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Stress and strain analysis of a bilayer composite beam with interlayer slip under hygrothermal loads

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1. Introduction

Timber-concrete, concrete-steel and concrete-concrete bilayer composite structures are widely used for the floor strengthening of buildings under renovation. Due to the relatively small mass, good oscillations' damping and the possibility to set up sound insulation, timber-concrete composite structures are used for the new dwelling house floors. Owing to simple technology and decrease of labour expenditure, concrete-steel bilayer beams are used for setting up the monolithic floor with retained formwork. Concrete-concrete layered structure is formed when old concrete is repaired with a new concrete layer.

For the above mentioned structures, the layer slip under external action is the main property which determines the calculation methods used for the design of such structures and calculation of stresses and strains. If glued layered structures such as wood beams may be calculated as a solid bar with axial strains in all layers being equal and the plane section hypothesis may be accepted for the entire layered cross-section, then due to specific joint the layers of timber-concrete, concrete-steel, concrete-concrete composite structures slip in regard to each other under external action. For these structures the classical methods of material mechanics cannot be applied.

The majority of calculation methods for stresses and strains of the above mentioned structures take into account only external loads, such as bending moment or shear forces. However it is well known that concrete, through hardening and drying, shrinks over time. Coefficients of wood, concrete and steel thermal and hygral expansion are different. Changes in layers temperature and humidity may generate additional stress. Therefore it is very important that stresses, strains and displacements of such structures would be simply and accurately calculable. In some cases a concrete layer of the layered structures may even crack because of strains caused by temperature and humidity. This undesired effect may be avoided selecting compatible materials or dividing the set-up concrete cover into deformation seams.

In the article a method allowing calculation of hygrothermal stress and deflection of bilayer timber-concrete, concrete-steel, concrete-concrete beams taking into account interlayer slip is presented. An algorithm allowing calculation of limiting thermal and hygral strains when the layers still do not crack and an algorithm for calculation of limiting beam length (when one of the layers cracks) are also presented in the article.

2. Main dependences

The calculation methods of layered structures may be divided into two large groups. One of them is based on certain hypotheses limiting usage of these methods. In other methods the layered structures are described as three-dimensional solid body subjected to three-dimensional stress and strain. Application of the following theories is not convenient due to the complicated solution for engineering calculation. The majority of practical problems may be solved quite precisely using much more simple calculation method based on various hypotheses [1-5]. One of the simplest methods considering slip of the layers is the theory of build-up bars [2]. This method is convenient because in most cases it is possible to write the final rigorous solution in the form of explicit function. The built-up bars theory is based on these assumptions.

1. Load-slip relationships for the interlayer connections are linear.
2. Layers' stress and elastic displacement relation is linear.
3. Thermal and hygral strains through the depth of each layer is uniform, i.e. independent from the X coordinate (Fig. 1, a.).
4. Thermal and hygral strains along the road are uniform, i.e. independent from X coordinate (Fig. 1, a);
5. Plane section hypothesis is valid for each separate layer.
6. Deflection due to shear strains in individual layers is negligible.

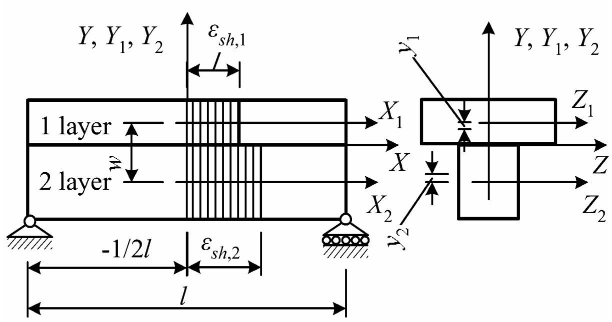

Fig. 1 The scheme of a layered beam, thermal and hygral strain ε_{sh} and coordinate locality: a – longitudinal section of a beam, b – cross-section of a beam

2.1. Layers' stress relationship

According to the built-up bars theory the total shear force and thermal and hygral strains of layers' joint are related by the second-order non-homogeneous differential equations with a constant coefficient [1]

$$\partial^2 T(x)/\partial x^2 = \xi \Delta T(x) + \xi\theta \qquad (1)$$

where $T(x)$ is total shear force acting in the joint of the layers, ξ is stiffness of layers joint; Δ is coefficient of layers compliance

$$\Delta = \frac{1}{E_1 A_1} + \frac{1}{E_1 A_1} + \frac{w^2}{D} \tag{2}$$

where E_i, A_i and D are the ith layer elasticity modulus, cross-section area and total flexural stiffness of both beams respectively; $D=E_1I_1+E_2I_2$ where I_i is the ith layer moment of inertia; w is the distance between the centroids of layers sections (Fig. 1, a.); θ is the difference of the layers thermal and hygral strains

$$\theta = \varepsilon_{sh,1} - \varepsilon_{sh,2} \tag{3}$$

where ε_{sh} are the layer thermal and hygral strains (Fig. 1, a.); ε_{sh} are positive if strains coincide with the direction of X axis (Fig. 1, a); otherwise ε_{sh} are negative. The solution Eq. (1) is as follows [1]

