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Numerical analysis of temperature distribution of cold cylindrical metal subjected to machining

S.A. Oke, O.I. Oyedokun, A.O. Bamigbaiye
University of Lagos, Akoka-Lagos, Nigeria, E-mail: sa_oke@yahoo.com

1. Introduction

Turning operation is a machining process, which involves removing metal in the form of chips, consequently, resulting into reduction in the diameter of the metal. Turning operation requires close dimensional accuracy. It is usually performed on various power-driven machines. These machines operate on either reciprocating or rotatory-type principle: either the tool or the work piece reciprocates or rotates. Turning operation generates a lot of heat on the metal being cut and on the cutting tool because of friction and motion between the cutting tool and the work piece. This invariably makes the chips very hot (since temperature is the average measure of heat energy) - [1, 2].

During this operation, energy is expended in two forms. The first is useful work done in turning down the metal. The second is heat energy generated which makes the work piece hot. This modelling concentrates mainly on the second form of energy – this energy is conducted in the material. The metal body does not just have a uniform temperature all over its surface and its internal parts but heat is conducted [3]. Heat conduction involves increasing the velocity of vibration of the metal molecules about their mean positions [4, 5]. The rate at which each particle vibrates depends on the amount of heat received. The metal will have a uniform temperature when all the particles of the metal at a particular time have the same vibration velocity (same agitation energy [6]). Turning operation is one of the processes that can be used to produce parts of accurate dimensions and smooth surface. Some of the types of turning operation include taper turning and straight turning.

The paper is organized into five parts. The introduction describes the motivation for the study. It also presents the problem definition, the research objective, and the expected contribution of the paper. Part two presents the investigation methodology used. It involves the development of a procedure that could be replicated in similar situations. In part three, a case study is presented in order to increase our understanding and verify the whole model. Hypothetical data is used to illustrate the model performance from an engineering perspective. Part four presents the discussion of results. In part five, conclusion to the study is made.

2. Methodology

The modelling is based on some assumptions, which are: (i) at turning operation there is no wobbling (ii) the cylindrical shaft is uniform in shape (iii) the shaft is heat conductor (iv) heat emission in the environment by the shaft is negligible. Mathematical principle and theory used for this modelling is complex analysis -applied potential theory [6]. Let us consider a cylindrical shaft which is a fair heat conductor, such that the rate at which the turning operation of the whole shaft length is faster than the rate at which heat is distributed. From the assumed conditions, it implies that there is no heat conduction centroidally. Let the initial radius of the cylindrical shaft be R while its length be l. Let R_1 be the radius of the boundary surface of the cylindrical shaft at anytime when it is turned. Let the vector angular velocity of the rotating cylindrical shaft be $\overrightarrow{\dot{\theta}}$ [7]. Linear velocity of the shaft is linear velocity of any point on the boundary surface at $\left|\vec{r}\right| = \left|\vec{R}_1\right|$, since it is pure rotation (Fig. 1).

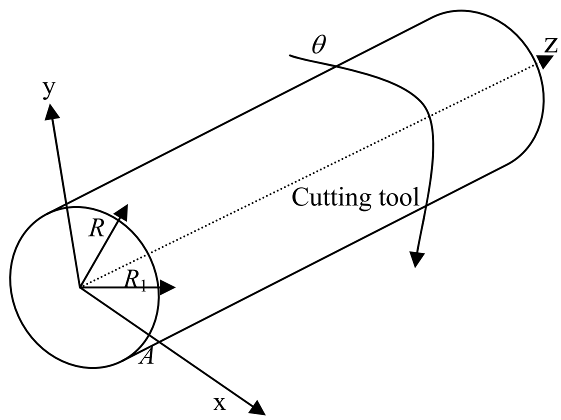

Fig. 1 Cylindrical shaft undergoing turning operation

Hence

$$\vec{V} = \vec{V}_o + \overrightarrow{\dot{\theta}} \wedge \overrightarrow{r_{OA}} = 0 + \dot{\theta}\left(-\hat{K}\right) \wedge R_1\hat{i} = \dot{\theta}R_1\hat{j} \quad (1)$$

