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Numerical modelling of creep functions of laminated composites

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1. Introduction

Fibered or reinforced as well as layered structures and their elements are widely used at present. Specific creep and creep characteristic which characterize viscoelastic properties of composite materials can be determined by modelling a composite element as a layered material loaded by axial force. However, it is possible to model like this not only the above-mentioned composites, but also concrete [1] and other composites [2, 3]. The limiting values of composite mechanical properties can be obtained by modelling the composite as a layered element loaded in a longitudinal and perpendicular direction to the joints of the layers. These limits are known as the Reus-Voight estimate [2, 3]. However, by modelling a composite as a layered element loaded in longitudinal direction to the joints of the layers it is also possible to determine the upper limits of creep characteristics and a specific creep of the composite.

A lot of methods have been developed that allow to forecast the elastic properties of composites according to the properties of the components, i.e. shape and joining of the layers: elasticity and shear modulus, coefficients of thermal and hygral expansion [2-7]. There are much fewer methods allowing to forecast the viscoelastic properties of materials. Many investigators model the composite viscoelastic properties by using operational methods of calculus [3, 8-14]. These methods are based on the equality of convolution of two functions to the product of mapping of these functions according to Laplace or two-side Laplace transformation. By application of this rule for integral equation we get operational equations with operators. In these equations instead of stress, deformation and specific creep or creep characteristic functions we have maps of these functions. Upon putting these operators into the known equations whose purpose is to calculate the elastic mechanical properties of composites we get analytical expressions of the function map of viscoelastic mechanical properties of composite materials. After finding the originals of these function we get an analytical expression of viscoelastic mechanical properties of the composites. When expressions of maps are complex, it is very difficult to obtain the originals of these maps. It may happen that the originals of these maps in explicit function forms do not exist [3,15]. In this case viscoelastic properties of composites can be obtained numerically by maps of functions. A number of methods have been developed for this purpose [3,15]. In practice we can obtain the originals of maps in explicit function forms when the expressions of the maps are simple. This is possible only if creep functions of the components of the composite are very simple.

Other authors model the viscoelastic deformations of layered composites by using the age-adjusted effective modulus method [16]. Equations obtained by using this method are with integral terms. Solution of these equations is complicated because integral terms include stress members which depend on time. Therefore, application of this method for engineering calculation may be complicated.

The finite element method is a universal method for modelling mechanical properties of composites [17,18]. It is also used to determine the precision of calculation of analytical methods which are used in modelling the mechanical properties of composites [9, 11, 19, 20]. However, due to its complexity, the method of labour expenditure of modelling and analysis of results of the finite elements may be inconvenient for engineering applications. In practice, it is important to have simple calculation methods allowing to make precisely calculations of viscoelastic properties of composites.

Therefore, we offer a relatively simple and practical calculation method which allows to model the viscoelastic properties of composites when the loading is applied by parallel to the joint of the layers.

2. Main dependences

A multilayered composite (Fig. 1) loaded with axial force N was studied. The directions of layers are parallel to the direction of loading.

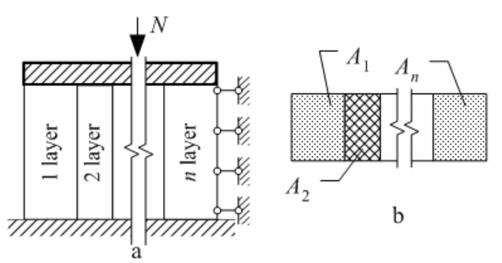

Fig. 1 The design scheme of a layered composite element – a and layered element cross-section – b

The linear creep deformation of composite can be described by the creep function which is widely used in the creep theory

$$\delta_c(t,t_0) = \frac{1}{E_c(t_0)} + C_c(t,t_0) = \frac{1}{E_c(t_0)}\left[1+\phi_c(t,t_0)\right] \qquad (1)$$

where $E_c(t_0)$, $C_c(t,t_0)$ and $\phi_c(t,t_0)$ are elastic modulus, specific creep and creep coefficient of the layered composite at time moment t_0 respectively. The creep coefficient and specific creep are related to the well-known relationship: $\phi_c(t,t_0) = E(t_0)C(t,t_0)$. According to Eq. (1), if we know two different functions which characterize creep function, i.e. $E_c(t_0)$ and $C_c(t,t_0)$, or $E_c(t_0)$ and $\phi_c(t,t_0)$, or $C_c(t,t_0)$ and $\phi_c(t,t_0)$, it is very easy to obtain the third unknown member.

