

## FE simulation of rupture of diaphragm with initiated defect

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### 1. Introduction

The shock tube diaphragm presents a key element of the laboratory facilities used to generate shock waves. Recently the shock waves have been used in many medical applications, material synthesis and condensed mater. Furthermore, shock tube-based research is expanding [1-5].

The flow in each shock tube facility is usually initiated by the rupture of a primary diaphragm that separates high pressure driver gas and low pressure driven gas. High pressure driver gas expands into the driven section, compressing and accelerating the lower pressure driven gas. A shock wave develops within the driven gas and propagates along the shock tube [2]. The device is operated by fast bursting the diaphragm, so rapidly expanding the compressed gas to flow along the low pressure channel [2].

The parameters of the shock wave such as pressure, velocity, etc., thus generated depend on the pressure level at which the rupture diaphragm breaks. Hence, the reproducibility of the shock depends on the reproducibility of the burst phenomenon of the rupture disk. Since the opening phenomenon of the rupture diaphragms is dominated by a complex factor, it is rather difficult to manufacture the rupture disk with the same specific performance. For the same reason, it is also difficult to design the rupture diaphragm with the specific performance [6].

It is well known, however, that the effect of diaphragm opening on shock wave formation in shock tubes is not trivial. In all cases, it was confirmed that this initiation process for shock tube operation was far from what was believed ideal. In the previous studies [1-3] of the shock tube flows, the main interest was in relating this nonideal diaphragm rupture pattern to the transient shock wave formation process. Furthermore, we must point out another drawback of the rupture disk – burst of the rupture diaphragm causes scattering of fragments. These fragments may cause damages to the facility or undesirable effect on the experiment. In spite of the data reported in the previous works, actual process of diaphragm rupture has not yet been thoroughly studied [2]. Many authors visualized diaphragm deformation and rupture using streak photography or a multiplespark camera [4, 6].

Apart from physical evidence, numerical simulation can be a powerful analysis tool for studying the process of diaphragm rupture in an expansion tube. In the last decades researcher simulated numerically the rupture of brittle diaphragms [1] and thin and thick diaphragms made of metal without incision [3, 7, 8] as well as diagrams made of extremely light and thin materials such as cellophane or mylar [1]. Because of high complexity different rupture models may be used to reproduce distinct rupture processes of these diaphragms [7].

The paper presents the simulation of nonlinear

static rupture of the ductile shock tube diaphragm with initiated defect under pressure by the finite element method (FEM). The main aim of the present investigation is to generate and test a finite element model suitable for describing rupture and prerupture behavior of the diaphragm.

The ANSYS code [9] was used for the analysis, while problem oriented pre- and post-process software was developed for modeling purposes.

The paper is organized as follows. Section 2 presents the description of basic data on the shock tube. Section 3 describes the FE models. Section 4 presents the numerical results of the influence of the defect depth on elastic – plastic deformation and Section 5 describes the data on diaphragm rupture. Conclusions are presented in Section 6.

### 2. Basic data

The shock tube used in the present investigation is a device which can produce shock waves up to the shock Mach number of 5. Pressure values of this tube could be 0-50 atm. It is employed for developing of major dynamical characteristics of the sensors used in Semiconductor Physics Institute of Vilnius [5].

The shock tube (Fig. 1) is made of a solid wall pipe of circular cross-section divided into two compartments separated by a diaphragm.

The shock tube diaphragm considered presents a circular plate with a defect which is initiated in the form of two perpendicular incisions (Fig. 1).

