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Analysis of compression zone parameters of cross-section in flexural reinforced concrete members according to EC2 and STR 2.05.05

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1. Introduction

It is known that for the analysis of normal sections of flexural, eccentrically tensioned and eccentrically compressed members according to acting regulations STR 2.05.05:2005 [1] and EC2 [2] different stress-strain (σ_c-ε_c) diagrams for concrete in compression may be used: parabola with descending branch, parabola-rectangle and bi-linear. Rectangular diagram for the stresses in concrete compression zone can be applied as well. For engineering applications, direct application of nonlinear stress diagrams is too complicated and thus inconvenient. For simplification of the analysis in many codes [1-7] rectangular stress diagram is substituted for nonlinear stress diagram in concrete compression zone. Coefficients applied for the substitution of these diagrams have to ensure this substitution to be equivalent, i.e. carrying capacity of the compression zone calculated using both nonlinear and rectangular stress diagrams should be the same.

Articles [8, 9] deal' with the substitution of rectangular stress diagram of compression zone in normal section of flexural members for nonlinear stress diagram. A general method making it possible to perform equivalent substitution of the diagrams was presented. Using mentioned method equivalency of the substitution of said diagrams was analyzed, i.e. concurrence of centers of the diagrams and equality of resultants of these diagrams. In article [9] equivalency of the substitution of rectangular stress diagram for parabola-rectangle one according to STR [1], EC2 [2], DIN [3] and SNB [5] was considered. It was determined that replacement of the diagrams according different methods of the codes for reinforced concrete structures is not quite equivalent. I.e. the area of nonlinear stress diagram to be replaced and that of the rectangular stress diagram are not equal and coordinates of the centers for these diagrams are not equal either. It will be observed that the most equivalent from all investigated diagram substitution methods was found to be the substitution of rectangular diagram for parabola- rectangle stress diagram according to DIN [3].

Though normal section of flexural members in codes [1-3] can be analyzed using the presented in EC2 σ_c-ε_c diagrams for concrete in compression: parabola, parabola- rectangle and bi-linear, but coefficients of rectangular stress diagram of concrete compression zone in the mentioned codes are different. Besides the mentioned codes in some methods [10, 11], coefficients ostensibly allowing equivalent substitution of rectangular diagrams for the said nonlinear diagrams are presented as well. These coefficients for the same σ_c-ε_c diagrams according to different methods are different as well. Therefore, in the article coefficients for rectangular stress diagram used in various regulations and methods are analyzed. These coefficients are compared with experimental data presented by other authors.

2. The main dependences

For rectangular cross-section members the compression zone resultant (F_c) and its moment (M_c) in relation to the stress resultant in tensile reinforcement of reinforced concrete flexural members are determined according to the following well-known general dependences

$$F_c = b\int_{d-x}^{d} \sigma_c(z)dz \tag{1}$$

$$M_c = b\int_{d-x}^{d} z\sigma_c(z)dz \tag{2}$$

where σ_c is concrete stress distribution function in concrete compression zone, z is a coordinate in coordinate system YOZ (Fig. 1), x is compression zone depth, b is the width of cross-section. It is obvious that direct application of nonlinear diagram for stress in compression zone is inconvenient since integration is required. For example the universal method of the integration proposed in [12, 13] is complicated in comparison with a case when a rectangular stress diagram is used. Therefore, for simplification of the calculation rectangular diagram is substituted for nonlinear stress diagram in compression zone.

In the case when rectangular stress diagram is used, resultant F_c of stress in concrete compression zone and its moment M_c about stress resultant in tensile reinforcement are determined according to such known general dependences

$$F_c = \eta f_c b\lambda x = \eta f_c b\lambda \xi d \tag{3}$$

$$M_c = \eta f_c b\lambda x(d - 0.5\lambda x) = \eta f_c b\lambda d^2 \xi(1 - 0.5\lambda\xi) \tag{4}$$