$$T(x) = C_1\, sinh(\lambda x) + C_2\, cosh(\lambda x) - \theta/\Delta \tag{4}$$

where $\lambda = \sqrt{\xi\Delta}$. The integration constants are obtained from a boundary condition stating that the total shear force equals to 0 at the beam end and the joint shear force equals 0 in the middle of the beam. If we assume that the origin of the XYZ coordinate' is in the middle (Fig. 1 a.), then the boundary conditions from which the integration constants are obtained are the following

$$\partial T(x)/\partial x \;\Big|_{x=0} = 0 \tag{5}$$

$$T(1/2\,l) = 0 \tag{6}$$

The Eq. (5) derivatives give

$$\partial T(x)/\partial x\Big|_{x=0} = C_1\, sinh(\lambda 0) + C_2\, cosh(\lambda 0) = 0$$

integration constant

$$C_1 = 0 \tag{7}$$

Inserting Eq. (7) into Eq. (6) we get

$$T(1/2\,l) = C_2\, cosh(1/2\,\lambda l) - \theta/\Delta = 0$$

We can obtain C_2 integration constant according to the expression

$$C_2 = \theta/\left(\Delta cosh(1/2\,\lambda l)\right) \tag{8}$$

The final expression of total shear in the joint is

$$T(x) = \frac{\theta}{\Delta}\left(\frac{cosh(\lambda x)}{cosh(1/2\,\lambda l)} - 1\right) \tag{9}$$

Axial force N_i and bending moment M_i in each layer are the following [1]

$$N_i(x) = \vartheta_i\, T(x),\, i = 1,2 \tag{10}$$

where $\vartheta_1 = 1, \vartheta_2 = -1$

$$M_i(x) = -wT(x)E_iI_i/D \tag{11}$$

and stresses are calculated by the well known formula

$$\sigma_i(x, y_i) = N_i(x)/A_i + y_i M_i(x)/I_i \tag{12}$$

where y_i is the distance between the ith layer centroid of the section and a considered point in the $X_iY_iZ_i$ coordinate (Fig. 1, a). The negative sign shows the material being under compression and positive sign shows the material to be under tension.

2.2. Calculation of stress and load in a joint

Commonly joint strength in scientific papers and design rules [6] are defined as a limiting joint force. For shear force calculation in a joint it is possible to use the known condition that the shear force in the joint is the derivative of the total shear force $T(x)$ in respect to x [1]

$$\tau(x) = \frac{\partial T(x)}{\partial x} = \frac{\theta}{\Delta}\frac{sinh(\lambda x)}{cosh(1/2\,\lambda l)} \tag{13}$$

The maximum shear force forms at the beam ends

$$\tau(1/2\,l) = \frac{\theta}{\Delta}\frac{sinh(1/2\,\lambda l)}{cosh(1/2\,\lambda l)} \tag{14}$$

The joint strength of layered structures is sufficient if the following condition is satisfied

$$f_j \geq \tau(1/2\,l) \tag{15}$$

where f_j is the joint strength. If the shear force τ is known, it is possible to calculate the number of connections n for one running meter

$$n = \tau(1/2\,l)/f_j\,. \tag{16}$$

2.3. Algorithm for obtaining extreme values of thermal and hygral strains

As it is known well, concrete shrinks while drying and hardening. If the first concrete layer tensile stress exceeds the strength limit due to shrinkage strain then it cracks before the start of the structure exploitation. In most cases the occurrence of cracks is undesired because of the reduction of stiffness. Therefore to avoid cracking of the layers, the materials with compatible properties should be selected. Below the method allowing calculation of the properties of layer materials not causing the cracks is provided.

As it may be seen from Eqs. (9) and (12) equations $T(x)$ and σ are maximum when x=0. Moreover, stresses through the height of each layer change linearly, therefore it is sufficient to check the stresses only at the joint and the outside of the layers. Therefore further analysis includes the investigation how stresses at middle of the beam depend on geometrical dimensions, mechanical properties and stiffness of the joint, taking that

$$x=0 \quad (17)$$

$$y_i=\pm h_i/2 \quad (18)$$

In general compressive and tensile strengths of materials are different, therefore in the formulas written above, strength must be taken keeping in mind σ_i. When σ_i is positive, the material is under tension and f_i must be the tensile strength – $f_{t,i}$, when σ_i is negative then the material is under compression and f_i must be the compressive strength– $f_{c,i}$. If we assume that the layers do not crack while the layers' stresses do not exceed the strength of their material, then according to the relationship (12) geometrical characteristics and values of material properties with which the layers do not crack may be calculated. The non cracking condition of the layers

$$\begin{aligned}&\sigma_i(x,y_i)\le f_{t,i},\ \text{when}\ \sigma_i(x,y_i)>0\\&\sigma_i(x,y_i)\ge -f_{c,i},\ \text{when}\ \sigma_i(x,y_i)<0\end{aligned} \quad (19)$$

Shortly (19) is written like this

$$\pm\sigma_i(x,y_i)\le f_i,\ f_i\in\{f_{t,i},f_{c,i}\} \quad (20)$$

where the positive sign + is when σ>0, the negative sign – when σ<0 $f_{t,i}$ is taken when the material is under tension, $f_{c,i}$ is taken when the materials is under compression.

