

# Numerical modeling of suspension cable kinematic displacements

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## 1. Introduction

Suspension cable is a specific structural element widely employed in engineering practice to cover large spans (e. g. bridge structures) or areas (roofing) [1-6]. An attractive feature of suspension cable or the system of suspension cables is the sufficiently high carrying capacity versus its weight. Due to specific feature – vanishing capacity to resist bending and compression, suspension cable responses to loading in specific way. It changes the primary shape in the way to resist loading only via one type of deforming – tension. Thus, the shape of loaded cable is prescribed by two factors, namely: cable adaptation to loading; deforming due to developed axial tensile internal forces. The displacements, caused by the above mentioned factors are denoted as kinematic (non-straining) and elastic ones. One must note that usually the first type displacements have an essential contribution to cable shape. The remaining ones actually are conditioned by the first ones as the distribution of axial forces depend on the cable shape.

Due to suspension cable adaptation to loading peculiarities, the relation load versus displacement is strongly nonlinear. One can mention that this geometrically nonlinear behavior (due its nature) differs from other structural members, possessing certain flexural stiffness (when large displacements are resulted by deformations). In cable case the large displacements usually are conditioned by it's adaptation to loading processes. Actually the cable has its primary form, compatible with certain loading (at least the weight force). An essential change of it's shape is caused by a supplement loading, differing in principle from the primary one. It can be concentrated loads, applied irregularly or additional asymmetrically distributed load. The latter case is the most general case, met in engineering practice and subsequently in codified design.

A codified design of a structure introduces certain restrictions in respect of it's displaced/deformed shape under loadings. These requirements are realized via maximal displacement and member curvature (second derivative of displacement) limitations up to fixed magnitudes. Thus, the accuracy when valuating actual maximal displacements, their locations is of the first significance to obtain reliable and economically efficient design project. One can also note, that the displacement constraints usually predominate in actual design of suspension cable/system of suspension cables.

The large list of investigations on cable behavior analysis [6-11] illustrates the relevancy of the problem and the lack of efficient and sufficiently exact methods suitable for engineering practice. The widely employed engineering (simplified) methods to estimate vertical total (kinematic

and elastic) displacements employ the superposition principle when splitting the actual loading to asymmetric and symmetric distributed loads [3, 12, 13]. But direct application of superposition principle for geometrically nonlinear cable behavior of specific nature results certain errors. As an alternative to reduce the error the certain equivalent load concept [6, 14] was introduced. Unfortunately, the clear instructions to identify this equivalent load are not fixed, leading to certain uncertainty when applying this method.

One can conclude that the list of investigations, assigned to efficient and reliable estimation of kinematic displacements (governing when conditioning suspension cable shape under asymmetric loading) is small so far. An investigation of the authors [15] presented an analytical method to estimate kinematic displacements. The method was sufficiently (comparing with only few exact solutions under existence) accurate and suitable for engineering design practice.

As an attractive and efficient alternative for cable analysis is the application of FEM techniques [16-18]. To confirm additionally the reliability of the proposed method [15] numerical modeling of suspension asymmetrically loaded cable kinematic displacements was performed by nonlinear finite element method package COSMOSM. Comparison of the results with these obtained by the proposed analytical and widely applied engineering methods was performed.

## 2. Cable shape valuation via analytical expressions of kinematic displacements

The shape of a cable, subjected by a distributed load  $q$  applied per total span, fits quadratic parabola. When a supplement asymmetric distributed load  $p$  (e.g. left middle span) is applied, the suspension cable changes its form adapting the loading (see Fig. 1). To distinguish pure kinematic displacements the axial stiffness  $EA \rightarrow \infty$  ( $E$  and  $A$  denoting cable elasticity modulus and cross-sectional area, respectively) is to be introduced. The elastic displacement magnitudes then approach to zero.