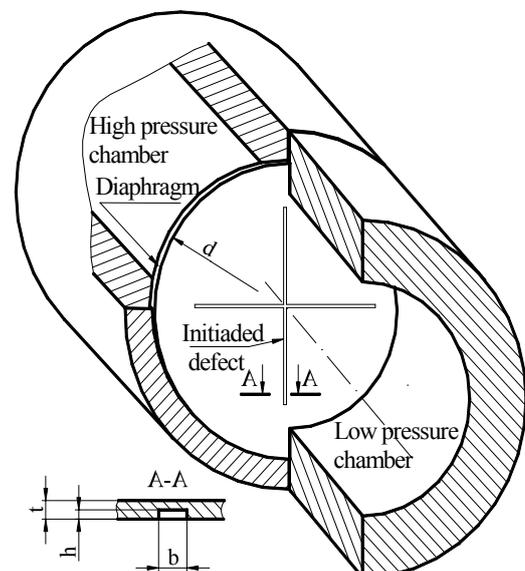


Fig. 1 The fragment of the gas shock tube

At the boundary the diaphragm is fixed between two cylinders assumed to have rigid surfaces. Finally, the supports of the diaphragm structure are assumed to be identical to the supports of the clamped plate.

The diaphragm should be considered as a thin plate of relatively small thickness  $\bar{t} = t/d = 1/120$ . Since  $\bar{t} < 1/100$ , then, according to nonlinear mechanics [10, 11] the geometrical nonlinear theory should be applied to mechanical analysis. Moreover, the deformation behaviour of the diaphragm is influenced by the accumulation of plastic strains.

The geometry of the diagrams is defined by global parameters, including the diameter and thickness, and very important local parameters, such as width  $b$  and depth of the initiated defect.

The material of the diaphragm is copper. It is assumed to be homogeneous and isotropic and its mechanical characteristics are obtained experimentally. Finally, the behavior of the copper used for diaphragms is described by the nonlinear stress  $\sigma$  – strain  $\varepsilon$  diagram (Fig. 2), experimentally evaluated from the tensile tests. These tests were performed in the Laboratory of Strength Mechanics of Vilnius Gediminas Technical University according to the standard EN 10002 [8].

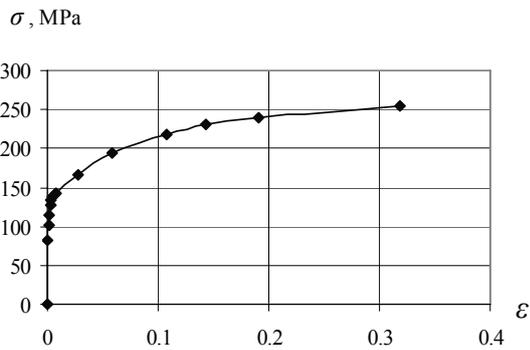


Fig. 2 Stress – strain diagram of diaphragm material

The Young's modulus  $E = 107.5$  GPa, yield point 135.0 MPa and maximum deformation  $\varepsilon_{\max} = 0.319$  was obtained from the experimental curve, while the Poisson's ratio  $\nu = 0.30$  for copper was taken from the paper [8].

### 3. FE discretization

Discretization of the diaphragm meets serious difficulties due to of cross-section incision. Actually, 3D stress and strain fields prevail not only in the vicinity of the defect, but also in the vicinity of the fixed boundary. The 3D approach will be much more suitable for describing plastic deformations.

The diaphragm structure has double axial in-plane symmetry; therefore, only a quarter of it is considered as a three-dimensional finite element domain (Fig. 3). The brick element, which is used in modelling and analysis of diaphragms by finite elements, has plasticity as well as large deflection and strain capabilities [9].

Two types of 3D meshing technique have been explored for modelling of plate as 3D body. The mapped mesh may be considered as a regular mesh. It is characterized by characteristic mesh dimension  $l_e$  applied for discretization of the vicinity of defect. It has been observed

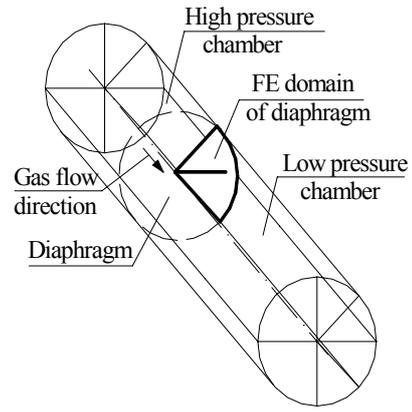


Fig. 3 Schematic view of the shock tube with FE domain of the diaphragm

that mapped meshing is highly inefficient because a realistic model with 8 layers dramatically increases the number of DOF [8].