If the stresses remain unchanged over time, then the composite deformation and stresses are related by the following equation

$$\varepsilon_c(t,t_0) = \sigma_c(t_0)\delta_c(t,t_0) = \frac{N(t_0)}{A_c}\delta_c(t,t_0) \tag{2}$$

where $\sigma_c(t_0)$ is the stress of composite, $N(t_0)$ is the axial loading force applied to the composite element at time t_0, A_c is the area of the composite cross-section (Fig. 1). It is not difficult to find the creep function and relationships of specific creep from Eq. (2)

$$\delta_c(t,t_0) = \left(\frac{1}{E_c(t_0)} + C_c(t,t_0)\right) = \frac{\varepsilon_c(t,t_0)A_c}{N(t_0)} \tag{3}$$

$$C_c(t,t_0) = \frac{\varepsilon_c(t,t_0)A_c}{N(t_0)} - \frac{1}{E_c(t_0)} \tag{4}$$

Therefore, if we know the composite deformations $\varepsilon_c(t,t_0)$ in a given time period, we can find the specific creep $C_c(t,t_0)$ of the composite. The purpose of the following investigation is development of a method for calculating the deformations of axial compressed layered composite in (t,t_0) time interval.

The proposed calculation method is based on the following assumptions:

1. the joints of the layers are absolutely rigid and all layers deform together;
2. the uniaxial stress state of the composite and the layers is assumed;
3. the layered composite does not bend and deformation arises only on axial strain;
4. the linear creep deformations of each separate layer of the composite and of the whole composite are assumed;
5. the creep functions of the layers under tension and compression are even.

If the stresses do not exceed a certain limit, the fourth assumption can be applied to many materials such as concrete, mortar, masonry, many plastics, wood, gypsum and etc.

When a composite is under uniaxial stress state, the axial deformation $\varepsilon_i(t)$ of the layers at the moment t of time can be described by the following integral equation [3,14,21]

$$\varepsilon_i(t) = \sigma_i(t_0)\delta_i(t,t_0) + \int_{t_0}^{t} \frac{\partial\sigma_i(\tau)}{\partial\tau}\delta_i(t,\tau)d\tau,\ i=1...n \tag{5}$$

where n is the number of layers, $\sigma_i(t)$ and $\delta_i(t,\tau)$ are stress and creep functions of the ith layer. After integration of the Eq. (5) into parts we get

$$\varepsilon_i(t) = \frac{\sigma_i(t)}{E_i(t)} - \int_{t_0}^{t} \sigma_i(\tau)\frac{\partial\delta_i(t,\tau)}{\partial\tau}d\tau \tag{6}$$

where $E_i(t)$ is the elastic modulus function of ith layer. Since the layers of the composite deform together, the deformation of each layer is equal to the axial deformation of the whole composite

$$\varepsilon_i(t) = \varepsilon_c(t) \tag{7}$$

where $\varepsilon_c(t)$ is axial deformation of the composite at t moment. The sum of forces of all layers is equal to the external axial loading. Generally, this loading varies with time

$$\sum_{i=1}^{n} A_i\sigma_i(t) = N(t) \tag{8}$$

where A_i is the area of the ith layer. Taking into account relationships (5)-(8) the system of equations interconnecting the deformation and stresses of the composite is designed

$$\begin{cases} \varepsilon_c(t) = \dfrac{\sigma_i(t)}{E(t)} - \displaystyle\int_{t_0}^{t} \sigma_i(\tau)\frac{\partial\delta_i(t,\tau)}{\partial\tau}d\tau \\ \displaystyle\sum_{i=1}^{n} A_i\sigma_i(t) = N(t) \end{cases} \tag{9}$$

This system has $n+1$ unknowns and is made up of $n+1$ equations, that is why it can be solved by well-known integral equation solution methods. It can be solved by taking out the mean of the stress function before the integral [22 - 25]. The n first equations of the Eqs. system (9) can be written as follows

$$\varepsilon_c(t) = \frac{\sigma_i(t)}{E(t)} - \sum_{j=1}^{m}\int_{t_{j-1}}^{t_j} \sigma_i(\tau)\frac{\partial\delta_i(t,\tau)}{\partial\tau}d\tau \tag{10}$$