Inserting Eqs. (12), (10) and (11) into (20) we get this inequality

$$f_i\ge\pm T(x)\left(\frac{\vartheta_i}{A_i}-\frac{wy_iE_i}{D}\right) \quad (21)$$

After putting Eq. (9) into Eq. (21) expression and taking Eq.e (17) into account we get this inequality of the non-cracking of layers

$$f_i\ge\pm\frac{\theta}{\Delta}\left(\frac{1}{cosh(1/2\,\lambda l)}-1\right)\left(\frac{\vartheta_i}{A_i}-\frac{wy_iE_i}{D}\right) \quad (22)$$

Then the inequalities allowing calculation of thermal and hygral strain which do not induce the cracking in the layers, are obtained

$$\theta\ge\Theta f_i,\quad \text{when}\quad \Theta f_i<0 \quad (23)$$

$$\theta\le\Theta f_i,\quad \text{when}\quad \Theta f_i>0 \quad (24)$$

here Θ is as follows

$$\Theta=\frac{\Delta}{\left(\frac{1}{cosh(1/2\,\lambda l)}-1\right)\left(\frac{\vartheta_i}{A_i}-\frac{wy_iE_i}{D}\right)} \quad (25)$$

In the Eqs. (23) and (24) f_i has these values $f_i\in\{f_{t,i},-f_{c,i}\}$. Since it is required that none of the layers cracks, then the solution of the system of Eqs. (23) and (24) is the difference of the shrinkage strains from the interval [$\theta_{min}\le\theta\le\theta_{max}$]. Here θ_{min} is the maximum negative, and θ_{max} is the maximum positive value of Eqs. (23) and (24) system. For the sake of clarity, the θ_{min} and θ_{max} algorithm of calculation is given in Fig. 2.

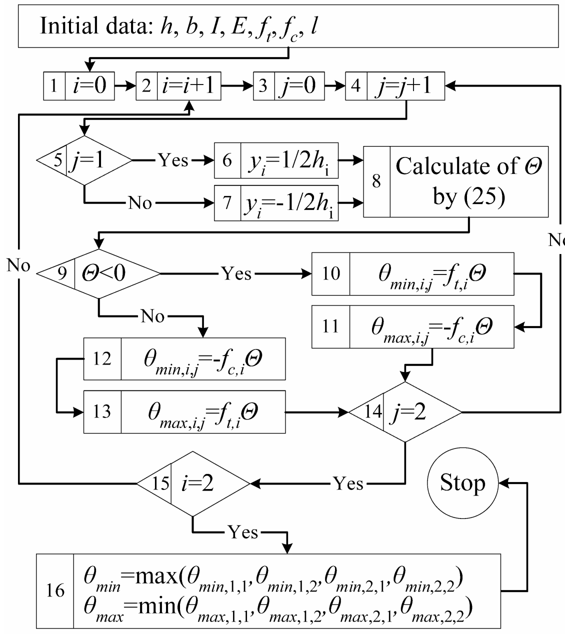

Fig. 2 Algorithm for calculation of the maximum θ_{max} and minimum θ_{min} values of thermal and hygral strains, that (if exceeded) will cause a single layer to crack

As it is shown by Eq. (25), the difference of shrinkage strains generating layer cracks is directly proportional to the strength of the materials. The influence of other factors on shrinkage strains is more complicated. As it may be seen from the Eq. (25), alteration of θ_{max} and θ_{min} due to l and ξ is similar, the difference is only that l is not under the root.

2.4. Algorithm for the calculation of limiting beam length, which, if exceed, will cause a single layer to crack

As it can be seen from the Eq. (12), stresses depend on the beam length. Thus to avoid undesired cracking of the layers, design structures may by divided into movement joints. Changing the inequality (22) into equality and solving it in respect to l, we obtain the limiting beam length l_{lim}, which, if exceeded will cause one of the layers to crack

$$l_{\lim}=2\,arccosh(K_i)/\lambda \quad (26)$$

where l_{lim} is the limiting beam length, which, if exceed, will cause one of the layers to crack. The coefficient K

$$K_i=1/(R_iG_i+1) \quad (27)$$

where G_i is as follows

$$G_i=\frac{\Delta}{\theta\left(\frac{\vartheta_i}{A_i}-\frac{wy_iE_i}{D}\right)} \quad (28)$$

The Eq. (26) may also be written as

$$l_{lim}\ge 2\,ln\left(K_i+\sqrt{K_i+1}\sqrt{K_i-1}\right)/\lambda \quad (29)$$

As it is well known, the *arccosh* function has real value only if the argument values are greater than or equal to 1. Therefore $K_i > 1$. From the inequality (26) we obtain, that when $f_i \to 0$ or $\theta \to \infty$ then $K_i \to 1$ and $l_{lim} \to 0$. Such a case is not possible in real structures, therefore $l_{lim} > 0$ at all times. When $f_i \to \infty$ or $\theta \to 0$ or $\xi \to 0$ then according to the Eq. (27) at $K_i < 0$ and Eq. (26) does not have a real solution. In this case the layer won't crack even if $l = \infty$.

For the sake of clarity, the algorithm for the calculation of l_{lim} is given in Fig. 3.