Then the cable shape can be analyzed separately for left (loaded by  $p$ ) and the right (free of this load) parts. Then the cable shape functions reads [15]

$$z_l(x) = \frac{M_l(x)}{H_{11}} = \frac{f_1}{\left(1 + \frac{\gamma}{2}\right)} \times \left[ \frac{4x}{l} - \frac{4x^2}{l^2} + \gamma \left( \frac{3x}{l} - \frac{4x^2}{l^2} \right) \right], \quad x \leq l/2 \quad (1)$$

and

$$\begin{aligned} z_r(x) &= \frac{M_r(x)}{H_{r1}} = \\ &= f_1 \left[ \left( \frac{4x}{l} - \frac{4x^2}{l^2} \right) + \gamma \left( \frac{x}{l} - 1 \right) \right] \left/ \left( 1 + \frac{\gamma}{2} \right) \right. \end{aligned} \quad (2)$$

$l/2 \leq x \leq l$

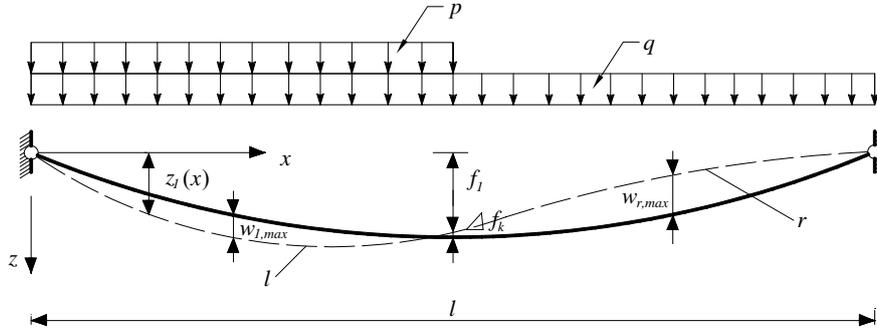


Fig. 1 Suspension cable design scheme

### 2.1. Cable left part vertical kinematic displacements

Cable left part can be determined by [15]

$$\begin{aligned} \omega_l(x) &= \frac{f_1}{1 + \gamma/2} \left( \frac{4x}{l} - \frac{4x^2}{l^2} + \gamma \left( \frac{3x}{l} - \frac{4x^2}{l^2} \right) \right) - \\ &- f_0 \left( \frac{4x}{l} - \frac{4x^2}{l^2} \right) \end{aligned} \quad (3)$$

It is obvious that for the middle span

$$\omega_l(x=l/2) = f_0 - f_1 = \Delta f_k \quad (4)$$

where  $f_0$  is the cable primary sag.

Find that kinematic displacement  $w_l(x)$  can be determined when  $f_1$  or  $\Delta f_k$  are already known. The latter values can be obtained by [15]

$$f_1 = f_0 \sqrt{\psi} \quad (5)$$

$$\Delta f_k = f_0 (\sqrt{\psi} - 1) \quad (6)$$

where

$$\psi = \frac{16 + 16\gamma + 4\gamma^2}{16 + 16\gamma + 5\gamma^2} \quad (7)$$

Having identified the cable sag  $f_1$ , the left part vertical displacements is obtained by

$$\omega_l(x) = f_0 \left[ \left( \frac{4x}{l} - \frac{4x^2}{l^2} \right) \left( \frac{1}{\xi} - 1 \right) + \frac{\gamma}{\xi} \left( \frac{3x}{l} - \frac{4x^2}{l^2} \right) \right] \quad (8)$$

where

here  $M_{l1}(x), H_{l1}$  are the moment and the tensile force of the left, loaded by  $(q+p)$  cable part, respectively;  $M_{r1}(x), H_{r1}$  are the moment and the thrusting force of the right cable part, respectively;  $\gamma = p/q$  is the ratio of asymmetric and symmetric loads intensities.

Fig. 1 denotes via  $f_1 = f_0 + \Delta f_k$  the cable sag in the middle span and via  $\Delta f_k$  the kinematic middle span displacement.

$$\xi = \sqrt{1 + \gamma + 5\gamma^2/16} \quad (9)$$

Find that kinematic displacements are directly dependent on the cable sag. The increase of the loads ratio  $\gamma$  results the increase of the  $\omega_l(x)$  magnitudes. The obtained expression (9) is convenient for usage, as it does not include  $f_1$ . It is obvious, that kinematic displacement for  $(x \leq l/2)$  is  $\omega_l(x) = \Delta f_k$ .