where f_c is compressive strength of concrete, d is the distance between the top of a beam and stress resultant in tensile reinforcement (Fig. 1, a), ξ is relative compression zone depth $\xi = x/d$. Coefficients η and λ by means of which the width and depth of rectangular stress diagram are changed in such a way that the areas of rectangular stress diagram and of equivalent to it nonlinear stress diagram would be equal, coordinates of gravity centers for these diagrams would be equal as well (Fig. 1). Other meaning of coefficients η and λ is as follows. Coefficient η is the ratio of areas of nonlinear diagram and of equivalent to it rectangular diagram while the depth of such rectangular stress diagram is equal to λx and the width is the same as that of nonlinear diagram, i.e. f_c [8, 9]. Coefficient η also can be treated as the ratio of widths of rectangular and of equivalent to it nonlinear stress diagram. Coefficient λ is the ratio of depths of rectangular and of equivalent to it nonlinear stress diagram [14] or the depth of rectangular stress diagram described in normalized coordinates, i.e. $\lambda = x_{eff}$ when $x = 1$. Some methods [10, 11, 15] and other, especially composed on the basis of DIN 1045, instead of separate coefficients η and λ present their product $\eta\lambda$. In many methods, e.g. [10, 11], instead of coefficient λ, a coefficient corresponding to 0.5λ is given.

Coefficients η and λ can be determined in such way [8, 9]

$$\eta = F_c^2 / 2S_c \tag{5}$$

$$\lambda = 2S_c / F_c \tag{6}$$

here F_c is compression zone resultant and S_c is the moment of resultant F_c about the layer of concrete under the highest compression. In the case of parabola with descending branch diagram for rectangular cross-section the compression zones F_c and S_c are such [8]

$$F_{c,parab} = f_c x \left[\frac{(k-1)^2}{(k-2)^2} - \frac{1}{2(k-2)\varepsilon_{c1}/\varepsilon_{cu1}} + \frac{\varepsilon_{c1}/\varepsilon_{cu1}(k-1)^2 \left(ln(\varepsilon_{c1}/\varepsilon_{cu1}) - ln(k+\varepsilon_{c1}/\varepsilon_{cu1}-2) \right)}{(k-2)^4} \right] \tag{7}$$

$$S_{c,parab} = f_c x^2 \left[\frac{(k-1)^2 \left(k + 2\frac{\varepsilon_{c1}}{\varepsilon_{cu1}} - 2 \right)}{(k-2)^3} - \frac{1}{6(k-2)(\varepsilon_{c1}/\varepsilon_{cu1})} + \frac{\varepsilon_{c1}/\varepsilon_{cu1}(k-1)^2 \left(ln(\varepsilon_{c1}/\varepsilon_{cu1}) - ln(k+\varepsilon_{c1}/\varepsilon_{cu}1-2) \right)}{(k-2)^4} \right] \tag{8}$$

where x is the depth of concrete compression zone, ε_{c1} and ε_{cu1} are the strain at the maximum stress and the ultimate strain of the compressive concrete for the parabola σ_c-ε_c diagram with descending branch respectively. These coefficients and other coefficient k can be found in EC2 and STR [1].

In the case of parabola-rectangle diagram for rectangular cross-section compression zones F_c and S_c are such [9]

$$F_{c,parab-rec} = f_c x \left(1 - \frac{\varepsilon_{c2}}{\varepsilon_{cu2}(n+1)} \right) \tag{9}$$

$$S_{c,parab-rec} = 0.5 f_c x^2 \left[(1-\varepsilon_{c2}/\varepsilon_{cu2})^2 - \frac{\varepsilon_{c2} n \left(\varepsilon_{c2}/\varepsilon_{cu2}(3+n) - 2n - 4 \right)}{\varepsilon_{cu2}\left(2+3n+n^2\right)} \right] \tag{10}$$

here ε_{c2} and ε_{cu2} are strain at the maximum stress and the ultimate strain of the compressive concrete for the parabola-rectangle σ_c-ε_c diagram respectively. These coefficients and factor n can be found in EC2 and STR [1] as well.