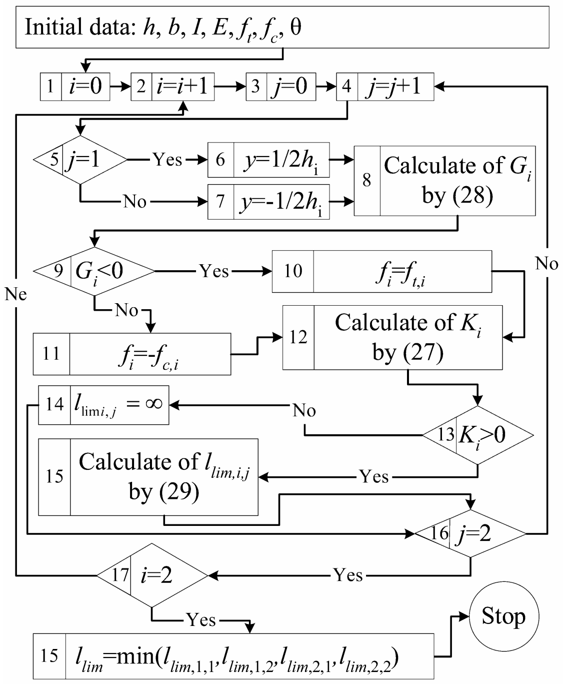

Fig. 3 Algorithm for the calculation of limiting beam length, which, if exceed, will cause one of layers to crack

If the beam does not bend or the bending strains are restrained, or bending stiffness of one of the layers is many times bigger than of the others, then the Eqs. (26) - (29) and the algorithm given in Fig. 3 can be used to calculate the distance between cracks. In this case the stress through the depth of each layer is even. Therefore the cracks through layer height and single layer sections can be considered as independent beams according to Eqs. (26) - (29). In general, according to Eqs. (26) - (29) and the algorithm in Fig. 3 only approximate calculation of the distance between cracks is possible. As maximum stresses are generated at the layers joint, a layer does not crack all the way through its height but cracks only partially. Single layer sections interact among each other; therefore the assumptions 1-5 are invalid.

As layers modulus of elasticity also has influence on its stress, for practical application it is helpful to have a simple expression according to which it would be possible to calculate the maximum elasticity modulus of the layers not causing cracks. However, it is impossible to express elasticity modulus of the layers, moments of inertia and cross–sectional areas from inequality (22) by explicit functions. Therefore, for the calculation of these values, the inequality (22) converted into equality may be used.

If a layer of a beam with the absolute stiffness joint does not crack, none of the layers will crack when the beam with a desired length and joint stiffness is provided. For the calculation of such layered beams we can apply the classical layered beam theory [4,5,7]. In this theory the accepted assumption is that the beam layers do not slip. For practical application simpler calculation methods based on the classical layered beam theory [3,7] are used.

Stresses calculated according to the build-up bars theory approach the stresses calculated without considering interlayer slip when the beam length or joint stiffness approach infinity i.e.

$$\lim_{\wp \to \infty} \sigma_i \to \sigma_{s,i},\ \wp \in \{l, \xi\} \tag{30}$$

where σ_i are stresses calculated according to Eq. (12), and $\sigma_{s,i}$ are stresses calculated considering that the layers do not slip. According to Eqs. (12) and (22) stresses in the middle of the beam

$$\sigma_i = \frac{\theta}{\Delta}\left(\frac{1}{cosh\left(1/2\,\lambda l\right)} - 1\right)\left(\frac{\vartheta_i}{A_i} - \frac{w y_i E_i}{D}\right) \tag{31}$$

when expression (30) is valid, then $\lambda \to \infty$ and

$$\lim_{\wp \to \infty}\left(1/cosh\left(1/2\,\lambda l\right)\right) \to 0,\ \wp \in \{\lambda, l\} \tag{32}$$

In this case the dependence of beam stresses without considering layers slip is the following

$$\sigma_{s,i} = -\frac{\theta}{\Delta}\left(\frac{\vartheta_i}{A_i} - \frac{w y_i E_i}{D}\right) \tag{33}$$

As it is well known the relative error is written as follows

$$\Delta e = \left(\sigma_i - \sigma_{s,i}\right)/\sigma_i \tag{34}$$

Because $\sigma_i < \sigma_{s,i}$ then $\Delta e < 0$. If Eqs. (33) and (12) put into Eq. (34) we get the final relative error expression

$$\Delta e = \frac{1}{1 - cosh\left(1/2\,\lambda l\right)} \tag{35}$$

As we can see from (35), the relative error of stresses depends only on mechanical properties of a beam, i.e. the beam length l, joint stiffness ξ, layers moment of inertia, compressive stiffness, and the distance between the layers centroid of section, but does not depend on the difference of the layers thermal and hygral strains Θ. Therefore in the Eq. (35), assuming the special stress error, according to the properties of the layered beams, we can determine the set of beams for designing of which necessarily take into account layer slip and the set of beams for calculation of which does not take into account the layer slip. According to the Eq. (35) it has been determined that the relative difference Δe increases when layers stiffness, i.e. the area of cross-section and the moment of inertia, increases and joint stiffness and the beam length decrease.

2.5. Calculation of deflections

Deflections of composite timer-concrete, concrete-steel, concrete-concrete beams depend not only on load but also on thermal and hygral strains. A method enabling to determine a bilayer beam deflections caused by thermal and hygral strains is presented hereunder.