By using the mean-value theorem for two functions [22 - 25], the Eq. (10) can be rewritten as follows

$$\varepsilon_c(t) = \frac{\sigma_i(t)}{E(t)} - \sum_{j=1}^{m}\sigma_{i,\xi}(t_j,t_{j-1})\int_{t_{j-1}}^{t_j} \frac{\partial\delta(t,\tau)}{\partial\tau}d\tau \tag{11}$$

where $\sigma_{i,\xi}(t_j,t_{j-1})$ is the mean value of the the stress function of the ith layer in time interval (t_j,t_{j-1}). However, in this case, beside the others, there is an additional requirement for stress functions, i.e. that the stress function in the interval of integration should not change its sign. This causes difficulties in the application because the integration intervals should be chosen so that the stress function sign does not change. It is assumed that for a very short time period (t_j-t_{j-1}) the mean of the stress functions is equal to the arithmetical stress mean value [22]

$$\sigma_{i,\xi}(t_j,t_{j-1}) = \left[\sigma_i(t_j) + \sigma_i(t_{j-1})\right]/2 \tag{12}$$

After putting (12) into (11) and integration of the obtained relationship according to the Newton-Leibniz rule we get

$$\varepsilon_c(t)=\frac{\sigma_i(t)}{E_i(t)}-$$
$$-\frac{1}{2}\sum_{j=1}^{m}\left(\sigma_i(t_j)+\sigma_i(t_{j-1})\right)\left[\delta_i(t,t_j)-\delta_i(t,t_{j-1})\right] \tag{13}$$

After an algebraic transformation, taking into account the condition (8), also that $t=t_m$, we obtain the final system of recursive linear equations which relates the stresses in layers and the axial forces

$$\begin{cases}\varepsilon_c(t_m)=\frac{1}{2}\sigma_i(t_m)\left[\frac{1}{E_i(t_m)}+\delta_i(t_m,t_{m-1})\right]- \\ \quad -\frac{1}{2}\sum_{j=0}^{m-1}\sigma_i(t_j)\left[\delta_i(t_m,t_{j+1})-\delta_i(t_m,t_{j-1})\right] \\ \sum_{i=1}^{n}A_i\sigma_i(t_m)=N(t_m)\end{cases} \tag{14}$$

The system of Eqs. (14) can be written in the matrix form as follows

$$[\mathbf{Q}(t_m)]\,\boldsymbol{\sigma}(t_m)=\mathbf{c}(t_m) \tag{15}$$

where

$$[\mathbf{Q}(t_m)]=\begin{bmatrix}-Q_1(t_m) & 0 & \cdots & 0 & 1\\ 0 & -Q_2(t_m) & \cdots & 0 & 1\\ \vdots & \vdots & \ddots & \vdots & \vdots\\ 0 & 0 & \cdots & -Q_n(t_m) & 1\\ A_1 & A_2 & \cdots & A_n & 0\end{bmatrix}$$
$$\boldsymbol{\sigma}(t_m)=[\sigma_1(t_m)\ \ \sigma_2(t_m)\ \ \cdots\ \ \sigma_n(t_m)\ \ \varepsilon_c(t_m)]^T$$
$$\mathbf{c}(t_m)=[u_1(t_m)\ \ u_2(t_m)\ \ \cdots\ \ u_n(t_m)\ \ N(t_m)]^T$$

Here Q_i and u_i, are calculated as follows

$$Q_i(t_m)=\frac{1}{2}\left[\frac{1}{E_i(t_m)}+\delta_i(t_m,t_{m-1})\right] \tag{16}$$
$$u_i(t_m)=-\frac{1}{2}\sum_{j=0}^{m-1}\sigma_i(t_j)\left[\delta_i(t_m,t_{j+1})-\delta_i(t_m,t_{j-1})\right]$$

When the determinant of the Eqs. systems (14) and (15) is not equal to zero, i.e. $|\mathbf{Q}(t_m)|\neq 0$, these systems have a single solution.

By solving the Eqs. systems (14), (15) or (19) when $m = 1$, $m = 2$, $m = 3$, ..., $m = z$ we obtain the stress and strain values at t_1, t_2, t_3, ..., t_z time moments. The stress $\sigma_i(t_0)$ at initial time moment t_0 can be obtained by using the dependence [26]

$$\sigma_i(t_0)=E_i(t_0)\,N(t_0)/B(t_0) \tag{17}$$

where B is the compressive stiffness of the layered composite calculated according to the formula [26]

$$B(t_0)=\sum_{i=1}^{n}E_i(t_0)\,A_i \tag{18}$$

In order to calculate the (14) or (15) equations systems it is more convenient to use computer programs which solve the linear equation systems in the matrix form. Instead of the recursive matrix (15), it is possible to form a single matrix the solution of which gives us the strain of the composite and the stresses of the layers. For this purpose the Eqs. system (14) is written down in the following matrix form