According to EC2 for rectangular stress diagram coefficients are determined by the following formulae

$$\left. \begin{aligned} &\eta_{EC2} = 1.0, \text{ when } f_{ck} \le 50 \text{ MPa} \\ &\eta_{EC2} = 1.0 - (f_{ck}-50)/200, \text{ when } f_{ck} > 50 \text{ MPa} \end{aligned} \right\} \tag{11}$$

$$\left. \begin{aligned} &\lambda_{EC2} = 0.8, \text{ when } f_{ck} \le 50 \text{ MPa} \\ &\lambda_{EC2} = 0.8 - (f_{ck}-50)/400, \text{ when } f_{ck} > 50 \text{ MPa} \end{aligned} \right\} \tag{12}$$

According to STR [1] the coefficients for rectangular stress diagram are described not in the same way as it is in EC2, ACI 318 or DIN 1045, but by the physical sense the coefficient for concrete design strength α and concrete compression zone deformability factor ω correspond with η and λ coefficients [14]. These coefficients are calculated using such formulae [1]

$$\left. \begin{aligned} &\eta_{STR} = \alpha = 0.9, \text{ when } f_{ck} \le 50 \text{ MPa} \\ &\eta_{STR} = \alpha = 0.9 - (f_{ck}-50)/200, \text{ when } f_{ck} > 50 \text{ MPa} \end{aligned} \right\} \tag{13}$$

$$\lambda_{STR} = \omega = a - 0.008 f_{cd} \tag{14}$$

here a is the coefficient allowed for concrete type: for normal-weight concrete $a = 0.85$, for fine grain Group A concrete $a = 0.80$, for fine grain Group B concrete $a = 0.75$, for light-weight concrete $a = 0.80$, f_{cd} is design concrete strength in MPa

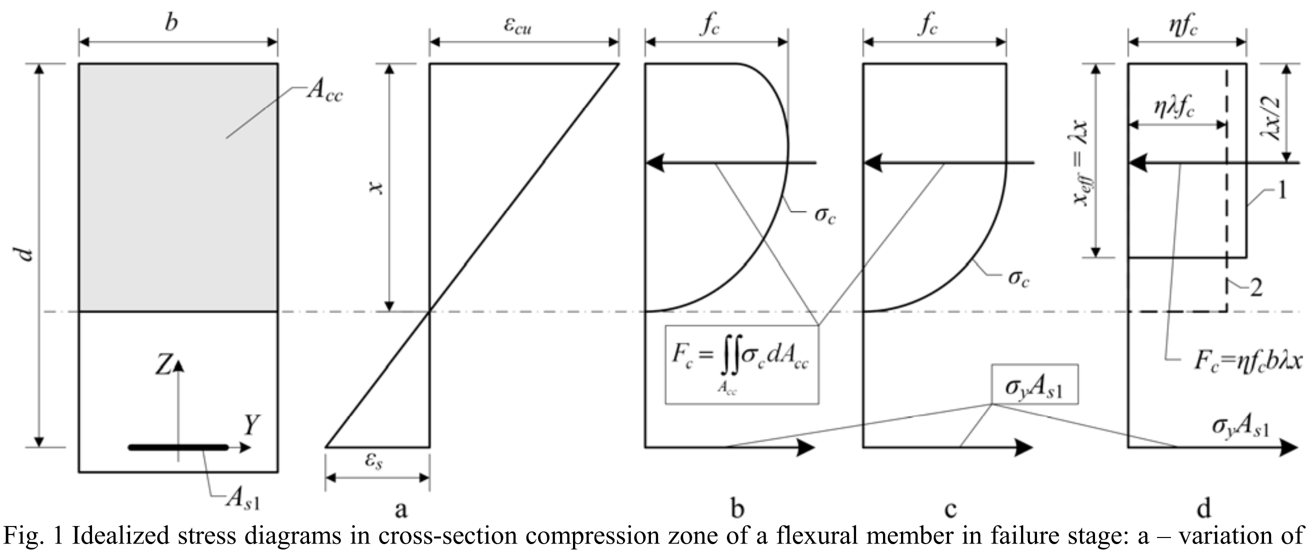

Fig. 1 Idealized stress diagrams in cross-section compression zone of a flexural member in failure stage: a – variation of deformations along cross-section height, b – parabola with descending branch diagram, c – parabola-rectangle diagram, d – rectangular stress diagrams: *1* – equivalent *2* – not equivalent