The second type of meshes, present swept or irregular meshes. This type of mesh is defined by two characteristic in-plane dimensions: the element length in the vicinity of defect  $l_e$  and the element length in the boundary  $l_{eb}$ . It is computationally favourably, however, implementation of irregular disconnecting of elements would be problematic.

For the purposes of rupture analysis, the combined three-dimensional finite element meshing technique comprising structured mesh in the vicinity of the initiated defect and free in-plane mesh was modified and further developed. In a frame of this approach, the diaphragm was considered as a three-dimensional multilayered plate. The governing parameter of the discretization is thickness of individual layer  $h_e$  (Fig. 4). This parameter is, actually, the thickness of a 3D brick element. It is defined as fraction to plate thickness  $t$  and to the variable defect depth  $h$ . The detailed geometry of the defect is presented in Fig. 4.

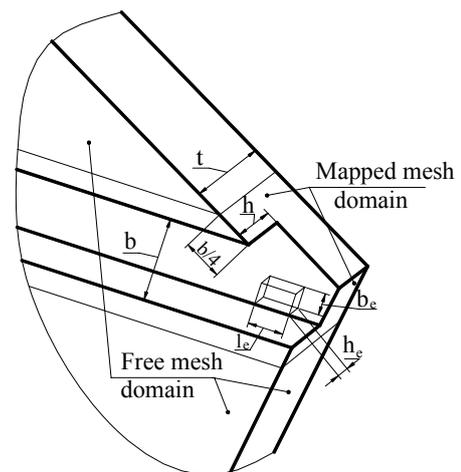


Fig. 4 Geometry of the defect

The remaining part is covered by the unstructured in plane mesh, the density of which is defined by a characteristic dimension of the element length  $l_{eb}$  at the boundary line. The example of the mesh fragment (Fig. 5) illustrates meshing concept of the defect area. The vie of the entire mesh is presented in Fig. 6.

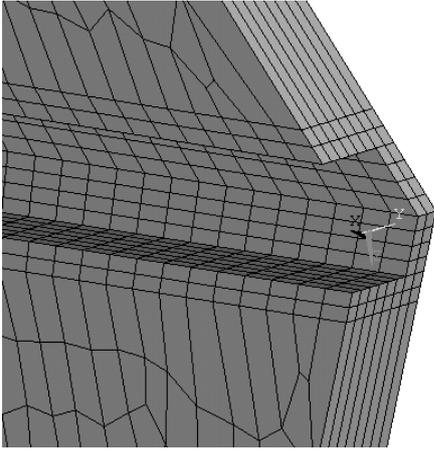


Fig. 5 Illustration of the defect discretization

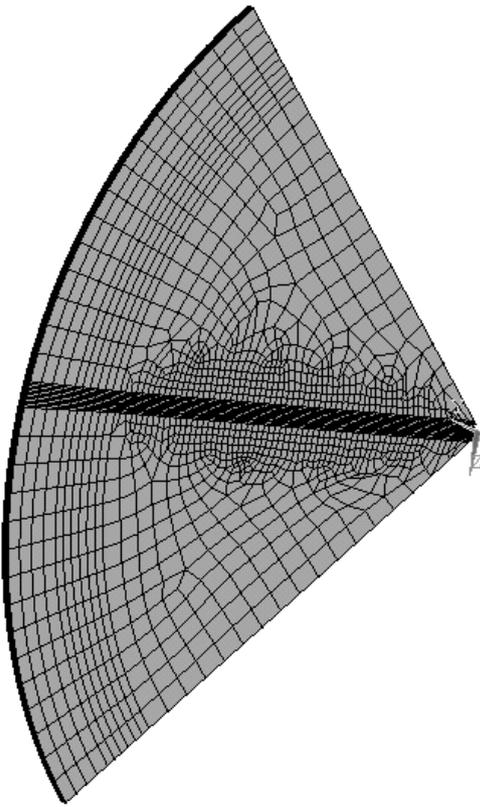


Fig. 6 Swept mesh of the diaphragm

Finally, the above discussed meshing was adopted for generation of variable depth of defect and later for automatic generation of rupturing elements.

