-resonance frequencies of its rotation.

The liquid reacts slower due to its inertia, the working bodies supplement each other. The balls smooth out the vibrations caused by the alteration of disbalance position and the liquid smooths out the vibrations caused by the transference of the balls. This feature shows an expediency of using combined liquid-ball types of ABD for the reduction of machines vibration which have disbalance changed in magnitude and position during the working cycle.

Proceeding from the results obtained, the combined ABD for the washing-wringing machine ”Volga-ll“ has been developed. The application of this machine allowed to reduce the oscillation amplitude at resonance 2.5 times and at the operating rotation frequencies by 3 times.

## 4. Conclusions

The theory for automatic balancing describes the work of automatic balancing units only at overcritical rotation of the rotor. Experience of using automatic balancing units shows that existing theories for automatic balancing describe processes with work bodies, particularly with liquids, during their operation with poor accuracy and need to be improved.

Developed mathematical model describes the operation of liquid ABU and considers hydraulic properties of the liquid, explains why liquids compensate rotor imbalance as at over-resonance frequencies of rotor rotation and at pre-resonance and resonance frequencies as well. This model is space model of liquid ABU partially filled with a liquid that considers gravity forces to evaluate effectiveness of balancing of vertical rotor.

Equation describing the condition of stationary motion of solid body with a liquid in its hollow according to the principle of least action in the form of Hamilton-Ostrogradski is taken as a basic. The research was conducted in the area of complete nonlinear equations of motion applying the methods of analytical mechanics.

Studying the stability of motion of solid body with liquid inside the approach related to Lyapunov’s ideas in the theory of balance figures of rotating liquid. Was used here stationary motion stability problem was assumed as a problem of functional minimum of potential energy of the system.

The analysis of theoretical research shows that liquid compensates rotor imbalance at pre-resonance, resonance and over-resonance frequencies of its rotation.

If pressure inside the liquid is a function of radius from revolution axis, and pressure on free surface of the liquid is constant then free surface of the liquid is revolvolution paraboloid, which axis is the one of absolute revolution of the system going through its gravity center. The shape of the paraboloid depends on geometrical and dynamical parameters of the system and on imbalance also. In the section made by horizontal plane the shape of free surface is a circle. Increasing volume of the liquid and correspondently its layer when the chamber radius is constant does not displace the system's gravity center, because the center of liquid's free surface coincides with it. Thus, only thin layer of the liquid not exceeding 2*e* takes part in changing imbalanced state of rotating system, what is the primary reason of low efficiency of liquid ABU. On the discovery of this fact the general idea of using multichamber liquid ABU is based.

The system's motion study on its stability proves that stationary motion is stable in condition (11). Physically this condition explains that the chamber should have flat shape, i.e. the height of the chamber should be much lower than its radius. This opens opportunities to make multichamber ABU not only as embedded type where chambers are embedded in each other but also as one where chambers are placed at different levels one over the others, or to have combinations of mentions types.

Incidental condition (11) is true in case of viscous liquid. In considered case when liquid rotates together with the hollow (in chamber) as one, viscosity of the liquid does not effect on motion of the system.

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V. Royzman, I. Drach

BESISUKANČIŲ ROTORIŲ SU SKYSČIO AUTOBALANSYRAIS AUTOMATINĖS BALANSAVIMO TEORIJOS PATIKSLINIMAS

R e z i u m ė

Automatinių įrenginių naudojimo patirtis rodo, kad esama automatinio balansavimo teorija netiksliai aprašo procesus, vykstančius darbo medžiagoje – skysčiuose, todėl ją reikia patikslinti. Be to, sudarant skysčio autobalansyro matematinį modelį, būtina atsižvelgti į hidraulines skysčio savybes. Automatinio balansavimo procesui pademonstruoti ir teorinių bei praktinių rezultatų analizei panaudotas greitasis darbo skysčių elgsenos videofilmavimas.

V. Royzman, I. Drach

IMPROVING THEORY FOR AUTOMATIC BALANCING OF ROTATING ROTORS WITH LIQUID SELF BALANCERS

S u m m a r y

Experience of using automatic balancing units shows that existing theories for automatic balancing describe processes with work bodies, particularly with liquids, during their operation with poor accuracy and need to be improved. Besides, there's a necessity to develop mathematical model, which would describe work of liquid ABU and consider hydraulic properties of liquid.

In order to demonstrate the process of self balancing as a whole and analyze contradictions between the theory and the practice, a high-speed video shooting of working bodies behavior is suggested to be used. For this purpose, we have elaborated a testing stand to investigate self balancing process with video and computer techniques applied.

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УТОЧНЕНИЕ ТЕОРИИ АВТОМАТИЧЕСКОЙ БАЛАНСИРОВКИ ВРАЩАЮЩИХСЯ РОТОРОВ ЖИДКОСТНЫМИ АВТОБАЛАНСИРАМИ

Р е з ю м е

Опыт использования автоматических устройств показывает, что существующая теория автоматической балансировки неточно описывает процессы, происходящие с рабочими телами, в частности жидкостями, во время их работы и требует уточнения. Кроме того, есть необходимость в разработке математической модели работы жидкостного АБУ с учетом гидравлических свойств жидкости.

Для демонстрации процесса автоматической балансировки и последующего анализа теоретических и практических результатов использовался метод скоростной видеосъемки поведения рабочих тел.

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