Table

Coefficients for rectangular stress diagram

Coeffi-cients	C8/10	C12/15	C16/20	C20/25	C25/30	C30/37	C35/45	C40/50	C45/55	C50/60	C55/67	C60/75	C70/85	C80/95	C90/105
parabola with descending branch diagram coefficients by [8] or (5) – (8)															
η_{parab}	0.852	0.832	0.844	0.858	0.863	0.875	0.869	0.864	0.880	0.878	0.907	0.905	0.87	0.85	0.839
λ_{parab}	0.934	0.924	0.907	0.889	0.872	0.855	0.847	0.839	0.821	0.813	0.773	0.746	0.719	0.709	0.700
parabola–rectangle diagram coefficients by [9], or (5), (6), (9), (10)															
$\eta_{parab\text{-}rect}$	0.973										0.947	0.921	0.877	0.845	0.826
$\lambda_{parab\text{-}rect}$	0.832										0.784	0.754	0.724	0.710	0.706
rectangular diagram according to STR 2.05.05:2005 [1] coefficients by (13) and (14)															
η_{STR}						0.90					0.875	0.850	0.800	0.750	0.700
λ_{STR}	0.807	0.786	0.765	0.743	0.717	0.690	0.663	0.637	0.610	0.583	0.560	0.536	0.492	0.449	0.408
rectangular diagram according to EC2 coefficients by (11) and (12)															
η_{EC2}	1										0.975	0.950	0.900	0.850	0.800
λ_{EC2}	0.800										0.788	0.775	0.750	0.725	0.700

$$\left.\begin{aligned} f_{cd} &= \frac{\alpha}{\gamma_c} f_{ck} = \frac{\alpha}{1.5} f_{ck}, \text{when } f_{ck} \le 50\text{MPa} \\ f_{cd} &= \frac{\alpha}{\gamma_c} f_{ck} = \frac{\alpha f_{ck}}{1.5/(1.1 - f_{ck}/500)}, \text{when } f_{ck} > 50\text{MPa} \end{aligned}\right\} \quad (15)$$

where α is coefficient depending on the stress diagram in concrete compression zone. For rectangular stress diagram $\alpha = 0.9$, for nonlinear stress diagram $\alpha = 1.0$. However, in our analysis coefficient α is the same as the coefficient η in Eqs. (3) and (4). Therefore in calculations of f_{cd} according to formulae (15) $\alpha = 1.0$ will be taken. Coefficients η and λ calculated according to formulae (5) - (15) when $a = 0.85$ are given in Table.

Wide known are formulae proposed by H. Rüsch [15] for the determination of coefficients $\eta\lambda$ and 0.5λ

$$\eta\lambda = \alpha_R = \varepsilon_{cu}(6 - \varepsilon_{cu})/12, \text{ when } \varepsilon_{cu} \le 2\% \quad (16)$$

$$\eta\lambda = \alpha_R = \frac{3\varepsilon_{cu} - 2}{3\varepsilon_{cu}}, \text{when } (2 \le \varepsilon_{cu} \le 3.5)\% \quad (17)$$

$$0.5\lambda = k_a = \frac{8 - \varepsilon_{cu}}{4(6 - \varepsilon_{cu})}, \text{when } \varepsilon_{cu} \le 2\% \quad (18)$$

$$0.5\lambda = k_a = \frac{\varepsilon_{cu}(3\varepsilon_{cu} - 4) + 2}{2\varepsilon_{cu}(3\varepsilon_{cu} - 2)}, \text{ when } (2 \le \varepsilon_{cu} \le 3.5)\% \quad (19)$$

here ε_{cu} is in %.

Performed analysis revealed that coefficients $\eta\lambda$ and 0.5λ calculated using Eqs. (16) - (19) correspond to the products $\eta_{parab\text{-}rect}\lambda_{parab\text{-}rect}$ and $0.5\lambda_{parab\text{-}rect}$ of coefficients η and λ for parabola- rectangle stress diagram within the interval of $(8 \le f_{ck} \le 50)$ MPa, i.e. when $\varepsilon_{c2} = \varepsilon_{cu2} = 3.5\cdot10^{-3}$. In the case of higher concrete classes coefficients $\eta\lambda$ and 0.5λ calculated using Eqs. (16) - (19) do not correspond with the products $\eta_{parab\text{-}rect}\lambda_{parab\text{-}rect}$ and $0.5\lambda_{parab\text{-}rect}$ of coefficients for parabola-rectangle stress diagram. For example, when $(8 \le f_{ck} \le 50)$ MPa $\eta\lambda$ and 0.5λ values calculated using Eqs. (17) and (19) correspond with the values of $\eta_{parab\text{-}rect}\lambda_{parab\text{-}rect}$ and $0.5\lambda_{parab\text{-}rect}$, $\eta\lambda = 0.973\cdot0.832 = 0.81$, $0.5\lambda = 0.416 = 0.832/2$ (Table). However, in the case of high concrete classes, e.g. when $f_{ck} = 90$ MPa: $0.5\lambda = 0.394 \ne 0.5\lambda_{parab\text{-}rect} = 0.706/2 = 0.353$, and $\eta\lambda = 0.744 \ne \eta_{parab\text{-}rect}\lambda_{parab\text{-}rect} = 0.826\cdot0.706 = 0.5683$. In methods [10, 11] for the substitution of rectangular stress diagram for parabola-rectangle stress diagram just the coefficients calculated according to Eqs. (16) - (19) are used.