$$[\mathbf{M}(t_m)]\,\mathbf{s}(t_m)=\mathbf{d}(t_m) \tag{19}$$

here $\mathbf{M}(t_m)$, $\mathbf{s}(t_m)$, and $\mathbf{d}(t_m)$ are

$$\mathbf{M}(t_m)=\begin{bmatrix}[\mathbf{Q}(t_1)] & 0 & \cdots & 0\\ [\mathbf{V}(t_2,t_1)] & [\mathbf{Q}(t_2)] & \cdots & 0\\ [\mathbf{V}(t_3,t_1)] & [\mathbf{V}(t_3,t_2)] & \cdots & 0\\ \vdots & \vdots & \ddots & \vdots\\ [\mathbf{V}(t_m,t_1)] & [\mathbf{V}(t_m,t_2)] & \cdots & [\mathbf{Q}(t_m)]\end{bmatrix}$$
$$\mathbf{s}(t_m)=[\boldsymbol{\sigma}(t_1)\ \ \boldsymbol{\sigma}(t_2)\ \ \cdots\ \ \boldsymbol{\sigma}(t_m)]^T$$
$$\mathbf{d}(t_m)=[\mathbf{y}(t_1)\ \ \mathbf{y}(t_2)\ \ \cdots\ \ \mathbf{y}(t_m)]^T$$

here $[\mathbf{Q}(t_k)]$ and $\boldsymbol{\sigma}(t_k)$ are the same as in (15), while $\left[\mathbf{V}(t_k,t_j)\right]$, and $\mathbf{y}(t_k)$ are

$$\left[\mathbf{V}(t_k,t_j)\right]=\begin{bmatrix}v_1(t_k,t_j) & 0 & \cdots & 0 & 0\\ 0 & v_2(t_k,t_j) & \cdots & 0 & 0\\ \vdots & \vdots & \ddots & \vdots & \vdots\\ 0 & 0 & \cdots & v_n(t_k,t_j) & 0\\ 0 & 0 & \cdots & 0 & 0\end{bmatrix}$$
$$\mathbf{y}(t_k)=\begin{bmatrix}-\sigma_1(t_k)\,v_1(t_k,t_0)\\ -\sigma_2(t_k)\,v_2(t_k,t_0)\\ \vdots\\ -\sigma_n(t_k)\,v_n(t_k,t_0)\\ N(t_k)\end{bmatrix}$$

here $v_i(t_k,t_j)=1/2\left[\delta_i(t_k,t_{j+1})-\delta_i(t_k,t_{j-1})\right]$

At the time moment t the elastic modulus of the layered composite along its layers can be obtained by using upper bound of Reuss' and Voigt's estimate or by using the so called rule of mixture [2-4]

$$E_c(t)=\sum_{i=1}^{n}E_i(t)V_i\Big/\sum_{i=1}^{n}V_i \tag{20}$$

where V_i is the volume fraction of the ith layer. By the given assumption the expression (20) is exact. For a greater convenience of calculation, the relationship (20) can be expressed in the following form

$$E_c(t)=\sum_{i=1}^{n}E_i(t)\,A_i\Big/\sum_{i=1}^{n}A_i \tag{21}$$

The elastic modulus of many materials, with respect to time, can be described by the equation

$$E(t)=E_\infty\left(1-\beta e^{-\alpha t}\right) \tag{22}$$

where E_∞ is the elastic modulus when $t\to\infty$; β and α are empirical values. By putting (22) into (21) and collecting of term, we obtain the following expression of elastic modulus of the layered composite along the layers

$$E_c(t) = E_{c,\infty}\left(1 - \frac{1}{B_{c,\infty}}\sum_{i=1}^{n} A_i E_{i,\infty}\beta_i\, e^{-\alpha_i t}\right) \quad (23)$$

where $E_{c,\infty} = \sum_{i=1}^{n} A_i E_{i,\infty} \Big/ \sum_{i=1}^{n} A_i$, $B_{c,\infty} = \sum_{i=1}^{n} A_i E_{i,\infty}$

