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# Swinging leg influence on long jump

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## 1. Introduction

At long jump event the aim is to jump the distance as long as possible. Fig. 1 shows how the results of record jumps changed at world championships [1]. Obvious is the fact that world records of the men during the last 25 - 30 years have changed only a few centimeters. It seems that the limit is already reached. Nevertheless trainers and researchers search for the ways to improve these results further.

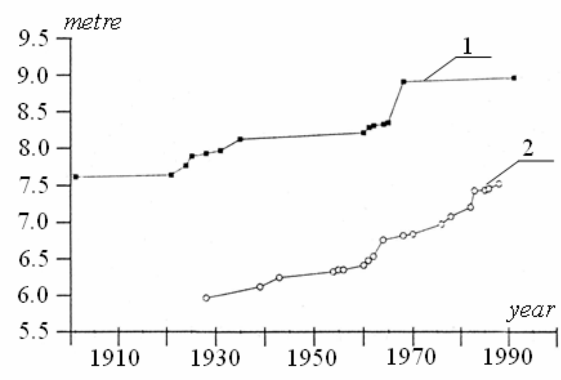



Fig. 1 Dynamics of long jump world records. *1* - men; *2* - women

The result of long jump is defined by inrun speed, take-off power, movement perfection at the flight phase and jump off. For the development of jump technique a lot of research works is performed. But too little attention is paid in them at the influence of swinging leg position during take-off phase. The take-off phase at long jump is a complicated process. It consists of impact type positioning of the take-off leg, elastic deformation, shift of the rotational axis, kinetic energy distribution of separate parts of the body due to both active and passive movements of the swinging leg, etc.

A lot of empirical data on long jump technique can be found in literature but they do not allow to define the relationship between the influencing parameters and results. That's why many authors researching long jump technique made mathematical modeling of take-off phase.

The most comprehensive model for long jump analysis is planar ten segment model. The solution of it suggests three interesting in methodical aspect conclusions [2]:

1. the most important part of all the body parts is of foot joint;

2. at take-off instant the body should develop the driving rotational moment;

3. at take-off instant the swinging leg should be as stretched as possible.

The investigation results using ten segment model partly coincide with the experimental results partly contradict them. It is worth mentioning that movement analysis using this model from the viewpoint of the variation of input and controlled parameters is complicated. Because of this reason and limitations of the aim as well the analysis of only swinging leg influence was performed – three mass model was constructed. The foot mobility in foot joint was taken into account in it. In such model the equations of kinetic moment and driving moment are written. Making variation of input and controlled parameters the influence of swinging leg on long jump is determined.

## 2. Model of swinging leg movement

For the analysis of swinging leg influence on long jump three mass linkage type model with rotational movement possibilities at points *A* and *K* which correspond foot and hip joints is constructed. The characteristic leg position and geometrical structure of the model is presented in Fig. 2.

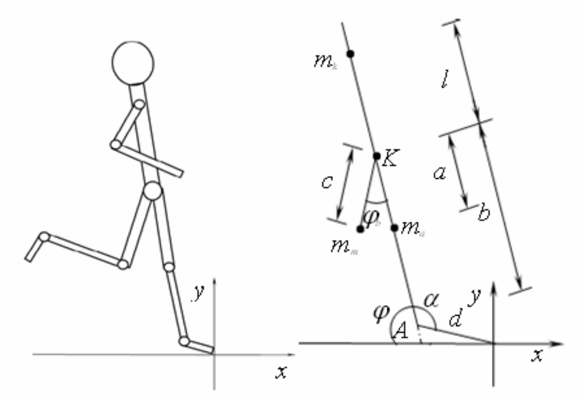
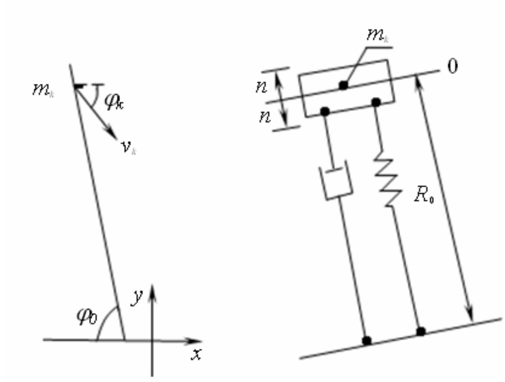