For a reversed rotation, $\vec{V} = -\dot{\theta}R_1\hat{j}$. Let K represent thermal conductivity of the shaft, with $\vec{V} = -K\nabla T$. Here, $T(x, y, z, t)$ is temperature, t is time. The amount of heat generated on the boundary surface as a result of turning can be expressed as $\iint_s \vec{v} \cdot \vec{n}\, dA$, where $\vec{v} \cdot \vec{n}\, dA$ is normal component of $\vec{V}$, dA is an elemental area of the material. Therefore, let J be a region on the shaft. Then

$$\iint_s \vec{v} \cdot \vec{n}\, dA = -K\iiint_J \nabla^2 T\left(dxdydz\right) \quad (2)$$

The total amount of heat in J is

$$H = \iiint_J \sigma T dm \quad (3)$$

where σ is specific heat of the shaft; m is mass. From physics $m = f(v, \rho)$ where v is volume, ρ is density, but $m = v\rho$. The total differential of the function gives

$$dm = \rho dv + v d\rho \quad (4)$$

since density of the shaft is constant, the mass of a small element considered will be

$$dm = \rho dv \quad (5)$$

Noting that $dv = dxdydz$ and $dm = \rho dxdydz$. By subtracting the value of dm in (3), we have

$$H = \iiint_J \sigma T dxdydz \quad (6)$$

hence, time rate of decrease of H is

$$\frac{-\partial}{\partial t}(H) = \frac{-\partial}{\partial t} \iiint_J \sigma \rho T dxdydz \quad (7)$$

Comparing Eqs. (7) and (2) gives

$$- \iiint_J \sigma \rho \frac{\partial T}{\partial t} dxdydz = -K \iiint_J \nabla^2 T dxdydz \quad (8)$$

This implies that

$$\rho \sigma \frac{\partial T}{\partial t} = K \nabla^2 T \text{, and } \frac{\partial T}{\partial t} = \frac{K}{\rho \sigma} \nabla^2 T \quad (9)$$

Note: the direction of rotation of the shaft does not affect temperature distribution when we consider a steady state heat conduction process. Considering steady state heat conduction

$$\frac{\partial T}{\partial t} = \frac{K}{\rho \sigma} \nabla^2 T = 0 \quad (10)$$

From Eq. (10), $\frac{K}{\rho \sigma} \neq 0$, which implies that

$$\nabla^2 T = \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = 0 \quad (11)$$

For a three dimensional Laplace equation,. Now, let $T(x, y, z)$ = constant (Isomtherms). Similarly, Laplace equation in cylindrical coordinates

$$\nabla^2 T = \frac{\partial^2 T}{\partial \gamma^2} + \frac{1}{\gamma} \frac{\partial T}{\partial \gamma} + \frac{1}{\gamma^2} \frac{\partial^2 T}{\partial \theta^2} + \frac{\partial^2 T}{\partial z^2} = 0$$

Let $T(\gamma, \theta, z)$ the cylindrical coordinates be a point function. But $z = \frac{\beta \theta}{2\pi}$, where β is the pitch for temperature distribution. Therefore

$$d\theta = \frac{2\pi dz}{\beta} \quad (12)$$

but

$$\frac{\partial \theta}{\partial T} = \frac{1}{\partial T / \partial \theta} \text{ (point function)} \quad (13)$$

From Eq. (12)

$$\frac{d\theta}{dT} = \frac{2\pi}{\beta} \frac{dz}{dT} \text{ or } \frac{\partial \theta}{\partial T} = \frac{2\pi}{\beta} \frac{\partial z}{\partial T} \text{ and } \frac{\partial T}{\partial \theta} = \frac{\beta}{2\pi} \frac{\partial T}{\partial z} \quad (14)$$

By putting the value of $\frac{\partial T}{\partial \theta}$ into the expression containing it for the Laplace equation in cylindrical coordinates, we have

$$\nabla^2 T = \frac{\partial^2 T}{\partial \gamma^2} + \frac{1}{\gamma} \frac{\partial T}{\partial \gamma} + \frac{1}{\gamma^2} \left(\frac{\partial^2 T}{\partial z^2} \right) \frac{\beta}{2\pi} + \frac{\partial^2 T}{\partial z^2} = 0 \quad (15)$$