As it is well known curvature is the second derivative of deflection. If a bilayer beam is only under thermal and hygral loads, then total bending moment is the following: $M(x)=M_1(x)+M_2(x)$, here $M_1(x)$ and $M_2(x)$ are give in Eq. (11). We obtain the curvature by dividing the total bending moment by total layers stiffness. Then we get the function of deflection by solving the following differential equation

$$\partial^2 v(x)/\partial x^2 = T(x) w/D \tag{36}$$

The function of deflection is obtained integrating by quadrature rule the differential Eq. (36)

$$v(x) = \frac{w}{D}\int\int T(x)\,dx + C_1 dx + C_2 \tag{37}$$

Inserting Eq. (9) into Eq. (37) and integrating the Eq. (37) we obtain the function of deflection

$$v(x) = \frac{w}{D}\left(\frac{\theta}{\Delta}\left(\frac{cosh(\lambda x)}{cosh(1/2\lambda l)\lambda^2} - \frac{x^2}{2}\right) + C_1 x\right) + C_2 \tag{38}$$

Integration constants are obtained from the boundary conditions. The first boundary condition is that the deflection at the beam end is equal to 0, i.e.

$$v(x)\big|_{x=-1/2l} = 0 \tag{39}$$

if insert Eq. (39) into Eq. (38) we get

$$C_2 = -\frac{w}{8D}\left(\frac{\theta(8 - l^2\lambda^2)}{\Delta\lambda^2} + C_1 l^2\right) \tag{40}$$

The second boundary condition states that layers rotation in the middle of a beam is equal to 0, i.e.

$$\phi(x) = \partial v(x)/\partial x\big|_{x=0} = 0 \tag{41}$$

Differentiating Eq. (38) in respect to x and inserting the obtained expression into Eq. (41) we get

$$C_1 = 0 \tag{42}$$

Also substituting Eqs. (40) and (42) into Eq. (38) we get the final expression of the deflection function

$$v(x) = \frac{w\theta}{D\Delta}\left(\frac{cosh(\lambda x)}{cosh(1/2\lambda l)\lambda^2} - \frac{x^2}{2} - \frac{1}{\lambda^2} + \frac{l^2}{8}\right) \tag{43}$$

The maximum deflection in the middle of a beam is the following

$$v(0) = \frac{w\theta}{D\Delta}\left(\frac{1}{cosh(1/2\lambda l)\lambda^2} - \frac{1}{\lambda^2} + \frac{l^2}{8}\right) \tag{44}$$

As in the case of stress calculation, while the beam length and joint stiffness increase, the deflections calculated according to Eq. (43) or Eq. (44) formulas approach to the deflections calculated as a solid beam without taking into account layer slip.

3. Analysis of results

To represent the obtained relationship, a real bilayer concrete-timber beam is taken. Distributions of layer stresses and deflections throughout the beam length are investigated according to the expressions obtained in this paper. Also the influence of joint stiffness, geometrical dimensions of the beam and the difference of thermal and hygral strain on the layers' stress is also investigated. The cross-section of the beam under investigation is show in Fig. 1. The first layer is made of lightweight concrete; the second layer is made of timber. The layers joint is made without an interlayer. It is made from two wood screws 156x5.9 mm in size. They are screwed into the timber 100 mm. Stiffness of one joint ξ_1, geometrical dimensions of the beam cross-section, concrete compressive and tensile strength f_c and f_t and modulus of elasticity E are taken from [8,9]. In the analysis performed, stiffness of the joint is taken to be two, three and ten connections in a metre, then joint stiffness equals ξ=54; 81 and 27 MN/m respectively. Compressive strength f_c and tensile strength f_t and elasticity modulus of timber E are taken from [10]. The values of the difference of layers thermal and hygral strains is taken approximate to the shrinkage strains of C20/25 class heavy concrete when relative air humidity is 60%, i.e. $\theta \approx -60\cdot 10^{-5}$. The length of the beam is taken to be equal to 3, 6, and 9 meters respectively. Geometrical characteristics of layers, material properties and other values are given in Table.

The bilayer beam stresses (calculated according to Eq. (12)) versus length and joint stiffness are shown in Fig. 4. As it may be seen from Fig. 4, the maximum stresses generate in the middle of the beam. Increasing the beam length or joint stiffness the stresses of layers approach the stresses of the beam with absolutely rigidly connected layers. As 2 and 5 assumptions are valid, stresses change linearly through each layer depth. It is clearly seen in Fig. 5.

As we can see from Fig. 5, the maximum stresses are generated in the middle of the beam. Analysis shows that when $\theta<0$ the first layer is always under tension at the joint and the second layer is always under compression at the joint, and this effect is independent from the layers parameters.

When $\theta>0$, then vice versa, the first layer is always under compression at the joint, and the second layer is always under tension at the joint. The outside layers may be under compression or under tension depending on parameters of the layers. Distribution of concrete–timber, concrete–steel and concrete–concrete bilayer beam stresses through the layer height under shear force load is alike as in Fig. 5 [8,11,12,13]. The sum of overall stresses of the layers at $\pm 1/2h_i$ point due to shear forces and thermal and hygral strain action increase. Therefore it is imperative that designing bilayer structures thermal and hygral strain for the layers' stress must be taken into account.