In practical design one must identify maximal deflection and its location point. Increasing the loads ratio  $\gamma$  from 1 to 10, the maximal deflection location point varies insignificantly [15]. Therefore it is enough to fix its location as the first quarter point, then the maximal kinematic displacement can be obtained by:

$$\omega_{l,max}(x) = 0.75 f_0 \left[ (1 + 2\gamma/3) / \xi - 1 \right] \quad (10)$$

### 2.2. Cable right part vertical kinematic displacements

Kinematic displacements of the right cable part can be obtained via expressions [15], reading

$$\omega_r(x) = \frac{f_1}{1 + 0.5\gamma} \left[ \frac{4x}{l} - \frac{4x^2}{l^2} + \gamma - \frac{x\gamma}{l} \right] - f_0 \left( \frac{4x}{l} - \frac{4x^2}{l^2} \right) \quad (11)$$

or

$$\omega_r(x) = f_0 \left[ \left( \frac{4x}{l} - \frac{4x^2}{l^2} \right) \left( \frac{1}{\xi} - 1 \right) + \frac{\gamma}{\xi} \left( 1 - \frac{x}{l} \right) \right] \quad (12)$$

Find the formulae formula (8) and (12) to be analogous. When varying the loads ratio  $\gamma$  from 1 to 10, the maximal deflection location point varies insignificantly. Therefore an approximate maximal vertical displacement location point can be taken at the end of the

third middle span quarter [15]. Then an approximate maximal kinematic displacement of the right part can be obtained by

$$\omega_{r,max}(x) = 0.75f_0 \left[ \left( \frac{1}{\xi} - 1 \right) + \gamma/3\xi \right] \quad (13)$$

Analyzing the left part displacement  $\omega_{l,max}$  and that of the right part  $\omega_{r,max}$ , one can find that the unloaded part displacements are larger in absolute magnitudes when compared with the ones of the loaded cable part. When increasing the loads ratio  $\gamma$ , the difference between these values in absolute magnitudes also increase.

### 3. Numerical modelling of kinematic vertical displacements

The asymmetrically loaded suspension cable shape evaluation via the numerical simulations by means of FEM package COSMOSM was performed. The aim of numerical simulations was estimating the reliability of the relations of proposed analytical method (see section 2) and that of the widely applied engineering methods.

For analysis of the structure response to loading the suspension cable of span  $l = 100\text{ m}$  and primary sag  $f_0 = 10\text{ m}$  was chosen (design scheme see in Fig. 1). The cable loading is splitted to symmetric  $q$  and asymmetric  $p$  distributed loads. The response of the cable was investigated in respect of loads ratios  $\gamma = p/q$ , varying it by  $\gamma = 1 - 10$ . The cable FEM design scheme was modeled by pin-jointed structure, created from straight bar finite elements. The actual distributed loading at nodes areas was replaced by resultant concentrated loads applied onto nodes (hinges). The nodes in FEM model were introduced to fit the exact primary cable contour curve of quadratic parabola. To estimate the influence of introduced number of finite elements (discretization level) on the results of numerical simulations, the calculations were performed modeling cable by 100 and 200 finite elements, respectively. Note that the number of elements of an actual cable approaches to infinity.

Actually, an application of FEM package results total values of displacements, caused by geometrically nonlinear adaptation of the structure to loading (kinematic displacements) and the action of internal forces (elastic displacements). Aiming to separate the kinematic displacements from total ones via numerical means, a sufficiently large axial stiffness  $EA = 51561.3\text{ MN}$  (resulting magnitudes of elastic components of cable total displacements to be significantly far from kinematic ones) of finite elements was chosen.