From the assumption made, heat conduction along z-axis is negligible, hence

$$\frac{\partial T}{\partial z} = 0; \frac{\partial^2 T}{\partial z^2} = 0 \text{, and } \frac{\partial^2 T}{\partial \gamma^2} + \frac{1}{\gamma} \frac{\partial T}{\partial \gamma} = 0 \quad (16)$$

Using variable separable of ordinary differential equation, and integrating both sides

$$\frac{\partial^2 T}{\partial \gamma^2} = -\frac{1}{\gamma} \frac{\partial T}{\partial \gamma} \text{ and } ln\left(\frac{\partial T}{\partial \gamma} \right) = -ln \gamma + \widetilde{a} \quad (17)$$

But $\widetilde{a} = e^{-\omega} \, ln \frac{a\gamma}{\omega}$, where $\frac{'a\gamma'}{\omega}$ is a constant; ω is a constant that is very small, therefore $e^{-\omega} \to 1$. Therefore $ln\left(\frac{\partial T}{\partial \gamma} \right) = ln\left(\frac{a\gamma}{\omega \gamma} \right)$, which implies that

$$\frac{\partial T}{\partial \gamma} = \frac{a\gamma}{\omega \gamma} \quad (18)$$

Integrating both sides give

$$T = g \, ln\gamma + b \quad (19)$$

where b is also a constant. Similarly, $g = \frac{a}{\omega}$ is a constant.

Let T_0 represent the room temperature, which is also initial temperature of the shaft. Let us consider a boundary condition; when $0 < r < 1$. $T = T_0$ at a time t and when $r = R_1$, $T = T_1$ and $T_0 = 0 + b$, $b = T_0$. Therefore $T = = a\gamma + T_0$. Also, when $\gamma = R_1$, $T = T_1$, $T_1 = aR_1 + T_0$, and $a = \frac{T_1 - T_0}{R_1}$. But T_1 is temperature at the boundary surface.

Thus

$$T = \left(\frac{T_1 - T_0}{R_1}\right)_\gamma + T_0 \quad (20)$$

Eq. (20) gives the mathematical model for temperature distribution in a cylindrical shaft that is fairly conductive to heat. From Eq. (20), it can be seen that temperature of the cylindrical shaft decreases from the boundary surface with the radius and time. In order to have a diagrammatic representation of this temperature distribution, let us assume that kinetic energy lost due to the loss in mass of turned shaft is equal to the heat generated, i.e. $\frac{1}{2}mv^2 = m\sigma(T_1 - T_0)$. But $v = \dot{\theta}R_v$, therefore

$$\frac{(\dot{\theta}R_1)^2}{2\sigma} = T_1 - T_0 \quad (21)$$

From Eq. (21) it can be seen that for greater heat to be generated and for a higher temperature gradient angular speed of the shaft must be increased while material must have low specific heat capacity. This has practical application. It will be wise and needful to lower the angular speed when turning a material microstructure of which can change within low temperature range. Eq. (21) is a useful formula for all manufacturers and metallurgists who should note so that the maximum speed that can be allowed to turn a cylindrical shaft should be such that the heat generated will not affect the microstructure of the material and consequently damage the shaft. Hence, putting Eq. (21) into (20) gives

$$T = \frac{(\dot{\theta}R_1)^2}{2\sigma(R_1)}(r) + T_0 \quad (22)$$

Eq. (22) is important for temperature distribution in a cylindrical shaft which is fairly conductive. From the above equation, it can be shown that temperature distribution in a fairly conductive (to heat) cylindrical shaft is radial, if angular speed is constant or not. But R_1 is not a constant, and Eq. (22) can be simplified further by considering $R_1 = R - n\alpha$. Also, α is the depth cut, n is the number of turnings performed. It is assumed that the depth cut is constant throughout the turning operations. Hence, Eq. (22) becomes

$$T = \frac{(\dot{\theta}(R - \alpha))^2}{2\sigma(a(R - \alpha))}(r) + T_0 \quad (23)$$

since n=1 (i.e. the turning is done once). Therefore, for a turning operation that is done on the shaft n times and when the heat added by each succeeding turning generates the same temperature, Eq. (23) becomes

$$T_1 = \frac{(\dot{\theta}(R - n\alpha))^2}{2\sigma(n(R - n\alpha))}(nr) + T_0 \quad (24)$$