#### 4. Prerupture behaviour of diaphragms with various defect depths

The prerupture deformation of the diaphragm with the initiated defect is formulated as nonconservative, geometrical and physically nonlinear analysis problem. The purpose of this chapter is to investigate prerupture behaviour of diaphragms with different defect depths for diaphragms with defect characteristics. Three models, having the values of the defect depth equal to  $h = 0.25t$ ,  $h = 0.5t$  and  $h = 0.75t$  were investigated, and the results were compared with a plate with zero defect.

Numerically obtained pressure-central deflection curves are presented in Fig. 7. The last curve with  $h = 0.75t$  demonstrates the limit behaviour of the diaphragm, where maximum pressure value  $p = 0.398$  MPa may be considered to be the limit pressure. This pressure corresponds to the opening of the diaphragm.

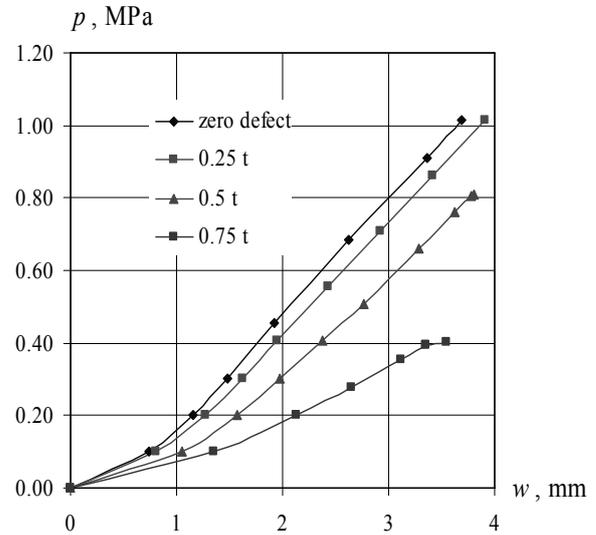


Fig. 7 Deflection curves of the central point

Examination of the influence of the defect has shown that the increase of defect depth by 25%, 50% and 75% at the same pressure increases deflections of diaphragms by 1.67, 2.18 and 2.31 times.

Distribution of plastic strains obtained according to von Mises strain is given in Fig. 8. The numerical results indicate the development of the local plastic zones characterising of the diaphragm behaviour. The most important plastic zone occurs in the defects at the centre of plate and propagates along the defect area. The second plastic zone is concentrated along the clamped boundary, which corresponds to the occurrence of a circular plastic hinge. This proves the necessity of using the suggested locally concentrated mesh concept.

The quantitative examination of the Mises strains indicates that the maximum stress value in diaphragms could be found in the centre of the diaphragm. Their absolute values and relative values compared to the clamped boundary increases along with the increase of the defect depth.

#### 5. Simulation of rupture

The deformation behavior of the diaphragm during rupture is, generally, a nonconservative process, involving geometrical, physical and structural nonlinearity as well. The detailed mathematical model depends, however, on the specified properties of the diaphragm and the incision level. Some aspects of numerical simulation may be found in [12-17].

A recent approach assumes rupture occurring under proportionally increasing quasistatic pressure. This assumption gives rather overestimated deflection values, however, gas flow and structure coupled behavior may be considered as a future challenge. Additionally it is expected that the rate of loading is higher compared to the

deformation rate during rupture. The prerule approach presents a large deflection elastic-plastic problem.

The problem is solved incrementally by controlling the pressure.

The FE model applied in prerule analysis was modified for describing rupture behavior. Disconnection of material is implemented through the one-dimension type element LINK8, of the ANSYS code [9].