One must note, that varying the loads ratio  $\gamma$  the intensities of symmetric  $q$  and asymmetric  $p$  loads were chosen in the way the resulting thrusting forces of the cable to remain almost constant (unchanged), i.e.  $H \cong \text{const}$ . Thus, the intensities of symmetric and asymmetric loads were determined by

$$q = 8Hf_0 / (1 + 0.5\gamma) \quad (14)$$

$$p = q\gamma \quad (15)$$

As for graphical illustrations of FEM simulations of cable shape the cable response vs three  $\gamma$  magnitudes are presented in Figs. 2-4.

To perform a reasonable comparison of the obtained results when employing the above mentioned methods and the FEM package, the maximal vertical displacements of asymmetrically loaded cable part were analyzed at the points  $x = l/4$  (left, loaded by  $p$ , cable part) and in the point  $x = 3l/4$  (right, free of the asymmetric loading  $p$ , cable part). The above mentioned coordinates of cable points are compatible with locations of extreme displacements. Coordinates of these points are also fixed in the widely applied engineering methods, aimed to identify extreme kinematic displacements. We remind the reader that the error when calculating extreme displacements employing these coordinates does not exceed 1.6% [11, 15]. The analysis results are presented in Tables 1-4, where superscripts  $t$ ,  $c$  and  $en$  refer to the proposed analytical, FEM package COSMOSM and engineering analysis methods, respectively.

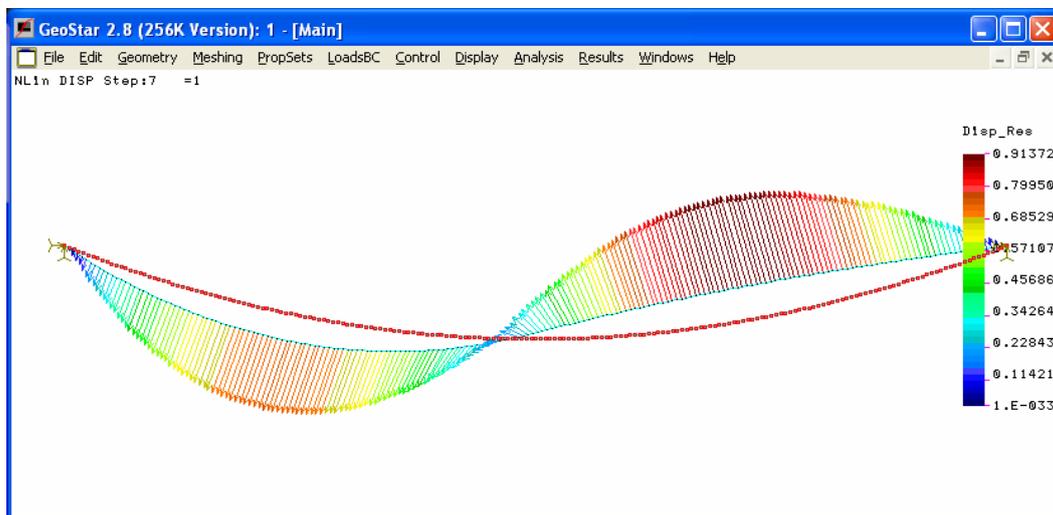
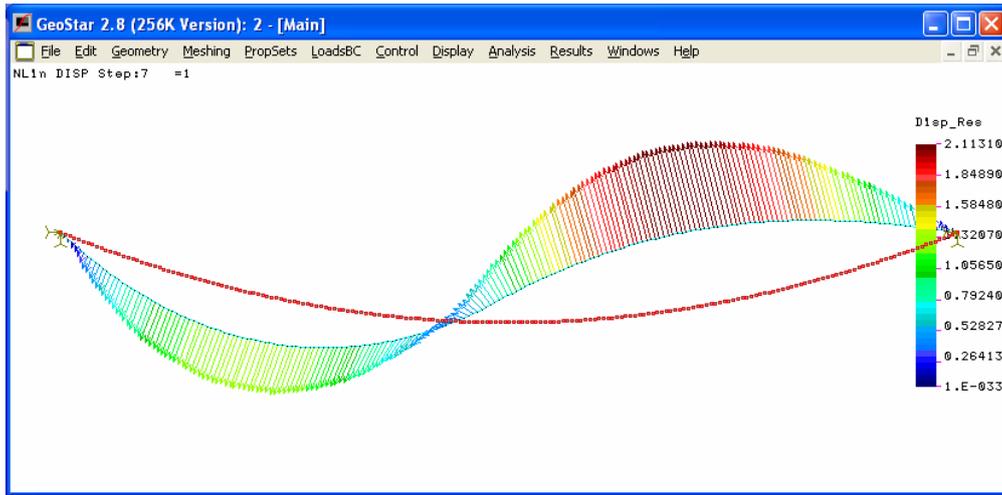
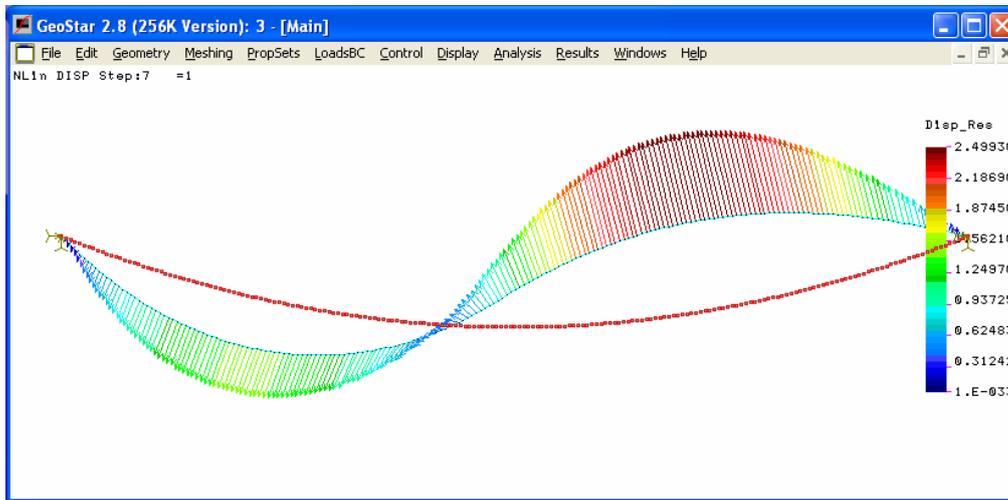