Eq. (24) is general equation for temperature distribution on a cylindrical shaft that is under turning operation, when the shaft is fairly heat conductive. We now want to proceed further to determine the equilibrium temperature of the shaft when thermal equilibrium occurs. Thermal equilibrium occurs in the shaft when temperature at the boundary surface is equal to the temperature at centroidal axis, i.e. particles of the shaft at the boundary surface (outer surface) and those at the centroidal axis have the same temperature. Now, let us assume that the molecules of the shaft behave as sea of gases, except that they are vibrating about their mean positions. Suppose velocity of the molecules, then

$$v = \alpha\sqrt{T} \quad (25)$$

Note that $v = \varphi\frac{\sqrt{T}}{\rho}$ where φ is a constant of proportionality. Now, let the velocity of the particles (molecules) at the boundary surface be v_1, while the velocity of the particles at the centroidal axis be v_0 and the velocity at equilibrium be v_2. From Eq. (25), we have $\frac{v_1}{v_2} = \frac{\sqrt{T_1}\rho_2}{\rho_1\sqrt{T_2}}$ and $\frac{v_2}{v_0} = \frac{\rho_0\sqrt{T_2}}{\rho_2\sqrt{T_0}}$. But $\frac{1}{2}m\left(v_1^2 - v_2^2\right) = m\,\sigma(T_1 - T_2)$, i.e. $v_1^2 - v_2^2 = 2\sigma(T_1 - T_2)$. By obtaining v_2 from the v_1/v_2 relationship above, and substituting it in the expression for ($v_1^2 - v_2^2$), we have

$$T_2 = \frac{\left(V_1^2T_1 - 2\sigma T_2^2\right)\rho_2^2}{\rho_1^2V_1^2 - 2\sigma T1\rho_2^2} \quad (26)$$

but from the expression $\vec{V} = -K\nabla T$, and $R_1 = R - n\alpha$,

$$T_2 = \frac{\left((\dot{\theta}(R - n\alpha))^2T_1 - 2\sigma T_1^2\right)\rho_2^2}{\rho_1^2\dot{\theta}^2(R - n\alpha)^2 - 2\sigma T_1\rho_2^2} \quad (27)$$

where, $T_1 = \frac{n\dot{\theta}(R - n\alpha)^2}{2\sigma} + T_0$.

Thus, the equilibrium temperature depends on angular speed, the number of turnings, room temperature and the shaft radius. Let us consider temperature distribution on a highly heat conductive cylindrical shaft. From the equation (15)

$$\nabla^2T = \frac{\partial^2T}{\partial\gamma^2} + \frac{1}{\gamma}\frac{\partial T}{\partial\gamma} + \left(\frac{\beta}{2\pi\gamma^2} + 1\right)\frac{\partial^2T}{\partial z^2} = 0 \quad (28)$$

but $T = f(\gamma, z)$, Therefore

$$dT = \frac{\partial T}{\partial\gamma}d\gamma + \frac{\partial T}{\partial z}dz \quad (29)$$

Integrating along the radial plane at a time, *t,* from Eq. (28)

$$\frac{\partial^2 T}{\partial \gamma^2} + \frac{1}{\gamma}\frac{\partial T}{\partial \gamma} = -\left(\frac{\beta}{2\pi\gamma^2} + 1\right)\frac{\partial^2 T}{\partial z^2} \quad (30)$$

Suppose $\frac{\partial^2 T}{\partial z^2} = A$ at a time t. Eq. (30) becomes

$$\frac{\partial^2 T}{\partial \gamma^2} + \frac{1}{\gamma}\frac{\partial T}{\partial \gamma} = -A\left(\frac{\beta}{2\pi\gamma^2} + 1\right) \quad (31)$$

Eq. (31) is solved using ordinary method of solution differential equation of the second order. Solution the left hand side of Eq. (31) gives similar result as of Eq. (16) i.e.