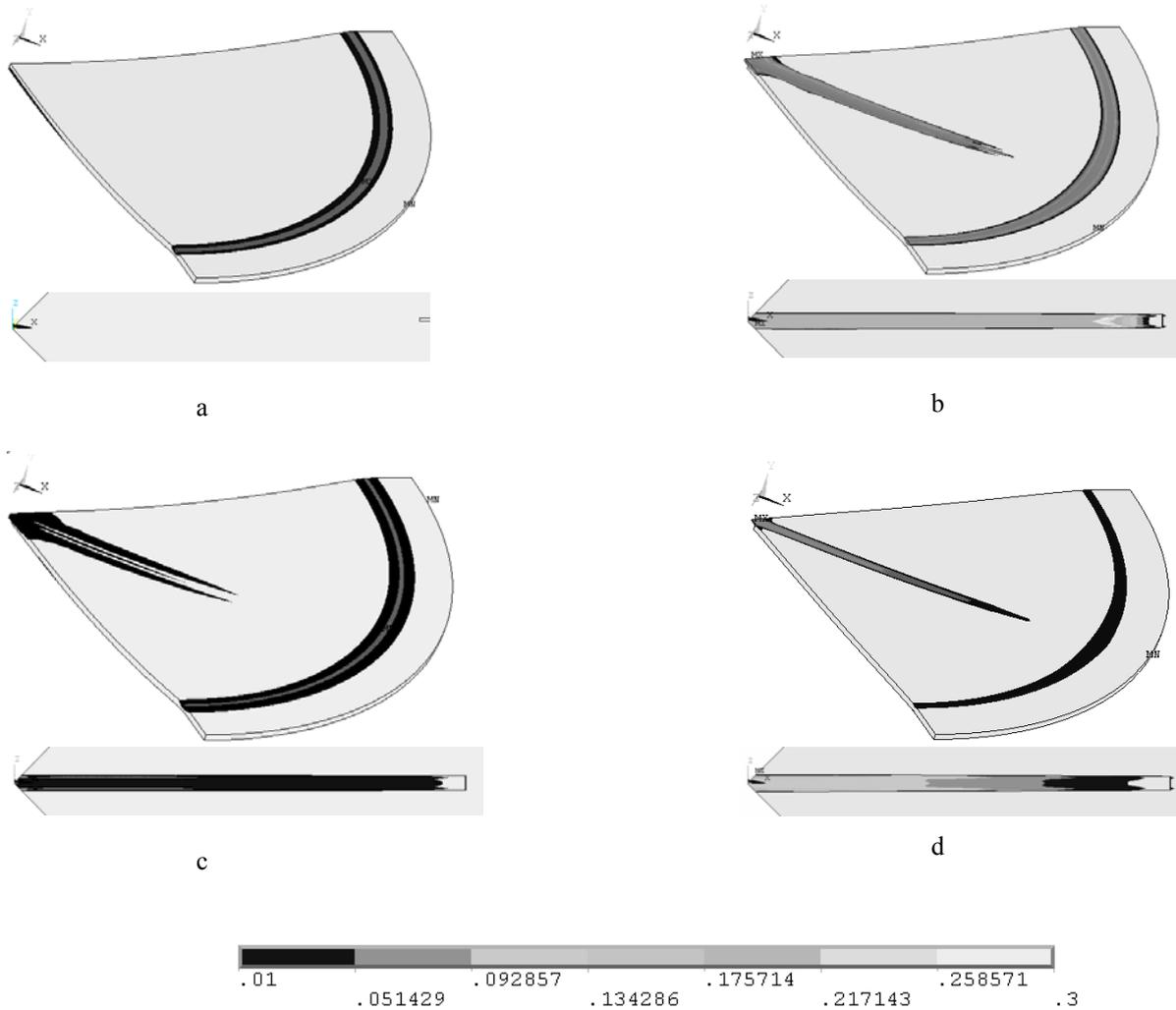


Fig. 8 Distributions of plastic strains of diaphragms with various defect depth: a - zero defect; b -  $0.25t$ ; c -  $0.5t$ ; d -  $0.75t$