Fig. 2 Cable kinematic displacements via COSMOSM in case  $\gamma = 1$

Fig. 3 Cable kinematic displacements via COSMOSM in case  $\gamma=5$ Fig. 4 Cable kinematic displacements via COSMOSM in case  $\gamma=10$ 

The calculation results of left part maximal displacements of asymmetrically loaded cable are presented in Table 1. The result analysis shows that the difference between maximal vertical displacements  $\omega_{l,max}$ , obtained by proposed in the paper and that of calculated by FEM package COSMOSM is not significant. The maximal relative difference is 4.90% in case of loads ratio  $\gamma=1$ . This accuracy is sufficient enough for practical calculations when valuating the changed shape of loaded cable. When increasing the  $\gamma$  magnitude up to 10, the latter difference gradually decreases till 2.8%. This result indicates a qualitative compatibility of changed cable shapes, determined by applying the proposed analytical and employed finite element method package.

One must note that vertical displacements, obtained via FEM package are less comparing to the ones, obtained by analytical method. This illustrates a little overestimation of cable adaptation displacements, conditioned by: an approximation of actual cable design scheme via certain number of the elements; accuracy when replacing the distributed loads by resultant concentrated ones; the FEM package accuracy when calculating such type of the structure; other. When analyzing the cable FEM design model, one can find the total primary length of the cable modeled by piece-wised curve to be a little bit less the ac-

tual one (being employed in analytical method) of smooth quadratic parabola cable curve. In addition, the cable thrusting force being identified via FEM model calculations (caused by concentrated resultant forces) is a little bit greater than the one, obtained by analytical method. This insignificant difference (approximately 1%) in respect of thrusting force increases with an increment of  $\gamma$ . The increased thrusting forces reduces the cable middle span vertical displacement  $\Delta f_k$ . It is evident that the primary cable length and thrusting force have direct influence on kinematic displacements [6, 9-11, 15].