$$T_L = B\ln r + E \quad (32)$$

but

$$\beta = \frac{\Phi}{\gamma} \quad (33)$$

where Φ represents the smallest coefficient of temperature distribution per m. Putting Eq. (33) into (32) gives

$$\frac{\partial^2 T}{\partial \gamma^2} + \frac{1}{\gamma}\frac{\partial T}{\partial \gamma} = -A\left(\frac{\Phi}{2\pi\gamma^3} + 1\right) \quad (34)$$

Suppose

$$T = -\left(\frac{D}{\gamma} - G\gamma^2\right) \quad (35)$$

Therefore

$$\frac{dT}{d\gamma} = -\left(\frac{-D}{\gamma^2} - 2G\gamma\right) \quad (36)$$

$$\frac{d^2T}{d\gamma^2} = -\left(\frac{2D}{\gamma^3} - 2G\right) \quad (37)$$

But Eq. (34) can be written as

$$\frac{d^2T}{d\gamma^2} + \frac{1}{\gamma}\frac{dT}{d\gamma} = -A\left(\frac{\Phi}{2\pi\gamma^3} + 1\right) \quad (38)$$

Putting Eqs. (36) and (37) into (38) gives

$$-\frac{D}{\gamma^3} + 4G = -\frac{A\Phi}{2\pi\gamma^3} - A \quad (39)$$

Comparing coefficients γ^{-3}: $-D = -\frac{A\Phi}{2\pi}$ i.e. $D = -\frac{A\Phi}{2\pi}$; γ^0: $+ 4G = -A$ i.e. $D = \frac{-A}{4}$.

Hence, solution of Eq. (34) becomes

$$T^r = B\ln r + E - \frac{A\Phi}{2\pi\gamma} + \frac{A\gamma^2}{4} \quad (40)$$

Integrating along the centroidal axis, from Eq. (30)

$$\frac{\partial^2 T}{\partial z^2}\left(\frac{\beta}{2\pi\gamma^2} + 1\right) = -\frac{\partial^2 T}{\partial \gamma^2} - \frac{1}{\gamma}\frac{\partial T}{\partial \gamma}$$

and

$$\frac{\partial^2 T}{\partial z^2}\left(\frac{\Phi}{2\pi\gamma^3} + 1\right) = -\frac{\partial^2 T}{\partial \gamma^2} - \frac{1}{\gamma}\frac{\partial T}{\partial \gamma} \quad (41)$$

For a constant value of

$$\gamma,\ \frac{\partial^2 T}{\partial \gamma^2} = +\mu;\ \frac{\partial T}{\partial \gamma} = +\psi\gamma$$

Therefore

$$\frac{\partial^2 T}{\partial z^2}\left(\frac{\Phi}{2\pi\gamma^3} + 1\right) = -(\mu + \psi) \quad (42)$$

$$\left(\frac{\Phi}{2\pi\gamma^3} + 1\right)\int \frac{\partial^2 T}{\partial z^2}dz = -(\mu z - \psi z) - c_1 \quad (43)$$

$$\left(\frac{\Phi}{2\pi\gamma^3} + 1\right)\int \frac{\partial T}{\partial z} = -(\mu z - \psi z) - c_1$$

$$\left(\frac{\Phi}{2\pi\gamma^3} + 1\right)\int \frac{\partial T}{\partial z}dz = -\frac{(\psi + \mu)}{2}z^2 - c_1 z + c_2 \quad (44)$$

$$\left(\frac{\Phi}{2\pi\gamma^3} + 1\right)T^z = -\left(\frac{\psi + \mu}{2}\right)z^2 - c_1 z + c_2 \quad (45)$$

Thus, according to Eq. (29)

$$T = \int \frac{\partial T}{\partial \gamma}d\gamma\gamma + \int \frac{\partial T}{\partial z}dz \quad (46)$$

$$T = B\ln r + E - \frac{A\Phi}{2\pi\gamma} + \frac{A\gamma^2}{4} + \frac{\left[(\psi - \mu)z^2 - c_1 z + c_2\right]2\pi\gamma^3}{(\Phi + 2\pi\gamma^3)} \quad (47)$$

where B, E, A, ψ, μ, c_1 and c_2 are constants. Furthermore, from equations $R_1 = R - n\alpha$ and assuming that $B\ln r + E \equiv$ $\equiv a\ln r + b$. Hence