Fig. 2 Leg position and swinging leg movement model. Antropometric parameters: *a* is distance between hip joint and mass center of the take-off leg; *b* is length of the take-off leg; *c* is distance between hip joint and mass centre of swinging leg; *d* is foot length. Controlled parameters: $\varphi$ is angle between take-off leg and horizontal plane; $m_m$ ir $m_a$ are concentrated masses of swinging and take-off legs respectively; $m_k$ is mass of the rest part of the body – head, neck, body, etc. This mass centre at the take-off phase changes its angular position in $\varphi$ direction, i.e. it is swinging. That means that the length *l* is variable parameter

### 3. Determination of mass centre position

The relationship between the angles and the rest mass centre velocity at initial position is shown in Fig. 3, a. Parameters presented in the diagram and other initial parameters necessary for the model analysis are obtained from jump kinograms and dynamograms. Angle $\varphi$ which defines spatial (angular) position of the model is determined as time function from the law of moments.




a b

Fig. 3 Calculation scheme

As it was mentioned above in the process of taking the take-off position the position of mass centre $m$ changes. The reason is elastic deformations in human body. Damping features are also characteristic for the "materials" of human body. Because of this the linkage type model can be changed by lumped parameter model consisting of mass damping and elastic elements (Fig. 3, b). Movement of such system along $R$ is described by second order differential equation [3]

$$\ddot{R} + 2\delta\dot{R} + \omega_0^2 R = 0$$

here $\delta$ is damping constant, $\omega_0$ is natural frequency of mass $m_k$ along $R$.

The distance of mass $m$ to the support $R$ is variable parameter and is determined as

$$R = R_0 - ne^{-\delta t}\sin(\omega_d t - \psi) \qquad (1)$$

The second member at the right hand side is the differential equation solution: $n$ is oscillation amplitude, $\omega_d$ is oscillation frequency of the damped system; $\psi$ is phase angle.

At the initial instant of time when $t = 0$, velocity $v_k$ and angle $\varphi$ are known (assumed).

Damping constant $\delta$ is determined from logarithm decrement

$$\ddot{R}(t) / \ddot{R}(t+T) = e^{\delta T}$$

The ratio of acceleration amplitudes and period $T$ are taken from dynamogram. When $T$ and $\delta$ are known according common formulas of vibration theory oscillation frequencies – natural and the frequency with damping are determined:

$$\omega_0 = \left(\delta + 4\pi^2 / T^2\right)^{1/2}, \ \omega_d = \left(\omega_0^2 - \delta^2\right)^{1/2} \qquad (2)$$

It is obvious that at the instant of impact take-off phase start the take-off acceleration equals to 0. Then from Eq. (1) making the second order derivative $\ddot{R}$ be equal 0 phase angle is determined

$$\psi = \arctan\frac{2\delta\omega_d}{\omega_d^2 - \delta^2}$$

When Eq. (1) is applied for the model presented in Fig. 2 it is necessary to take: $R = l + b$ and $R_0 = l_0 + b$ .

### 4. Driving moment determination

The calculation procedure is the following.