According to [3] when $(8 \le f_{ck} \le 50)$ MPa $\lambda = 0.95$, $\eta = 0.8$. One can see that coefficients for the same stress diagram differ not only in regulatory literature but in other methods of analysis as well.

3. Analysis of coefficients for rectangular stress diagram

Coefficients for rectangular stress diagram presented in previous chapter are obtained analytically. However, coefficients for rectangular stress diagram in ACI 318 [4, 16] are obtained experimentally by means of eccentric compression of columns in such a way that one face of the column always remains un-deformed (Fig. 2).

In many publications, e.g. [16 - 19], experimentally determined products of coefficients k_1k_3 and relative coordinates k_2 of compression zone resultant are calculated using such formulae [17, 19]

$$k_3k_1 = (F_1 + F_2)/(f_c bx) \quad (20)$$

$$k_2 = 1 - \frac{F_1a_1 + F_2a_2}{(F_1 + F_2)x} \quad (21)$$

here $k_1 = f_{av}/\sigma_{max}$ is the ratio of average stress to the maximum stress, $k_2 = 0.5x_{eff}/x$ is the ratio of resultant coordinate to compression zone depth, $k_3 = \sigma_{max}/f_c$ is the ratio of the maximum stress to cylindrical strength [16, 17]. It can be seen from (20) that $k_1k_3 = f_{av}/f_c$. If according to ACI 318 it is taken that the area of rectangular stress diagram in compression zone is equal to k_3f_c then the depth of rectangular stress diagram is equal to k_1x. It indicates that coefficient k_1 is not a ratio between the depths of nonlinear and of rectangular equivalent to it stress diagrams, i.e. $k_1 \ne x_{eff}/x$ when $x_{eff} = 2k_2x$. However, in [16] it is just treated that $k_1 = x_{eff}/x$ when $x_{eff} = 2k_2x$. In reference [20] coefficient k_1 is treated otherwise, as the ratio between areas of nonlinear and rectangular stress diagrams when rectangular stress diagram depth is equal to the depth of nonlinear stress diagram. According to this consideration the coefficient corresponds with the product of coefficients η and λ, i.e. $k_1 = \eta\lambda$ (Fig. 1, d).

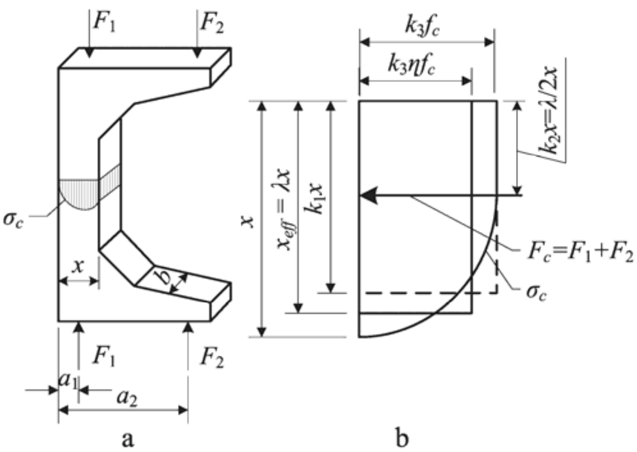

Fig. 2 Column under eccentric compression for the determination of coefficients for rectangular stress diagram – a; concrete stress diagram in cross-section of the column – b