If we know the elastic modulus of a layered composite and the total deformation, we can calculate the specific creep of the composite according to the Eq. (4). It should be noted that, according to the above-mentioned methods, deformations are found numerically and that is why evaluation of the specific creep by applying Eqs. (14), (15) and (19) is found only at the loading moment t_0. If the modulus of elasticity and specific creep of layered materials vary with respect to time, in order to calculate specific creep $C_c(t,\tau)$ of a multilayer composite for different loading moments, i.e. $C_c(t,t_1)$, $C_c(t,t_2)$, ..., $C_c(t,t_{z-1})$, we have to calculate the specific creep according to Eqs. (14), (15) and (19) by taking the loading start $t_1 > t_2 > t_3 > ... > t_{z-1}$ at time moments. The obtained numerical values of a composite specific creep can be approximated by analytical functions. According to method [27] it is possible to approximate the specific creep and relaxation coefficients of a non-aging composite. According to methods [28-30] it is possible to determine the specific creep of an aging composite.

3. Analysis of results

The proposed method was compared to the known analytical method [31] which allows calculation of deformation of a two-layered composite. According to this method, the creep function of layers is as follows

$$\delta_i(t - t_0) = 1/E_i + C_{\infty,i}\left(1 - e^{-\gamma_i(t-t_0)}\right), i = 1,2 \quad (24)$$

Fig. 2 plots the specific creep of a two-layered composite and its layers with respect to time, calculated according to exact analytical method [31] (dotted curve) and numerically according to (14) - (19) (diamond-shape dots) when the number of integration steps are equal to m=1. Fig. 2 a) plots curves when E_1=10 GPa, E_2=3 GPa, $C_{\infty,1}$=1·10^{-9} Pa, $C_{\infty,2}$=3·10^{-9} Pa, γ_1=0.2 and γ_2=0.4, whereas Fig. 2 b) plots curves when E_1=200 GPa, E_2=30 GPa, $C_{\infty,1}$=0.4·10^{-9} Pa, $C_{\infty,2}$=3·10^{-9} Pa, γ_1=0.2, γ_2=0.4. In both cases the axial loading and areas of the layers are as follows: N=0.2 Pa, A_1=A_2=0.1 m^2. As we can see from Fig. 2, only one integration step is enough to calculate the two-layer composite deformations rather precisely.

In numerical calculations based on Eqs. (14) - (15), it is very important to select such steps of time intervals which, on the one hand, would ensure that the error of integration would not be too large, and, on the other hand, would cause as few iterations as possible. When loading remains constant over time, stresses in layers change most quickly during the initial stage of loading, later stresses in layers change more slowly, and in the end they become steady. Therefore, when we apply rational calculus, the width of steps should depend on the stress changes. The simplest method to select the width of an integration step more rationally is to divide the measurement period into intervals of certain uneven width according to a preset rule.

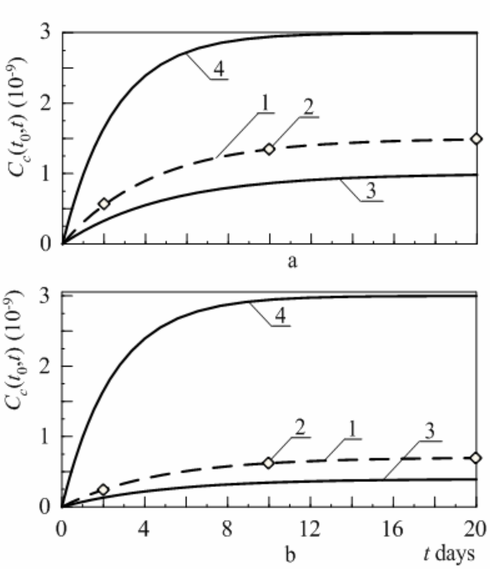

Fig. 2 Specific creep of two different composites calculated by the analytical method [31] - *1* and by the numerical method according to Eqs. (14)-(19), when the number of steps m=1, *2*, *3* and *4* are specific creep of the first and second layer respectively. A more detailed explanation is given in the text

The width of these intervals gradually increases from the shortest at the beginning of the loading to the longest at the loading end. The simplest dependency for calculation of the width of integration steps can be as follows

$$t_j = t_0 + t_m\,(m/j)^k \;,\; j = 1, 2, ..., m \quad (25)$$

where m is the number of steps, t_0 is the age of material at the loading, degree k = 1,2,3,…,n. It is clear that the bigger is k, the shorter are the initial time periods of loading and the longer are the final periods of loading.