As it is shown in Fig. 6, if joint stiffness ξ and beam length l are increasing, θ_{max} and θ_{min} asymptotically tend to particular values at which the beam layers with absolutely rigid joints would crack. Therefore, for longer and more rigid joint beams, the absolute value of θ_{max} and

Table

Beam parameters [8,10,11]

Layer's substitute	b, m	h, m	E, GPa	ξ_1, MN/m	f_c, MPa	f_t, MPa	Θ
1 concrete layer	0.50	0.10	12.7	27	29	2.8	$-60\cdot10^{-5}$
2 wood layer	0.12	0.15	10		15	7	

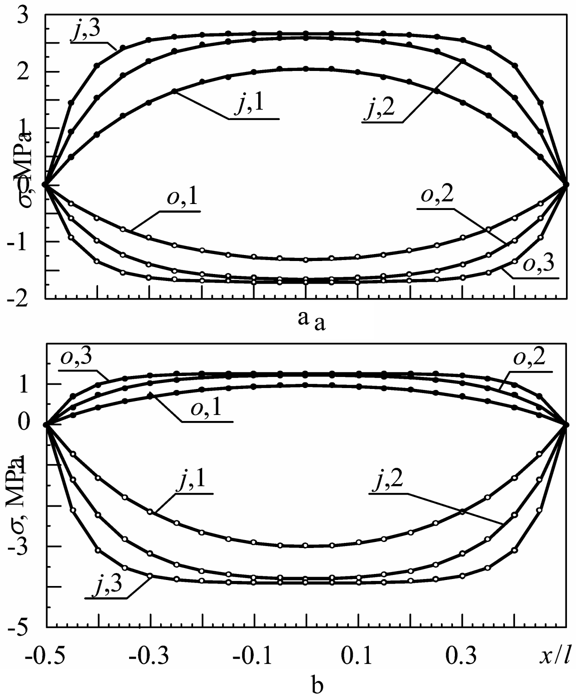

Fig. 4 Change of the layers' normal stresses through a beam length: a – in the first layer, b – in the second layer, j – at the joint of layers, o – on the outside of the layers, *1* - l=3 m, ξ=81N/m; *2* - l=6 m, ξ=81 MN/m; *3* - l=6 m, ξ=270 MN/m, the parameters of the beam are given in Table

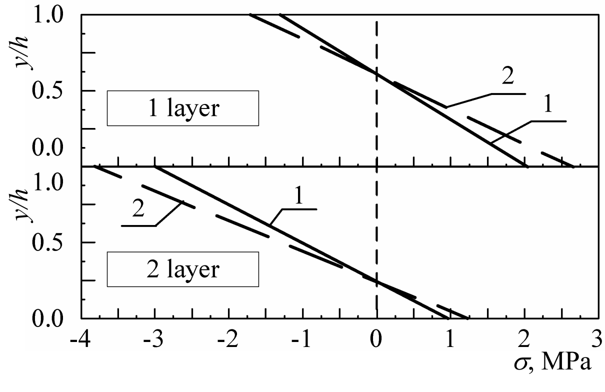

Fig. 5 Variation of stresses through each layer height when x=0. The parameters of the layers are the same as in Fig. 4

θ_{min} is much less than for short beam. The layer slip and the shorter beam length may reduce the risk of cracking.

As illustrated in Fig. 7 when E_1 increases, absolute values of θ_{max} and θ_{min} decrease to a certain limit. Therefore in some cases the increase of the elasticity modulus raises the risk of cracking. It must be emphasized that in most cases for cement materials, the elasticity modulus and the strength are closely related. Usually the increase of the elasticity modulus of cement based materials increases the material strength. In some cases the in

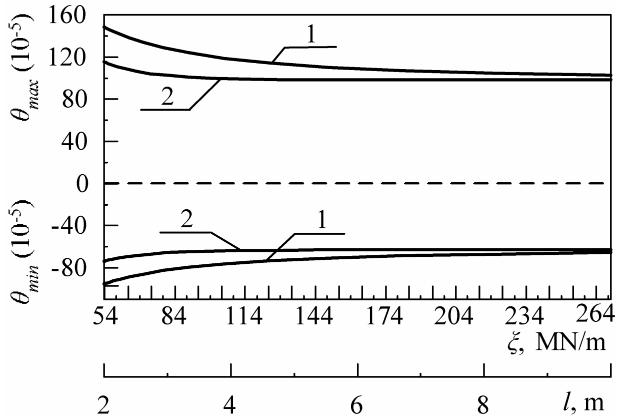

Fig. 6 Dependence of minimum θ_{min} and maximum θ_{max} thermal and hygral strain which does not induce the crack in the layers, on joint stiffness ξ when l=3 m – *1* and the length of beam l – *2*, when ξ=54 MN/m, the parameters of the beam are given in the Table

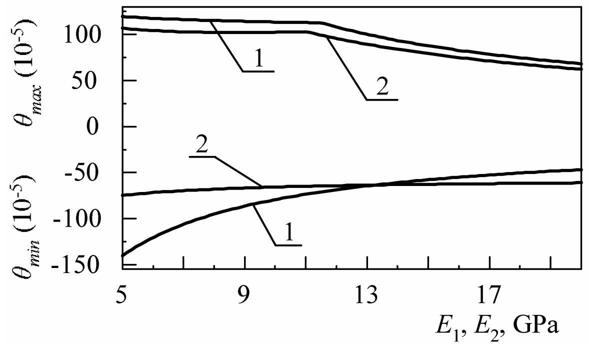

Fig. 7 Dependence of the minimum θ_{min} and the maximum θ_{max} thermal and hygral strain which do not cause the layers to crack on elasticity modulus of layers: *1* - when the elasticity modulus of the first layer varies, *2* - when elasticity modulus of the second layer varies, the parameters of the beam are given in the Table l=3 m ξ=270 MN/m