We will prove that $k_1 \neq x_{eff}/x$ when $x_{eff} = 2k_2x$. For the sake of briefness we will introduce notation $F_{tot} = F_1+F_2$. From (20) we obtain

$$k_1 = F_{tot}/(k_3 f_c bx) \tag{22}$$

If according to [16] $x_{eff} = 2k_2x$ and $k_1 = x_{eff}/x$ then $k_1 = 2k_2$ and taking in to account (22) and $k_1 = 2k_2$ equation (21) can be written in such form

$$\frac{1}{2}\frac{F_{tot}}{k_3 f_c bx} = 1 - \frac{F_1 a_1 + F_2 a_2}{(F_1+F_2)x} \tag{23}$$

Or after rearrangement, in such form

$$\frac{1}{2}\frac{F_{tot}^{\ 2}}{k_3 f_c bx} = F_{tot} - \frac{F_1 a_1 + F_2 a_2}{x} \tag{24}$$

Let us examine separate member $F_{tot}^2/(k_3 f_c bx)$. Since $F_{tot}/(bx) = \sigma_{max}$, and $f_c = \sigma_{max}/k_3$ then $F_{tot}^2/(k_3 f_c bx) = F_{tot}$. Then equation (24) becomes such

$$0.5F_{tot} = F_{tot} - (F_1 a_1 + F_2 a_2)/x \tag{25}$$

Rearrangement of (25) gives $0.5F_{tot}x = (F_1a_1 - F_2a_2)$. Since $a_1 = 0.5x$ and $F_{tot} = F_1+F_2$ then (25) after rearrangement and collecting of terms may be written in the form of $F_2a_2 = 0.5F_2x$ either $a_2 = 0.5x$ or $a_2 = a_1$. It shows that (23) is correct only when $a_2 = a_1$, i.e. both forces are applied in the center of column cross-section. Naturally it is not correct. Then the statement that $k_1 = x_{eff}/x$ when $x_{eff} = 2k_2x$ as it is considered in [16] is not correct. Thus $k_1 \neq x_{eff}/x$, when $x_{eff} = 2k_2x$. Actually $k_1x < x_{eff} = \lambda x = 2k_2x$ (Fig. 2). An important conclusion can be made here that coefficients k_3k_1 and k_2 calculated using Eqs. (20) and (21) do not provide equivalent replacement of the diagrams. We shall stress that the rectangular diagram is equivalent to the nonlinear diagram only when the area or resultant of the rectangular stress diagram is equal to the area or resultant of the nonlinear stress diagram and coordinates of gravity centers of these diagrams are equal. It should be stressed that coefficients for rectangular diagrams are calculated according to Eqs. (20) and (21) in many references [16, 19, 21]. Coefficients for rectangular stress diagram are being determined for compression zone of high strength concrete, fiber concrete beams. Nevertheless we will stress once more that k_3f_c wide and k_2x deep rectangular stress diagram is not equivalent to nonlinear stress diagram according to the definition of equivalency of the diagrams presented in this article. Although ACI 318 and other references [4, 16, 17, 19, 21] consider that rectangular stress diagram mentioned above is equivalent. Actual physical resultant of such rectangular stress diagram does not coincide with actual resultant of nonlinear stress diagram.

We will show how it is possible to calculate coefficients for equivalent rectangular stress diagram when values of coefficients k_1k_3, k_3 and $k_2 = \lambda/2$ presented in many publications are given. Let the maximum stress in nonlinear stress diagram is $\sigma_{max} = f_ck_3$. Then resultant of compression zone

$$F_c = F_{tot} = F_1 + F_2 = k_3 f_c k_1 xb \tag{26}$$

Resultant of equivalent rectangular stress diagram is such

$$F_c = F_{tot} = F_1 + F_2 = k_3 \eta f_c \lambda xb \tag{27}$$

Comparison of (26) and (27) after collecting of terms gives

$$\eta = k_1/\lambda \tag{28}$$

Since $\lambda = 2k_2$ [8, 9] then (28) will take the form

$$\eta = \frac{k_1}{\lambda} = \frac{k_1}{2k_2} = \frac{k_1k_3}{2k_2k_3} \tag{29}$$

Theoretically calculated coefficients for stress diagrams are given in Table and experimentally determined coefficients for rectangular stress diagram according to [16, 17, 19] are shown in Fig. 3. Line 3 in Fig. 3, a is drawn using Eq. (29) when coefficients k_1 and k_2 are calculated according the following formulae given in [17]

$$k_1 = 0.94 - 5.578\cdot10^{-3} f_{cm} \tag{30}$$

$$k_2 = 0.5 - 1.813\cdot10^{-3} f_{cm} \tag{31}$$

where f_{cm} is the average strength of concrete ($7 \le f_{cm} \le 55$) MPa. Line 3 shown in Fig. 3, a is drawn according to (31).