Fig. 3 shows the convergence of a relative error of creep deformations of an axial compressed reinforced concrete element to 0 depending on the number m of integration steps and the degree k of the equation (25). The relative error of composite specific creep $\Delta C_c(m)$ and stress $\Delta\sigma_i(m)$ of its layers can be calculated in percents by the following relationship

$$\Delta C_c(m) = \frac{C_{c,m}(t,t_0) - C_{c,500}(t,t_0)}{C_{c,500}(t,t_0)}100 \quad (26)$$

$$\Delta\sigma_i(m) = \frac{\sigma_{i,m}(t,t_0) - \sigma_{i,500}(t,t_0)}{\sigma_{i,500}(t,t_0)}100, i = 1,2 \quad (27)$$

where $C_{c,m}(t,t_0)$, $C_{c,500}(t,t_0)$, $\sigma_{i,m}(t,t_0)$, $\sigma_{i,500}(t,t_0)$ are specific creep of the composite and stress of its layers calculated according to Eqs. (14) and (15) for m and 500 (i.e. m=500) iterations. The creep function of concrete was taken according to EC2 [32]. The parameters of this function are as follows: relative air humidity RH=80%, cement hardening coefficient - s=0.25, average compressive strength of concrete - f_{cm}=38 MPa, scale coefficient - h_0=133. The second layer - steel reinforcement, E_2= =200 GPa. Ratios of the cross-section areas of the layers - A_2/A_1=0.03. The age of the concrete at loading moment t_0=1 day, the duration of the loading - t=100 days.

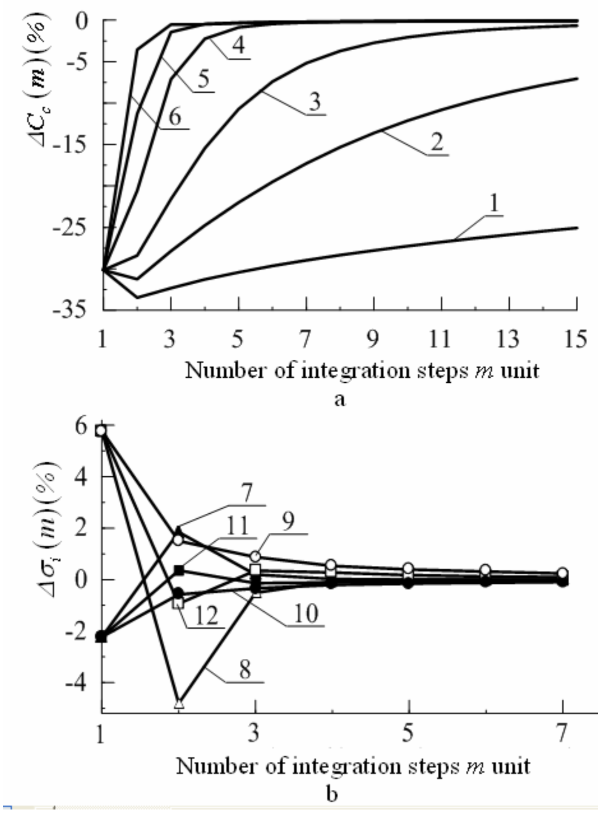

Fig. 3 Convergence of the relative errors of specific creep (a) and the stresses of the layers $\sigma_1(m)$ and $\sigma_2(m)$ (b) of an axially compressed reinforced concrete element to 0 depending on the number m of integration steps and the degree k of equation (25). *1* - when $k = 1$; *2* - when $k=2$; *3* - when $k=3$; *4* - when $k=5$; *5* - when k=7; *6* - when $k=10$; *7*, *8* - when $k=5$; *9*, *10* - when $k=7$; *11*, *12* - when $k=10$. Other explanations are given in the text

As we can see from Fig. 3, when the degree of equation (25) is equal to $k=5$, 7 or 10 and the number of integration steps is equal to $m=4$, specific creep of the composite and stresses of its layers, calculated according to proposed methods, are fairly accurate for practical applications. The relative error of calculation in this specific case did not exceed 2.23 %.

Fig. 4 shows the convergence of relative errors of specific creep and the stresses of layers of a similar element to 0, depending on the loading duration t. These relative errors were calculated by Eqs. (26) and (27). In this case it was taken that number of integration steps is equal to $m=4$ (Fig. 4, a)). When the duration of loading is equal to 5000 days, the relative error of calculation does not exceed 2 %. In practical calculations of reinforced concrete elements it is possible to assume that 5000 days of loading duration are equal to infinity. Therefore, when the number of integration steps is equal to $m=4$ and the degree k of formula (25) is equal to $k=10$, the strain and the stresses of a reinforced concrete element can be calculated rather exactly according to Eqs. (14), (15). In the case when the number of integration steps is equal to $m=20$ (Fig. 4, a)), the relative error of specific creep of composite does not exceed 1.05%.