One must note that calculation error of suspension cable kinematic displacement obtained when applying engineering method is rather big when comparing with the results obtained either by FEM package or by the proposed analytical method. In case of loads ratio  $\gamma=1$  it is 15.4% when comparing with accurate analytical method and is 21.4% when comparing with FEM package COSMOSM. In case of  $\gamma=10$  the error increases up to 54.8% and up to 59.3%, respectively. The graph of engineering method error of maximal kinematic displacement magnitude vs loads ratio  $\gamma$  is presented in Fig. 5.

The results of maximal kinematic displacements, being developed of cable part, free of load  $p$ , are pre-

sented in Table 2. When analyzing the results one can find

Table 1

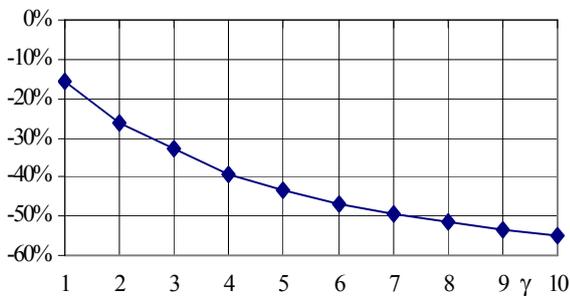
Comparison of left (loaded) cable kinematic displacements

$\gamma$	$\omega_{l,max}^t, \text{ m}$	$\omega_{l,max}^c, \text{ m}$	$\Delta\omega_{l,max}^c, \%$	$\omega_{l,max}^{en}, \text{ m}$	$\Delta\omega_{l,max}^{en}, \%$
1	0.722	0.687	4.9	0.833	-15.4
2	0.989	0.950	3.9	1.250	-26.4
3	1.120	1.081	3.5	1.500	-32.9
4	1.196	1.156	3.4	1.667	-39.4
5	1.245	1.204	3.3	1.786	-43.5
6	1.278	1.238	3.1	1.875	-46.7
7	1.302	1.263	3.0	1.944	-49.3
8	1.320	1.282	2.9	2.000	-51.5
9	1.335	1.295	3.0	2.046	-53.3
10	1.346	1.308	2.8	2.083	-54.8

Table 2

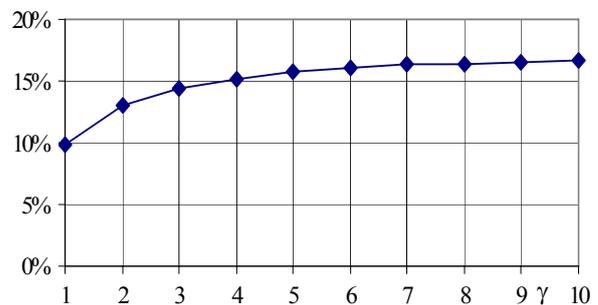
Comparison of right (unloaded) cable kinematic displacements

$\gamma$	$\omega_{r,max}^t, \text{ m}$	$\omega_{r,max}^c, \text{ m}$	$\Delta\omega_{r,max}^c, \%$	$\omega_{r,max}^{en}, \text{ m}$	$\Delta\omega_{r,max}^{en}, \%$
1	-0.924	-0.874	5.4	-0.833	9.8
2	-1.437	-1.367	4.9	-1.250	13.0
3	-1.753	-1.680	4.2	-1.500	14.4
4	-1.966	-1.889	3.9	-1.667	15.2
5	-2.119	-2.039	3.8	-1.786	15.7
6	-2.233	-2.152	3.6	-1.875	16.0
7	-2.322	-2.241	3.5	-1.944	16.3
8	-2.393	-2.311	3.4	-2.000	16.4
9	-2.452	-2.366	3.5	-2.046	16.5
10	-2.50	-2.422	3.1	-2.083	16.7