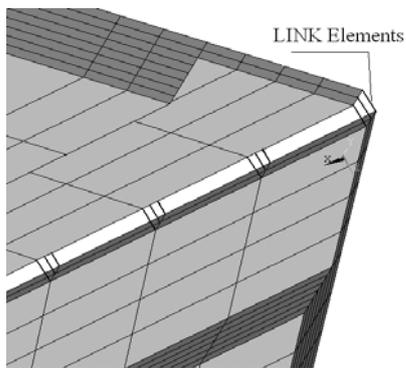


Fig. 9 Mesh fragment with special LINK elements

LINK8 is a link which may be used in a variety of engineering applications. The element allows simulation of nonlinear elastic-plastic behaviour and disconnecting under the given strength condition of the element. This element can be used to model trusses, sagging cables, links, springs, etc. The 3-D link element is a uniaxial tension-compression element with three degrees of freedom at each node: translations in the nodal  $x$ ,  $y$ , and  $z$  directions. Simi-

lar to a pin-jointed structure, no bending of the element is considered.

The element is defined by two nodes, cross-sectional area, initial strain, and the material properties. The element's  $x$ -axis is oriented along the length of the element.

The essential parameters of the element are its length and cross-sectional area. The length is maximally small ( $l = 0.1$  mm). The area may be expressed as follows

$$A_e = A/n$$

where  $A$  is the area of a longitudinal section of diaphragm defect, while  $n$  is the number of spring elements in FE model.

Application of link elements is one of the tools used in fracture mechanics, where crack propagation path is known in advance. Disappearing elements are small enough in order do not violate energy balance.

The diaphragm rupturing process is now discussed. Specifically, we examine the effect of opening process on the diaphragms with defects. Then the numeri-

cal simulations are presented to investigate the effect of incomplete diaphragms opening on the diaphragm petaling [17].

The deformation behaviour of the diaphragm and the influence of various models may be exposed by comparing different curves representing pressure  $p$  and central deflection (Fig. 10). It has been found that the displacement along  $Y$  axis of the central point of diaphragms dramatically increases after the rupture of special elements.

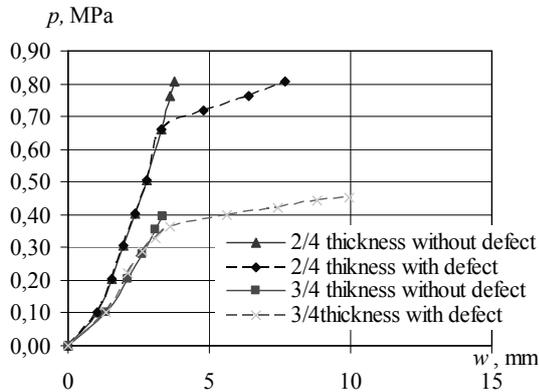


Fig. 10 Deflection curves of the central point of various diaphragms

A schematic view of distribution of diaphragm displacements after rupture is shown in Fig. 11.

The plastic zone occurred at the crack tip propagates towards the support and touches another zone. Finally, the plastic strains are concentrating in the vicinity of

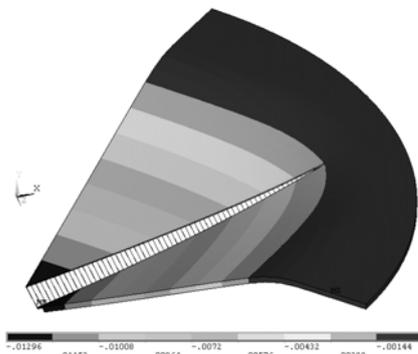


Fig. 11 The deformed diaphragm

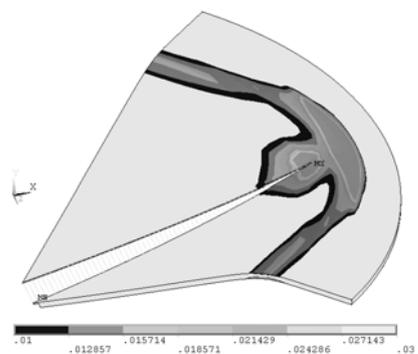


Fig. 12 Mises strains of the diaphragm after rupture ( $h=0.75t$ )

clamped boundary. This highly concentrated zone may be considered as a distorted circular plastic hinge (Fig. 12). This new composition with different localization centers

and quite equal values may be sensitive to small imperfection occurring during fracture.