The performed analysis shows that the elasticity modulus of a composite is higher than the lowest elasticity modulus of a layer and smaller than the highest elasticity modulus of a layer, i.e. (min $E_i(t)$) < $E_c(t)$ < (max $E_i(t)$). If the specific creep of a composite is higher than the lowest specific creep of a layered material and smaller than the highest specific creep of layered material, i.e. (min $C_i(t,\tau)$) < $C_c(t,\tau)$ < (max $C_i(t,\tau)$), the creep function of the composite will be between the lowest and highest creep functions of the layered material, i.e. (min $\delta_i(t,\tau)$) < $\delta_c(t,\tau)$ < (max $\delta_i(t,\tau)$), (Fig. 2, curves 1, 2, 3).

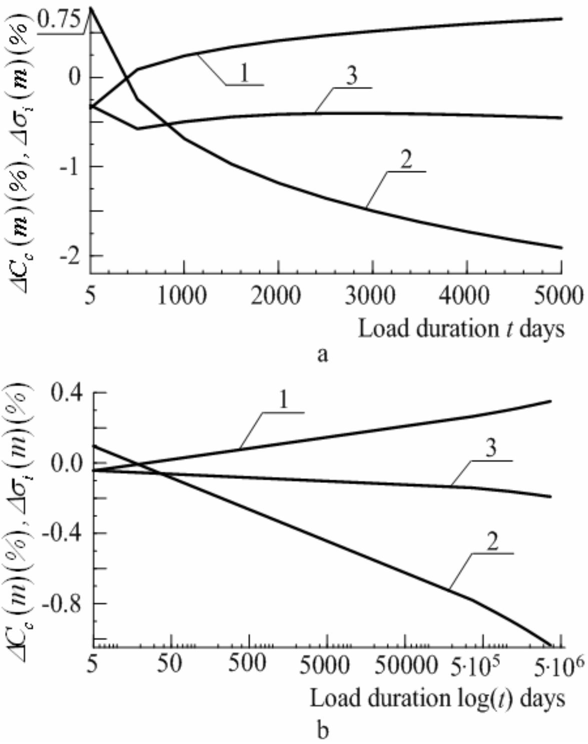

Fig. 4 The relative error of specific creep and the stresses of the layers of an axially compressed reinforced concrete element depending on the duration of loading when $k=10$ and $m=4$ - a or when $m=20$ - b. *1* and *2* are the relative errors of the stresses of the 1-st and the 2-nd layers respectively, *3* is the relative error of composite specific creep. Other explanations are given in the text

Another possible variant is that at a certain moment of loading specific creep of the composite will be higher than the highest specific creep of the layered material, i.e. (max $C_i(t,\tau)$) < $C_c(t,\tau)$ although $\delta_c(t,\tau)$ < max $\delta_i(t,\tau)$. Such a possibility is given in Fig. 5, when $E_1=10$ GPa, $E_2=1$ Gpa, $C_{\infty,1}=C_{\infty,2}=1\cdot10^{-9}$ Pa, $\gamma_1=0.2$, $\gamma_2=0.8$, areas of the layer $A_1=A_2=0.1$ m^2.

As we can see in Figs. 2 and 5, the creep deformations of the composite under constant loading always settle during a shorter period of time than the longest settling time of creep deformations of a composite material and longer than the shortest settling time of creep deformation of the same material.

The given examples show that if we select the properties of composite layers improperly, the creep and total deformations of a composite may be significantly increased and in this way its long term behaviour may be worsened.

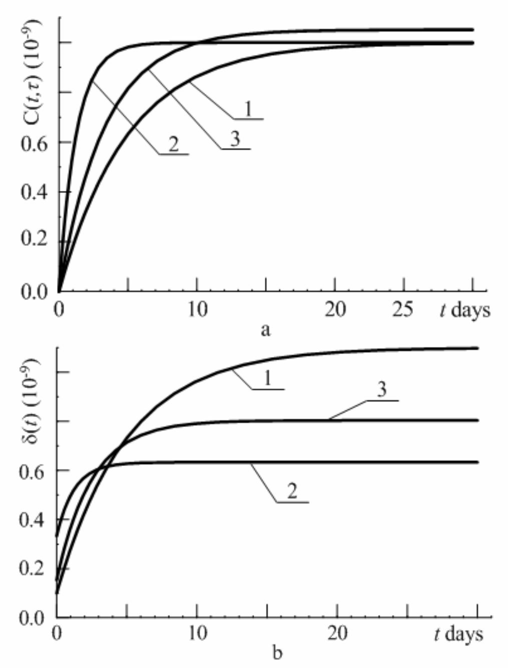

Fig. 5 Specific creep - a) and creep function - b) of a two layer composite calculated by numerical method according to Eqs. (14) and (15) - *3*, *1* and *2* are creep functions and the specific creep of the 1st and the 2nd layers respectively. The shape of the creep functions of the layers is Eq. (24), more details are given in the text