$$T_z = \frac{(\dot{\theta}R_1)^2}{2\sigma R_1}\gamma + T_0 \quad (48)$$

$$T = \frac{(\dot{\theta}R_1)^2}{2\sigma R_1}(\gamma) - \frac{A\Phi}{2\pi\gamma} + \frac{A\gamma^2}{4} + \\ + \frac{\left[(\psi - \mu)z^2 - c_1 z + c_2\right]2\pi\gamma^3}{(\Phi + 2\pi\gamma^3)} + T_0 \quad (49)$$

where $R_1 = R - \alpha$ (for a turning). Hence, for n-turnings $R_1 = R - n\alpha$

$$T_1 = \frac{n\dot{\theta}^2 (R - n\alpha)^2}{2\sigma(R - n\alpha)}(\gamma) + \frac{An}{2}\left(\frac{-\Phi}{\pi\gamma} + \frac{\gamma^2}{2}\right) + \\ + \frac{\left[(\psi - \mu)z^2 - c_1 z + c_2\right]2\pi\gamma^3}{\left(\Phi + 2\pi\gamma^3\right)} + T_0 \qquad (50)$$

From Eq. (50), it can be deduced that for a highly heat conductive cylindrical shaft, the temperature decreases as the radius decreases and the temperature decreases while its lengths increases. This form of temperature distribution is conical.

3. Case study

In order to show practical application of the mathematical relations just derived, it is necessary to give corresponding practical examples. Let us consider the example that follows. A lathe-machine operator is turning down a cylindrical shaft, which is being prepared to be fitted into an automobile. The cylindrical shaft has a diameter of 0.1m and is 0.3 m length. The shaft was made to rotate at an angular speed of 600 rpm while the turning operation was going on. The operator turned the dial on the lathe machine at an angle of 80^o before the operation start. The operator ensured that the turning was done ten times and at each time, the dial is turned 80^o. Determine: (i) temperature at the outer surface of the shaft, (ii) temperature at the distance of 0.02 m away from the boundary surface and equilibrium temperature after the ten times, if the temperature in the workshop is 20^{o}C. Let us consider this problem when the shaft is made of (i) steel (ii) aluminium. [Hint: 40^o turn ≡ 1 mm cut] (Take σ_{steel} = 440 J/kgK; σ_{Al} = =880 J/kgK)

The problem is solved in the following ways:

(a) Considering steel: Steel is a fair conductor of heat, therefore, the mathematical relation for temperature distribution is Eq. (50), i.e.

$$T_1 = \frac{n\dot{\theta}^2 (R - n\alpha)^2}{2\sigma(n(R - n\alpha))}(nr) + T_0$$

Note that angular speed, $\dot{\theta}$ = 62.83 rads^{-1}; Depth cut, $\alpha = \frac{80^o}{40^o} \times 1$ = 2 × 10^{-3} m; Radius = 0.05 m; σ_{steel} = = 0.44 J/gK; Room temperature, T_0 = 293 K.

(i) At the boundary surface $r = R_1 = (R - n\alpha)$

$$T_1 = \frac{n\dot{\theta}^2 (R - n\alpha)^2}{2\sigma(n(R - n\alpha))}(n(R - n\alpha)) + T_0 = 60.37^o\text{C}$$

(ii) At the distance of 0.02 m from the boundary surface, i.e. r = 0.01 m (R_1 – 0.02), T_1 = 33.46°C

The equilibrium temperature

$$T_2 = \frac{\left(\dot{\theta}^2 (R - n\alpha)^2 - 2\sigma T_1^2\right)\rho_2^2}{\left(\theta^2 (R - n\alpha)2\rho_1^2 - 2\sigma\rho_2^2 T_1\right)}$$

Given that the inertia at T_1, ρ_1 = 0.1 N/particle and at T_2 = 0.4 N/particle, T_2=56.57°C.

(b) Considering aluminium: Aluminium is a very good heat conductor. Therefore, Eq. (50) gives the mathematical relation for temperature distribution. Suppose that: A = 103 K/m^2; φ = 0.2 Km^{-2}; c_1 = 0.09 K/m; β = = (0.001)^7 K/m; μ = 0.4 K/m^2; Φ = 7 π × 10^{-20}/m; c_2 = 0.