1. Mass centre coordinates and the first order derivatives are determined

for mass $m_a$

$$X_a = d\cos(\varphi + \alpha) - (b - a)\cos\varphi$$

$$Y_a = -d\sin(\varphi + \alpha) - (b - a)\sin\varphi$$

$$\dot{X}_a = -d(\dot{\varphi} + \dot{\alpha})\sin(\varphi + \alpha) + (b - a)\dot{\varphi}\sin\varphi$$

$$\dot{Y}_a = -d(\dot{\varphi} + \dot{\alpha})\cos(\varphi + \alpha) + (b - a)\dot{\varphi}\cos\varphi$$

for mass $m_k$

$$X_k = d\cos(\varphi + \alpha) - (b + l)\cos\varphi$$

$$Y_k = -d\sin(\varphi + \alpha) + (b + l)\sin\varphi$$

$$\dot{X}_k = -d(\dot{\varphi} + \dot{\alpha})\sin(\varphi + \alpha) - \dot{l}\cos\varphi + (b + l)\dot{\varphi}\sin\varphi$$

$$\dot{Y}_k = -d(\dot{\varphi} + \dot{\alpha})\cos(\varphi + \alpha) - \dot{l}\sin\varphi + (b + l)\dot{\varphi}\cos\varphi$$

for mass $m_m$

$$X_m = d\cos(\varphi + \alpha) - b\cos\varphi + c\cos(\varphi - \varphi_b)$$

$$Y_m = -d\sin(\varphi + \alpha) + b\sin\varphi - c\sin(\varphi - \varphi_b)$$

$$\dot{X}_m = -d(\dot{\varphi} + \dot{\alpha})\sin(\varphi + \alpha) + b\dot{\varphi}\sin\varphi - c(\dot{\varphi} - \dot{\varphi}_b)\times$$
$$\times\sin(\varphi - \varphi_b)$$

$$\dot{Y}_m = -d(\dot{\varphi} + \dot{\alpha})\cos(\varphi + \alpha) + b\dot{\varphi}\cos\varphi - c(\dot{\varphi} - \dot{\varphi}_b)\times$$
$$\times\cos(\varphi - \varphi_b)$$

2. According formulas $X_c = \frac{\sum m_i X_i}{m}$ and $Y_c = \frac{\sum m_i Y_i}{m}$ coordinates of mass centres $m_a$, $m_k$ and $m_m$ are determined

$$X_C = \frac{1}{m}\left[md\cos(\varphi + \alpha) - c_1\cos\varphi + c_2\cos(\varphi - \varphi_b)\right]$$

$$Y_C = \frac{1}{m}\left[-md\,sin(\varphi+\alpha) + c_1 \,sin\,\varphi - c_2\,sin(\varphi-\varphi_b)\right]$$

here $m = m_m + m_a + m_k$, $c_1 = mb - m_a a + m_k l$, $c_2 = m_m c$.

3. For the mass centre kinetic moment or angular impulse $L$ is determined.

The formula of kinetic energy of a point is applied for this task – considering mass centre movement about support point [3]

$$L = m\left(xv_y - yv_x\right)$$

here $x = X_C$, $y = Y_C$, $v_x = \dot{X}_C$, $v_y = \dot{Y}_C$.

Substituting the necessary parameters to the formula of kinetic moment or the expression of kinetic moment angular impulse for the mass centre

$$L = \dot{\varphi}\left(c_3 - 2dc_1\cos\alpha + 2c_2P\right) - \dot{\alpha}\left[dm - c_1\cos\alpha + c_2\cos(\alpha+\varphi_b)\right]d - m_k d\dot{l}\sin\alpha + m_m\dot{\varphi}_b(P+c)$$

4. The first order derivative of kinetic moment in time is found. It is equal to the searched driving moment $dL = M$. After rearrangement it is obtained

$$\dot{L} = -\ddot{\varphi}\left(c_3 - 2dc_1\cos\alpha + 2cP\right) - \dot{\varphi}\left[dc_1\dot{\alpha}\sin\alpha - c_2Q + m_k\dot{l}(b+l-d\cos\alpha)\right]\cdot 2 - \ddot{\alpha}\left[dm - c_1\cos\alpha + c_2\cos(\alpha+\varphi_b)\right]d - \ddot{\alpha}^2\left[c_1\sin\alpha - c_2 \times \sin(\alpha+\varphi_b)\right]d - dm_k\left(\ddot{l}\sin\alpha + \dot{l}\dot{\alpha}\cos\alpha\right) - c_2\left[-d\dot{\alpha}\dot{\varphi}_b\sin(\alpha+\varphi_b) - \ddot{\varphi}_b(P+c) + \dot{\varphi}_bQ\right] \quad (3)$$