4. Conclusions

1. Having compared the known analytical method to the proposed numerical method, it was determined that the developed numerical method allows calculating the deformations of a layered composite element and stresses of its layers fairly precisely.

2. If the creep function of concrete is according to EC 2, the number of integration steps is equal to four and the coefficient k of formula (25) is equal to k=10, the reinforced concrete is also under constant axial compressing loading, the relative error of the stresses and deformations of the concrete and reinforcement is less than 5 %.

3. When the number of integration steps is equal to twenty and the coefficient k of formula (25) is equal to k=10, the relative error of specific creep of the above mentioned reinforced element, is less than 1.5%.

4. In some cases the specific creep of layered composites can be higher than the highest specific creep of the layers materials.

5. The creep function of some layered composites at a given moments of time can be greater than the highest creep function of the layers of materials.

6. When a layered composite is under constant axial loading, its creep deformations settle more quickly than the longest settling time of the creep deformation of the layered materials of this composite and more slowly than the shortest settling time of the creep deformation of its layers.

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SLUOKSNIUOTŲJŲ KOMPOZITŲ VALKŠNUMO PARAMETRŲ SKAITINIS MODELIAVIMAS

R e z i u m ė

Darbe pasiūlyta sluoksniuotųjų kompozitų tiesinių valkšnumo parametrų modeliavimo metodika sluoksnių sujungimo kryptimi. Parengtoji metodika paremta skaitiniu integralinės Volterra antros eilės lygties integravimu prieš integralo ženklą iškeliant įtempių vidurkį. Žinomą tikslų analizinį dvisluoksnio kompozito deformacijų apskaičiavimo metodą palyginus su skaitiniu, nustatyta, kad sudarytu skaitiniu metodu galima tiksliai apskaičiuoti sluoksniuoto kompozito deformacijas. Nustatyta, kad kai kuriais atvejais kompozito valkšnumo matas gali būti didesnis už kiekvieno atskirai paimto sluoksnio medžiagos valkšnumo matą, taip pat kai kuriais atvejais, tam tikrą trumpą laiko tarpą, kompozito valkšnumo funkcija gali būti didesnė už kiekvieno atskirai paimto sluoksnio medžiagos valkšnumo funkciją.

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NUMERICAL MODELING OF CREEP FUNCTIONS OF LAMINATED COMPOSITES

S u m m a r y

This research paper proposes the simulation method of linear creep parameters of laminated composites when the composite is loaded along the joint of the layers. The developed method is based on the numerical integration of the second kind Volterra integral equation taking out the stress mean before the integral. Having compared the creep deformations calculated by the known exact analytical solution and by the proposed method it has been determined that the proposed method allows to calculate creep deformation of composite materials fairly close to the exact value. It has been determined that in some cases the specific creep of a composite may be higher than the specific creep of each layer of the composite material Also, in some cases for a certain short period of loading time the creep function of a composite may be higher than the creep functions of each layer of the composite material.

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ЧИСЛЕННОЕ МОДЕЛИРОВАНИЕ ПАРАМЕТРОВ ПОЛЗУЧЕСТИ СЛОИСТЫХ КОМПОЗИТОВ

Р е з ю м е

В статье приведена методика численного моделирования параметров линейной ползучести слоистых композитов, когда нагрузка действует вдоль слоев. Предложенная методика основана на численном интегрировании интегрального уравнения Волтерра второго рода, когда перед интегралом выносится среднее значение напряжения. После сравнения результатов полученных с помощью предложенного метода и известного аналитического решения в явном виде, установлено, что приведенная методика позволяет точно рассчитать деформации ползучести слоистых композитов. Установлено, что в некоторых случаях мера ползучести слоистого композита может быть больше меры ползучести материала каждого отдельно взятого слоя, а также в некоторых случаях в короткий промежуток времени функция ползучести слоистого композита может быть больше функции ползучести материала каждого отдельно взятого слоя.

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