The results described in this chapter explain the experimental facts [8] that rupture of the diaphragms with smaller defects occasionally yields to undesired irregular rupture pattern of the diaphragm in the contour.

## 6. Conclusions

Different types of diaphragms with various defect depths have been used to identify diaphragm plastic zones and to simulate the rupture process of these diaphragms by numerical modelling.

1. The combined FE meshing technique for constructing a three-dimensional diaphragm model, comprising the structured mesh in the vicinity of defect and free meshing in the remaining part has been suggested and tested. It was justified that this meshing was suitable for generation of variable defect depth along with implementation of rupturing elements.

2. The influence of the defect depth on prerule behaviour and limit pressure of the diaphragm was studied. It was found, that increasing of the defect depth stimulates formation of a new plastic zone in the centre of the plate as well as their spreading along the defect area but simultaneously reduces the concentration of plastic zones on the contour.

3. It was observed, that the rupture of the diaphragm with a small defect depth causes high inhomogeneous concentration of plastic strains and occurrence of single plastic zone on the contour with distorted shape. This explains sensitivity of the experimentally observed rupture to defect geometry and the occurrence of another contour rupture mechanism.

4. The numerical results obtained in the investigation prove, that the suggested model accurately describes the rupture behaviour of the diaphragm under proportionally increasing pressure and it will be later applied for simulating the rupture process of the diaphragm, loaded with the time-dependent pressure.

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#### DIAFRAGMOS SU INCIJUOTU DEFEKTU MODELIAVIMAS BAIGTINIŲ ELEMENTŲ METODU

#### R e z i ū m ė

Pasiūlytas BE metodas slėgio veikiamos smūginio vamzdžio diafragmos netiesinio deformavimo ir irimo elgsenai tirti. Kombinuotas trimatis baigtinių elementų modelis tinka deformacijoms ir irimui aprašyti atliekant šį tyrimą. Nustatyta, kad plastinių deformacijų zonos po apkrovimo koncentruojasi ties įtvirtinimu ir išilgai defekto. Jų

dydis ir pasiskirstymas priklauso nuo apkrovos ir defekto gylio. Irimo analizei modelyje pritaikyti specialūs ryšio elementai. Diafragmos su mažesnio gylio defektu irimas sukelia didesnes plastinių deformacijų koncentracijas kontūre ir gali būti nepageidaujamo nereguliaraus irimo, prasižedančio ties diafragmos kontūru, priežastis.

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#### FE SIMULATION OF RUPTURE OF DIAPHRAGM WITH INITIATED DEFECT

#### S u m m a r y

Nonlinear FE deformation and rupture analysis of the shock tube diaphragm behavior under pressure is presented. The combined three-dimensional finite element model is suitable for describing deformation and rupture in the investigation. Plastic deformation zones after loading are concentrated along the clamped boundary and along the defect area. Their values and distribution depend on the load value and defect depth. The FE model for rupture analyses is simulated with special link elements. The rupture of diaphragms with smaller defect causes higher plastic strains in the contour and may be results of undesired rupture of the diaphragm on the contour.

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#### МОДЕЛИРОВАНИЕ ДИАФРАГМЫ С ИНИЦИИРОВАННЫМ ДЕФЕКТОМ МЕТОДОМ КОНЕЧНЫХ ЭЛЕМЕНТОВ

#### Р е з ю м е

Предложен нелинейный анализ методом КЭ деформирования и разрушения диафрагмы в ударной трубе. Комбинированная трехмерная модель конечных элементов может быть использована для характеристики деформирования и разрушения. Установлено, что зоны пластических деформаций после нагрузки концентрируются на зажатом крае и вдоль дефекта. Их величина и распространение зависят от величины нагрузки и глубины дефекта. Модель КЭ для анализа разрушения имитирована с помощью специальных элементов. Разрушения диафрагмы с менее глубоким дефектом вызывают большие концентрации пластических деформаций на контуре и могут быть причиной нежелательного нерегулярного разрушения диафрагмы в зоне контура.

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