Fig. 5 Left cable part maximal kinematic displacement evaluation relative error vs  $\gamma$ 

that displacement magnitudes of the left cable part are close to these calculated by COSMOS/M package and by the proposed analytical method. The largest difference of the results, obtained by above methods in respect of vertical left part displacements  $\omega_{r,max}$  is 5.40%, corresponding the case  $\gamma=1$ . When increasing  $\gamma$  magnitude up to  $\gamma=10$  the error gradually reduces up to 3.10%. When applying the engineering method the displacement evaluation error of unloaded (free of load  $p$ ) right part is a little bit less than the error, obtained when valuating cable right part displacements. In case of  $\gamma=1$  the above mentioned error is 9.80% and 4.69%, respectively, comparing with the result obtained by COSMOSM package and analytical method. The graph of engineering method error when estimating maximal right cable part displacement vs loads

ratio  $\gamma$  is presented in Fig. 6. The graph shows that the  $\gamma$  increment causes gradual increment of the error, conditioned by the application of the engineering method. In case of  $\gamma=10$ , the error is 16.7% when compared with the proposed analytical method and 14% - when compared with the result obtained by FEM package COSMOSM. One must note, that relatively smaller errors of engineering method are conditioned by larger displacements of right unloaded cable part (see data of Tables 1 and 2).

Fig. 6 Right cable part maximal kinematic displacement evaluation relative error vs  $\gamma$ 

As it was mentioned earlier, the essential difference amongst engineering and proposed analytical methods is conditioned by a proposition of engineering method, stating the middle span displacement is zero, i.e.  $\Delta f_k = 0$ . Numerical FEM cable simulations resulted the latter to be nonzero magnitude, i.e.  $\Delta f_k \neq 0$ . The cable middle span

vertical kinematic displacement, estimated by FEM, is directed up analogously as the one, estimated by analytical method. The middle span kinematic displacement vs  $\gamma$  are presented in Table 3. Take a notice that this displacement increment is compatible with the increment of  $\gamma$ . We remind the reader that kinematic displacement magnitudes identified by analytical method are greater than the ones, obtained by FEM per all  $\gamma$  variation range. The difference (in percentage values) of displacement magnitudes, obtained by analytical and FE methods is in average 5%. The largest difference is fixed in case of  $\gamma=1$  (Table 3).

Analyzing displacement magnitude, obtained via analytical method and numerically (via FEM package COSMOSM), one can find that shapes of loaded cable are qualitatively close/fitting. Kinematic displacements of the cable right part are greater than those of loaded left part (see Tables 1 and 2). A ratio of maximal kinematic displacements of cable right and left parts vs ratio  $\gamma$  is presented in Table 4. One can find that the  $\gamma$  increment results the subsequent gradual increment of the latter ratio (this is valid when analyzing the cable by analytical

method and by COSMOSM package). An employment of engineering method results a constant magnitude of this ratio vs increment of  $\gamma$  (see Table 4).

Table 3

Comparison of middle span cable kinematic displacements

$\gamma$	$\omega^t_{l,max}$ , m	$\omega^c_{l,max}$ , m	$\Delta\omega^c_{l,max}$ , %
1	-0.136	-0.122	10.1
2	-0.299	-0.288	3.6
3	-0.422	-0.404	4.2
4	-0.513	-0.490	4.5
5	-0.583	-0.556	4.6
6	-0.637	-0.607	4.7
7	-0.680	-0.648	4.7
8	-0.715	-0.682	4.7
9	-0.745	-0.704	5.5
10	-0.769	-0.731	5.0

Table 4

Comparison of kinematic displacements of cable loaded and unloaded

$\gamma$	$\omega^t_{r,max}/\omega^t_{l,max}$	$\omega^c_{r,max}/\omega^c_{l,max}$	$\Delta\omega$ , %	$\omega^{en}_{r,max}/\omega^{en}_{l,max}$	$\Delta\omega^{en}$ , %
1	1.280	1.272	0.6	1.000	21.9
2	1.453	1.439	1.0	1.000	31.2
3	1.565	1.554	0.7	1.000	36.1
4	1.644	1.634	0.6	1.000	39.2
5	1.702	1.694	0.5	1.000	41.3
6	1.717	1.738	0.5	1.000	42.8
7	1.783	1.774	0.5	1.000	43.9
8	1.813	1.803	0.6	1.000	44.8
9	1.837	1.827	0.5	1.000	45.6
10	1.857	1.852	0.3	1.000	46.1