In Fig. 3, a circles and rhombs (○ and ◊) indicate coefficients η calculated using formula (29) when k_2, k_1k_3 and k_3 are taken from [16, 19]. In Fig. 3, b the values of coefficients k_2 or $\lambda/2$ taken from [16, 19] are denoted by circles and rhombs (○ and ◊) respectively. In these publications coefficients k_2, k_1k_3 and k_3 are calculated according to formulae (20) and (21), test diagram is shown in Fig. 2. It should be stressed that the nonlinear denoted in this figure by ACI 318 is empirical one [4, 16]

$$\left.\begin{array}{l} \alpha_1 = \eta_{ACI} = 0.85 \\ \beta_1 = \lambda_{ACI} = 0.85, \text{ if } f_{cm} \le 27.6\,\text{MPa} \\ \text{if } f_c > 27.6 \text{ MPa} \\ \beta_1 = \lambda_{ACI} = 0.85 - 0.05/6.9(f_{cm} - 27.6) \ge 0.65 \end{array}\right\} \quad (32)$$

We shall stress that according to ACI 318 η_{ACI} and λ_{ACI} are denoted by symbols of α_1 and β_1.

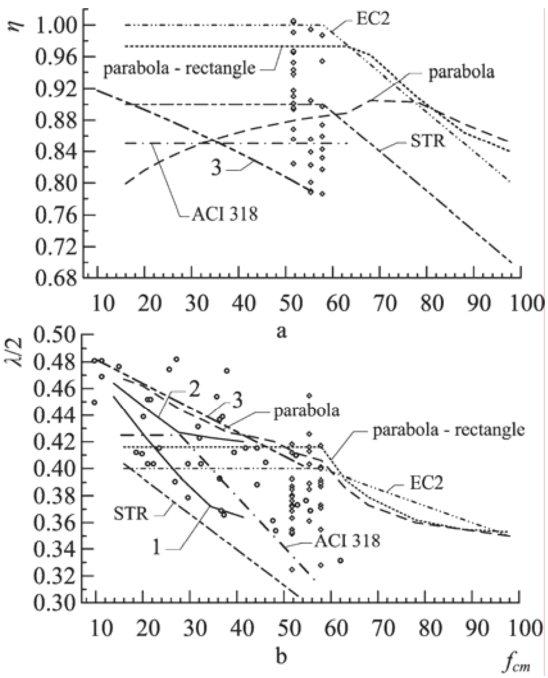

Fig. 3 Ratios of rectangular to nonlinear stress diagram widths – a and ratios between coordinates of gravity centers of these diagrams – b. *1 - 3* – empirical dependences obtained according to experimental data, *1*, *2* according to [16], *3* according to [17], ○ and ◊ – experimental data of various authors

It can be seen in Fig. 3, b that within the interval of ($7 \le f_{cm} \le 55$) MPa line 3 according to Eq. (31) of [17] practically corresponds with theoretically calculated nonlinear $\lambda_{parab}/2$. In this figure also it can be seen that the dependence denoted by line 3 may be quite accurately extrapolated towards the side of the higher concrete strength values. For comparison with (31) linear approximation of coefficient λ_{parab} given in [8] is presented: $\lambda = 0.987\text{-}3.1\cdot10^{-3}f_c$, then $\lambda/2 = 0.493\text{-}1.55\cdot10^{-3}f_c$. It is seen that the said dependences differ not much.

Fig. 3, a shows that the difference between empirically determined and theoretically calculated values of coefficient η is quite substantial. Coefficient η_{parab} increases with the growth of concrete class from 16 to 75 MPa while other coefficients do not change or decrease. Coefficient η_{parab} differs quite greatly from dependence denoted by line 3.

Fig. 3 shows that concrete compression zone coefficients η and $\lambda/2$ vary in general within quite wide limits for the same strength of the concrete. In reference [19] on the basis of experimental data it is stated that coefficients η and λ are influenced by the scale factor. Therefore it is complicated to give unambiguous answer to the question what accurate values of coefficients η and λ should be. It is clearly seen in the Fig. 3 that values of coefficient $\lambda_{STR}/2$ are the least ones. When f_{cm} = 90 MPa, $\lambda_{STR}/2 = 0.204$, and it is much less in comparison with other coefficients.