(i) Temperature at the boundary surface and at z = = 0.3 m

$$T_1 = \frac{n\dot{\theta}^2 (R - n\alpha)^2}{2\sigma(R - n\alpha)}\gamma + \frac{An}{2}\left(\frac{\gamma^2}{2} + \frac{-\Phi}{\pi\gamma}\right) + \\ + \left(\frac{(-\psi - \mu)z^2 - c_1 z + c_2}{\left(\Phi + 2\pi\gamma^3\right)}\right) n2\pi\gamma^3 + T_0$$

Considering the fact that $\Phi \to 0$; c_2 = 0, $\gamma = (R - -n\alpha)$

$$T_1 = \frac{n\dot{\theta}^2 (R - n\alpha)^2}{2\sigma} + \frac{An\gamma^2}{4} + \\ + \left((-\psi + \mu)z^2 - c_1 z\right)n + T_0 = 41.63^\circ\text{C}$$

Temperature at the first point of contact, i.e. at $z = 0$

$$T = \frac{n\dot{\theta}^2 (R - n\alpha)^2}{2\sigma} + \frac{An\gamma^2}{4} + T_0 = 42.44^\circ\text{C}$$

This problem is considered at the same time $t = 1$ s.

(ii) At the distance of 0.02 m from the boundary surface, after 10-turns, when z = 0.3 m; r = 0.01 m

$$T^1 = \frac{n\dot{\theta}^2 (R - n\alpha)\gamma}{2\sigma} + \frac{An\gamma^2}{4} + \\ + \left(-(\psi + \mu)z^2 - c_1 z\right)n + T_0 = 26.17^\circ\text{C}$$

Considering at contact point and at r = 0.01, z = 0

$$T_1 = 26.98^\circ C\left(i.e.\ \frac{n\dot{\theta}^2 (R - n\alpha)\gamma}{2\sigma} + \frac{An\gamma^2}{4} + T_0\right)$$

The equilibrium temperature = T_2. Given that at T_1, inertia $\rho_1 \longrightarrow 0.01$ N/particle and at T_2 ρ_2 = = 0.015 N/particle

$$T_2 = \frac{\left(\dot{\theta}^2 (R - n\alpha)^2 T_1 - 2\sigma T_1^2\right)\rho_2^2}{\left(\dot{\theta}^2 (R - n\alpha)^2 \rho_1^2 - 2\sigma\rho_2^2 T_1^2\right)} = 40.6^\circ\text{C}.$$

4. Discussion of results

Temperature at the boundary surface of steel shaft is higher than that of aluminium because steel has a lower heat capacity than aluminium. As a result of constant supply of heat to both surfaces, the temperature at the lower heat capacity material is higher. Also, the calculation just performed obviously showed that temperature difference between points in a good heat conductor is very small and that at a very finite time, all particles in the conductor will be at equilibrium. For example, looking at aluminium at a very small finite period t (say 1 s) the temperature difference between its ends is only 0.81°C (i.e. the temperature gradient for a second). Temperature difference between two points in the fairly heat conductive material (steel) is higher than in good heat conductor (aluminium) when we consider radial distribution. This depicts that the temperature distribution in aluminium, though, conical is even and fast. For temperature distribution in steel also, the reason for this behaviour could be the fact that aluminium particles have low inertia for vibration while those of steel have high inertia for vibration.

5. Conclusions

The mathematical model can now make us to conclude that temperature distribution in a cylindrical shaft depends on the conductivity of the shaft material mostly and the nature of the material particles (whether the particles have high or low inertia for vibration). This model can be improved. It can be used or applied to real-life situation by considering an unsteady heat conduction process, heterogeneous material and that wobbling occurs during some of the turning operations.