crease in strength may compensate the increasing stresses-caused by the raised elasticity modulus. For example, according to [14] if the elasticity modulus of 28-day-old heavy concrete increases from 27 GPa (C12/15) to 30 GPa (C20/25) or relatively 11.11 %, the tensile strength of 0.05 quantile increases from 1.1 MPa to 1.5 MPa or relatively 36.4 %. Meanwhile the stresses of the first layer (the beam geometrical dimensions are given in the table ξ=270 MN/m, l=3 m, θ=60·10^{-5}) increase accordingly from $\sigma_1(-1/2h_1)$=4.4 MPa to $\sigma_1(-1/2h_1)$=4.7 MPa, or relatively 6.82%. Therefore the increase of the elasticity modulus does not necessarily cause a large risk for the layers to crack. However if with the increase of the elasticity modulus the strength does not increase, then the increase of the elasticity modulus reduces the limiting value of thermal and hygral strain difference (Fig. 7).

It must be emphasized that if flexural stiffness of the second layer is very large comparing to the first layer or the beam cannot bend (for example it is restrained by flexural strains) with the increase of E_2 or A_2 stresses of the first layer always increase at any point. Therefore in general, the influence of the elasticity modulus on θ_{max} and θ_{min} cannot be uniquely determined.

The dependence of limiting beam length on the strength of the first layer, the difference of thermal and hygral strain θ and joint stiffness are shown in Fig. 8.

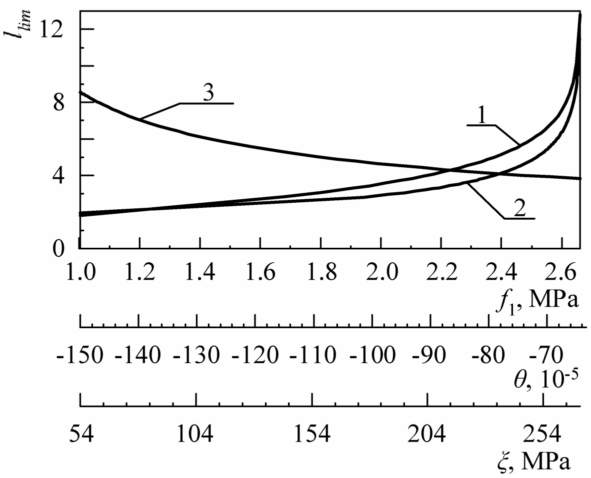

Fig. 8 Dependence of the limiting beam length on the strength of the first layer f_1 – *1*, ξ=54 MN/m, the difference of thermal and hygral strain θ – *2*, ξ=54 N/m and the stiffness of the joint ξ – *3*, θ=-64·10^{-5}, the parameters of the beam are given in Table

As it can be seen, with the increase of the layers strength and the decrease of the difference of the absolute values of thermal and hygral strains, the length of the limiting beams asymptotically tends to a specific limit.

The beam deflections are show in Fig. 9. As we can see from the Fig. 9, if joint stiffness increases, the deflection also increases. The graph also shows that due to shrinkage strains, the real beam deflections may achieve half of the limiting deflection v_{lim}≈6/200=3 cm. Therefore, owing to thermal and hygral strains, the deflections of the bilayered beams under shear load may be considerably increased.

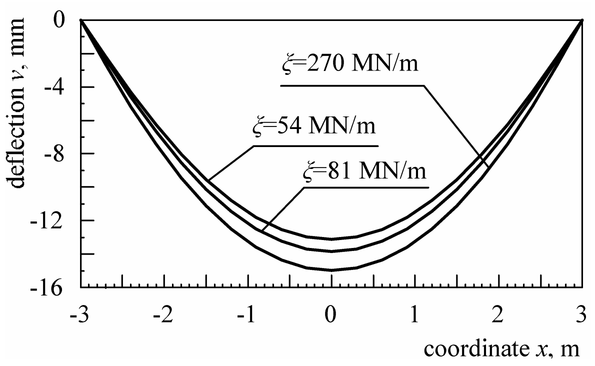

Fig. 9 Dependence of deflections of the bilayer beam on joint stiffness and the difference of the layers' thermal and hygral strain when l=6 m, other beam parameters are given in the Table

For the calculation of such structure deflections it is necessary to consider thermal and hygral loads as well.

4. Conclusions

The conclusions listed hereunder are written for the concrete-timber, concrete-steel and concrete-concrete bilayered beams under thermal and hygral deformation action.

A layer slip decreases its stresses and beam deflections. Therefore for more precise estimates of stresses and strains state, in majority of cases it is necessary to take into account the layer slip.

Designing beams it is necessary to consider the influence of thermal and hygral strain on the stresses and deflections of the layers. Due to these strains, stresses of the layers may exceed the strength limit. Deflection of the beam under shear force always increases and may be larger than the allowable deflection set by structural requirements.

With the increase of joint stiffness and beam length the hygrothermal stresses asymptotically approach to the stresses of the beam with absolutely rigid joint.

If flexural strains are not restrained and hygrothermal strains are uniformly distributed through the height of each layer, the maximum stresses occur at the joint of layers in respect to the beam height. The minimum stresses occur at the outside of the layers. The maximum stresses generate in the middle of the beam in respect to the beam length and minimum stresses generate at the end of the beam.

The length of the beam and the stiffness of the joint, owing to which the cracks occur, are principally dependent on the difference of layer thermal and hygral strain and the material strength of layers.