A qualitative compatibility amongst deformed axes curves of asymmetrically loaded cable, obtained by analytical method and FEM method (COSMOSM package), is ensured by fitting maximal kinematic displacements ratios of right and left parts. Analyzing data of Table 4 one can find that difference of the latter ratios does not exceed 1% and this difference gradually decreases vs increment of  $\gamma$ . The engineering method errors in respect of this ratios are essential and reach up to 46% (see Table 3 and Fig 7).

Basing on the analysis of numerical simulation results one can state that cable shape, identified via engineering methods, differs qualitatively (cable form and extreme displacement magnitudes) from an actual one. The method results equal maximal magnitudes of both cable parts and zero cable middle span displacement, i.e. to be independent on  $\gamma$ .

The results the proposed analytical method for kinematic displacement determining were also compared with the ones, obtained via corrected engineering method [6, 14]. The performed calculations illustrated compatibility for maximal displacements of the cable loaded part. However, one must note that corrected engineering method

is not suitable for real engineering design as is it valid for limited discrete number of  $\gamma$  magnitudes.

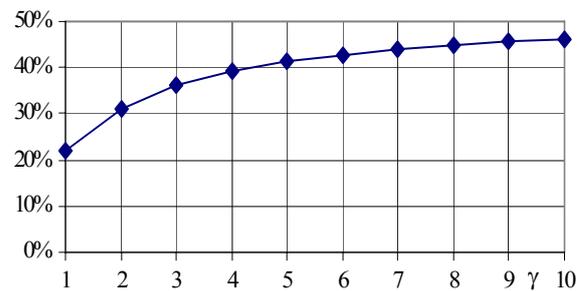


Fig. 7 Ratio error of extreme displacements (defined by engineering methods) vs  $\gamma$

#### 4. Concluding remarks

Numerical modeling of suspension cable kinematic displacements was performed applying the FEM package COSMOSM, and proposed by the authors analytical and the widely employed engineering methods. Analysis of obtained results yielded:

1. The results obtained by proposed analytical method and FEM package COSMOSM fit sufficiently well

(maximal error does not exceed 5%).

2. The variance of obtained results is conditioned by FEM peculiarities when modeling actual cable behavior.

3. Actually the asymmetric load causes cable middle span lifting, when the engineering methods neglect this phenomenon, that results the significant displacement evaluation errors.

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## SKAITINIS KABAMOJO LYNO KINEMATINIŲ POSLINKIŲ MODELIAVIMAS

### Reziumė

Atliktas skaitinis kabamojo lyno kinematinųjų poslinkių modeliavimas naudojant baigtinių elementų, autorių pasiūlytą analizinį ir inžinerinius metodus. Pasiūlyto analizinio metodo rezultatai gana gerai sutampa su BEM, o taikant inžinerinius metodus gaunamos nemažos paklaidos.

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## NUMERICAL MODELING OF SUSPENSION CABLE KINEMATIC DISPLACEMENTS

### Summary

Modelling of cable kinematic displacements is performed by FE, proposed by authors analytical and engineering methods. The proposed analytical and FE methods results fit sufficiently well, when engineering methods yield significant errors.

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## ЧИСЛЕННОЕ МОДЕЛИРОВАНИЕ КИНЕМАТИЧЕСКИХ ПЕРЕМЕЩЕНИЙ ВИСЯЧЕЙ НИТИ

### Резюме

Выполнено численное моделирование кинематических перемещений нити предлагаемым аналитическим и инженерными методами, а также МКЭ. Результаты показали хорошее совпадение МКЭ с предлагаемым аналитическим и большие погрешности инженерных методов.

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