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S.A. Oke, O.I. Oyedokun, A.O. Bamigbaiye

CILINDRO FORMOS ŠALTO METALO TEMPERATŪROS PASISKIRSTYMO SKAITINIS MODELIAVIMAS PRIKLAUSOMAI NUO APDIRBIMO BŪDO

R e z i u m ė

Straipsnyje pateikiamas modelis, sudarytas remiantis tuo, kad kai kurios medžiagos turi keletą temperatūros zonų, kuriose keičiasi jų mikrostruktūra ir tai turi įtakos jos mechaninėms savybėms. Modelis atskleidžia matematinę priklausomybę tarp medžiagos temperatūros ir sukimosi greičio, jei apdirbimo metu ji buvo sukama. Darbas remiasi kompleksinės potencinės energijos taikymo teorijomis, šilumos laidumo lygtimis, kinetinės ir šiluminės energijos lygtimis. Gauti rezultatai rodo, kad pažeidimai ir irimas gali turėti įtakos medžiagos mikrostruktūrai, jeigu pjovimo greitis nėra kontroliuojamas, kadangi nėra žinoma medžiagos specifinė šiluminė talpa. Modelis gali būti naudojamas medžiagos pažeidimo laipsniui nustatyti, esant pastoviam šilumos laidumui, homogeniškai medžiagai ir tolygiam sukimuisi. Modelio negalima taikyti apdirbant heterogeninės mikrostruktūros medžiagas, taip pat jei apdirbimo procese detalės nesisuka. Modelis bus naudingas metalurgams, technologams ir inžinieriams konstruktoriams gaminant gaminius, kurių medžiagų mechaninės savybės gamybos proceso metu nekinta. Darbe pateikta nauja matematinė priklausomybė, kuri suteikia papildomos informacijos, kaip išvengti deformacijų ir medžiagų mechaninių savybių pokyčių.

S.A. Oke, O.I. Oyedokun, A.O. Bamigbaiye

NUMERICAL ANALYSIS OF TEMPERATURE DISTRIBUTION OF COLD CYLINDRICAL METAL SUBJECTED TO MACHINING

S u m m a r y

The paper presents a model based on the fact that some materials (alloys and metal precisely) have some temperature ranges at which the nature and microstructure of the material changes affect the mechanical properties of the materials. The model provides a mathematical relation between the temperature of the material at a specific time and the speed of rotation of the material if it was to be turned in a turning operation. The work is based on the theories of complex applied potential, heat conduction equation, the kinetic energy equation, and heat energy equation. The results obtained show that damages and destruction can be caused to the microstructure of the material if the speed of cut is not controlled due to lack of knowledge of the specific heat capacity of the material. The model can be used to check the extent of damage done on the material. The model is limited to a steady heat conduction process, a material that is homogeneous, and a turning operation where there is no wobbling occurring. The model may not apply to engineering materials with

heterogeneous microstructure, and a machining process that does not involve the rotation of the work piece. However, the model would assist metallurgists, machine operators, and design/manufacturing engineers in producing products with unaltered mechanical properties. The work is a new mathematical relation that provides an additional information for manufacturing industries on how to avert alteration or changes in the mechanical properties of materials that are being turned down and in similar operations.

С.А. Оке, О.И. Оуедокун, А.О. Бамигбайуе

ЧИСЛЕННОЕ МОДЕЛИРОВАНИЕ РАСПРЕДЕЛЕНИЯ ТЕМПЕРАТУРЫ В ЦИЛИНДРИЧЕСКОМ ХОЛОДНОМ МЕТАЛЛЕ В ЗАВИСИМОСТИ ОТ СПОСОБА ОБРАБОТКИ

Р е з ю м е

В статье представлена модель, основана на свойстве материалов, имеющих зоны температуры, в которых меняется микроструктура материала и это влияет на механические свойства материала. Модель предлагает математические зависимости между температурой материала и скоростью резания во время обработки. Работа основана на комплексном применении теории потенциальной энергии, уравнений теплопроводности, а также кинетической и тепловой энергии. Полученные результаты показывают, что повреждения могут влиять на микроструктуру материала, если не контролируется скорость резания из-за недостатка информации о специфической теплоемкости материала. Модель можно использовать для оценки состояния гомогенного материала, который имеет постоянную теплопроводность и равномерно вращается при обработке. Модель невозможно использовать при обработке материалов с гетерогенной микроструктурой, а также при обработке не вращающихся деталей. Модель может быть полезна металлургам, технологам, а также инженерам конструкторам и производственникам при изготовлении изделий, механические свойства которых в процессе изготовления не меняются. В работе представлена новая математическая зависимость, дающая дополнительную информацию для исключения изменений механических свойств при обработке.

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