The applied designations

$$c_3 = md^2 - m_a(b-a)^2 + m_k(b+l)^2 + m_m(b+c)^2$$

$$P = d\cos(\alpha+\varphi_b) - b\cos\varphi_b$$

$$Q = d(\dot{\alpha}+\dot{\varphi}_b)\sin(\alpha+\varphi_b) - b\dot{\varphi}_b\sin\varphi_b$$

The moment determined according Eq. (3) equals to the weight force created moment

$$\dot{L} = -mg\;X_C = g\left[md\cos(\varphi+\alpha) + c_1\cos\varphi - c_2\cos(\varphi-\varphi_b)\right]$$

This is the differential equation searched for the determination of position angle $\varphi(t)$. It can be solved by numerical methods. The solution and modeling results as well as conclusions will be presented in the next publication.

**5. Modelling results**

The model presented allows investigating the take-off process by changing input and controlled parameters independently on the fact whether such jumps can be or can not be performed at present. The influence of the main jump elements – impact at take-off in foot extension joint, position of the swinging leg and their interaction is determined. The influence of the effect on swinging leg dynamics is expressed by dynamo graphs. Such dependences can not be derived (obtained) from the view obtained by film method.

Theoretical jump lengths depending on angular acceleration controlled by a sportsman can be determined also. By this method parameter variation directions allowing to achieve the greatest jump lengths can be determined and the sustainability resulting from such variations can be evaluated. For the first time angular impulses are determined.

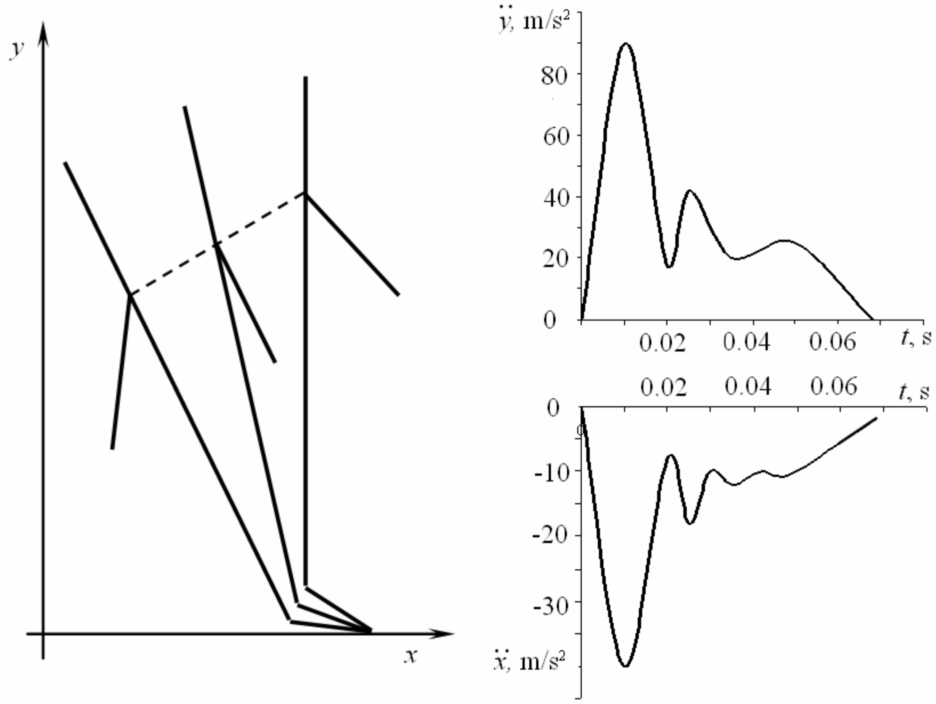



Fig. 4 Modelling positions and dynamogram

Parameter variation (modelling) shows that seeking for higher jump length the swinging leg should perform several functions:

1. The swinging leg being of the lowest mass (about 15% of total body mass) and the body part of the highest mobility accumulates the significant part of kinetic energy.