The relative error of the stresses calculated considering layer slip and the stresses determined not taking into account layer slip does not depend on the difference of thermal and hygral strain.

References

1. **Ржаницын А.Р.** Составные стержни и пластинки. -Москва: Стройиздат, 1986.-316с.
2. **Болотин В.В., Новичков Ю.Н.** Механика многослойных конструкций.-Москва: Машиностроение, 1980.-375с.
3. **Bareišis, J.** Static and Cyclic Strength of Structural Composites.-Summary of habilitation thesis.-Kaunas: Technologija, 1995.-36p.
4. **Vasiliev, V.V., Morozov, E.V.** Mechanins and Analysis of Composite Materials.-Amsterdam: Elsevier, 2001.-412p.
5. **Berthelot, J.M.** Composite Materials: Mechanical Behaviour and Structural Analysis.-New York: Springer-Verlag, 1999.-645p.
6. Eurocode 5 (EC5). Design of timber structures – Part 1.1: General rules and rules for buildings. European prestandard ENV 1995-1-1. European committee for standardization, Brussels, Belgium, 1995.-121p.
7. **Zabulionis, D., Samofalov, M.** Analysis of mechanical state of laminated beams subject to hygrothermal load. -Proc. of the 10th Int. Conf. Mechanika-2004.-Kaunas: Technologija, 2004, p.138-144.
8. **Steinberg, E., Selle, R., Faust, T.** Connectors for timber-lightweight concrete composite structures.-J. of

Structural Engineering, 2003, v.129, No11, p.1538-1545.

9. **Gurkšnys, K., Kvedaras, A., Valiūnas, B.** Results of the test of timber-concrete composite joisted floor sub-components inter-connections.-Proc. of the 8th Int. Conf. "Modern building materials, structures and techniques" May 19-21, 2004, Vilnius, [6].
10. **Ozelton, E.C., Baird, J.A.** Timber Design Manual.-Oxford: Blackwel Science, 2002.-542p.
11. **Faust, T., Selle, R.** Properties of composite joints of timber-lwac composite structures.-Lacer, 1999, No4, p.231-240.
12. **Rug W., Liner K.** Holz-Beton-Verbundbauweise in der Praxis. Beton und Stahlbetonbau, 2004, No7, p.578-586.
13. **Stojić, D., Cvetković, R.** Analysis of a composite timber-concrete structures according to the limit states design and innovative methods in coupling of a timber and concrete. Facta Universitatis Series: Architecture and Civil Engineering, 2001, v.2, No3, p.169-184.
14. Eurocode 2 (EC2). Design of concrete structures – Part 1: General rules and rules for buildings. European prestandard ENV 1992-1-1. European committee for standardization, Brussels, Belgium, dec. 1991.-253p.

D. Zabulionis

TEMPERATŪRINIŲ IR DRĖGMINIŲ DEFORMACIJŲ SĄLYGOJAMO DVISLUOKSNĖS SIJOS ĮTEMPIŲ IR DEFORMACIJŲ BŪVIO ANALIZĖ ĮVERTINANT SLUOKSNIŲ PRASLYDIMĄ

R e z i u m ė

Darbe pateikta dvisluoksnės sijos temperatūrinių ir drėgminių deformacijų sąlygojamo įtempių ir deformacijų būvio analizė įvertinant sluoksnių praslydimą. Sudarytos priklausomybės remiasi sudėtinių strypų teorija. Pateiktos ribinių temperatūrinių ir drėgminių deformacijų bei kritinio sijos ilgio, kurį viršijus vienas iš sluoksnių supleišėja, apskaičiavimo metodikos. Pagal gautas priklausomybes atlikta realios dvisluoksnės sijos įtempių, deformacijų ir poslinkių analizė. Nustatyta, kad temperatūrinės ir drėgminės deformacijos gali sukelti didelius dvisluoksnių sijų įtempius, deformacijas ir įlinkius.

D. Zabulionis

STRESS AND STRAIN ANALYSIS OF A BILAYER COMPOSITE BEAM WITH INTERLAYER SLIP UNDER HYGROTHERMAL LOADS

S u m m a r y

The work includes stress and strain analysis of a bilayer composite beam with interlayer slip under hygrothermal loads. An algorithm allowing calculation of limiting thermal and hygral strains and an algorithm for calculation of limiting beam length (if exceeded, it will cause one of the layers cracking) are presented in the article. According to the proposed relationships the analysis of a real bilayer beam stresses, strains and deflections has been performed.

Д. Забулёнис

РАСЧЕТ НАПРЯЖЕНИЙ И ДЕФОРМАЦИЙ ДВУХСЛОЙНОЙ БАЛКИ ОТ ДЕЙСТВИЯ ТЕМПЕРАТУРНЫХ И ВЛАЖНОСТНЫХ ДЕФОРМАЦИЙ С УЧЕТОМ ПОДАТЛИВОСТИ СОЕДИНЕНИЯ

Р е з ю м е

В статье предложена методика расчета напряжений и деформаций двухслойной балки от действия температурных и влажностных деформаций с учетом податливости соединения. Предложенные зависимости получены на основе теорий составных стержней. Представлены методики, позволяющие рассчитать предельные температурные и влажностные деформации в слоях, а также предельную длину, при которых в слоях появляются трещины. На основе полученных зависимостей сделан анализ напряженно-деформированного состояния двухслойной балки.

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