2. The accelerated motion of the swinging leg generates additional pressure to take-off leg. The force proportions at which the body is effected by the highest force impulse and more energy it gets at the same period of contact with the ground can be determined. The effect is the greater the higher is the acceleration of swinging leg and the higher is its inertia moment.

3. If motion of the swinging leg is stopped a part of its kinetic energy is transferred to the body due to what the foot joint is unloaded and it can be extended faster. Because of this reason the body mass center trajectory is lifted and the higher flight angle is achieved.

Conclusions 2 and 3 contradict each other (as they have the opposite effect) in respect to the jump length. According to the conclusion 2 the modelling shows that accelerating of the swinging leg allows higher jump length. It can be considered that the moments caused in the foot joint are such that angular accelerations at the swinging leg accelerating and decelerating phases are of the same values.

At present only realistically achieved values of the moments at foot joint are available. The modelling results suggest the necessity to make these values objective. The influence of low foot angle changes was impossible to determine up to now (without performing the proposed modelling).

The resolution to components of the motion of separate masses enabled by the model proves in vertical dynamograph that the force maximum at the leg support to the ground instant is of mechanical origin (mass of the parts, inertia, movement) but not the result of muscle work. This force maximum balances the reaction of the foot joint of take-off leg at the analysed phase. Reactive moment is an essential element at take-off. The leg support phase should last as long as possible. A slight concave in the dynamograph indicates that that the reaction phase starts at the right moment.

The modelling shows that the jump length is sensitive to the coordination of the body parts. Optimization of general motion and flight maintaining by this model is impossible. For the analysis of knee joint influence the model improvement is foreseen.

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MOJAMOSIOS KOJOS ĮTAKA ŠUOLIUI Į TOLĮ


R e z i u m ė

Straipsnyje nagrinėjama mojamosios kojos padėties įtaka šuolio į tolį rezultatui. Tam tikslui sudaromas strypinis trijų masių modelis, leidžiantis įvertinti pėdos sąnario judrumą. Nustačius kūno korpuso svyravimų dažnius, apskaičiuojamos atskirų masių centrų koordinatės, jų pirmosios išvestinės, randamos bendrojo masių centro koordinatės. Nustatoma kinetinio momento lygtis, kurios pirmoji laiko išvestinė žymi ieškomą varantįjį sukimo momentą.


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SWINGING LEG INFLUENCE ON LONG JUMP


S u m m a r y

The influence of swinging leg on long jump result is analyzed in the paper. For this purpose three mass linkage type model is constructed the mobility of foot joint is evaluated in which. After natural frequencies determination mass centre coordinates of the separate body parts, their derivatives of the first order and the coordinates of general mass centre are expressed. The equation of kinetic moment the first order derivative in time of which is the searched driving moment is obtained.


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ВЛИЯНИЯ МАХОВОЙ НОГИ НА ПРЫЖОК В ДЛИНУ


Р е з ю м е

В статье анализируется влияние положения маховой ноги на результаты прыжков в длину. Для этой цели предложена стержневая трехмассовая модель, оценивающая подвижность сустава ступни. На основе вычисленных частот качания туловища прыгуна определяются координаты отдельных центров масс и их первые производные, устанавливаются координаты общего центра масс. Приводится уравнение кинетического момента и его первая производная по времени, являющаяся искомым движущим моментом вращения.


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