We shall demonstrate that such small values of coefficient λ_{STR} for equivalent stress diagram are impossible when hypothesis of plane sections is valid. It is known that in the case of high strength concrete stress diagram of compression zone is close to triangle. For the limit case it can be assumed that the stress diagram of compression zone concrete is triangle. Gravity center coordinate for such diagram is equal to $1/3x$, here x is the depth of triangular stress diagram. Gravity center coordinate for equivalent diagram has to coincide with the gravity center coordinate of triangular stress diagram and the areas of these diagrams have to be equal as well. Then the depth of equivalent rectangular diagram is $x_{eff} = 2/3x$. Hence ratio λ_{triang} between depths of triangular and of equivalent to it rectangular stress diagrams is obtained

$$\lambda_{triang} = x_{eff} / x = 2/3 \approx 0.667 \quad (33)$$

Equality of equivalent diagrams is as follows

$$1/2\, f_c x = 2/3\, f_c \eta_{triang} x \quad (34)$$

Hence it is found that the ratio of widths of the diagrams η_{triang} is such

$$\eta_{triang} = 3/4 = 0.75 \quad (35)$$

Thus it is obvious that the limit values of ratios for widths and depths of the diagrams are 0.75 and 0.667, and in all cases the product of these ratios cannot be less than $0.75\cdot0.667 \approx 0.5$. However $\eta_{STR} < 0.75$ when $f_{ck} > 80$ MPa, while $\lambda_{STR} < 0.667$ when $f_{ck} \ge 35$ MPa and λ_{STR} is substantially less than 0.667 in the case of high concrete classes (Table). According to ACI 318 also $min(\lambda_{ACI}) = 0.65 < 0.667$. The smallest value of η according to Canadian code [22] is 0.67, it is less than the limit value $\eta_{triang} = 0.75$ as well. Some in ACI 318 denoted values of coefficients α_1 and β_1 or $2k_2$ experimentally determined and theoretically defined and corresponding coefficients η and λ [19, 23, 24] are less than the values of (33) and (35) respectively

Carrying capacity of reinforced concrete member depends on limit depth of compression zone when failure of flexural member takes place due to crushing of concrete and simultaneous reaching the yield stress limit in reinforcement. Therefore, hereafter we are going to compare limit values of compression zone depths determined on the basis of the hypothesis of plane sections and according to formulae presented in STR [1].

4. Analysis of compression zone limit

In cross-section the limit state calculations of flexural, eccentrically compressed and tensioned members according to EC2 and STR [1] limit values of concrete compression strains ε_{cu2} or ε_{cu3} are applied corresponding to whether the parabola-rectangle diagram of σ_c-ε_c or bilinear diagram of σ_c-ε_c is used. Deformation of tensile reinforcement is applied equal to ε_{ud} which corresponds to the highest stress in reinforcement. When accurate value of ε_{ud} is not known EC2 recommends to take $\varepsilon_{ud} = 0.02$. However, when this value of ε_{ud} is used the limit value of relative compression zone depth does not exceed 0.15. It is very small value. Therefore in many methods, e.g. [7, 10, 25, 26], for the case of nonprestrressed tensile reinforcement the limit value of compression zone depth is calculated taking deformations corresponding to the yield limit of reinforcement $\varepsilon_{yd} = f_{yd}/E_s$. Here f_{yd} and E_s are design strength and elasticity modulus of tensile reinforcement. On the basis of assumption used in EC2 that hypothesis of plane section is valid the limit value of relative compression zone depth ξ_{lim} is calculated in this way [7, 10, 25, 26]

$$\xi_{lim,EC2} = \frac{1}{1+\varepsilon_{yd}/\varepsilon_{cu}} = \frac{1}{1+f_{yd}/(E_s\varepsilon_{cu})} \tag{36}$$

here $\xi_{lim,EC2} \in \{\xi_{lim,parab}, \xi_{lim,parab\text{-}rect}\}$, ε_{cu} is concrete in compression ultimate (limit) strain $\varepsilon_{cu} \in \{\varepsilon_{cu1}, \varepsilon_{cu2}\}$, $\varepsilon_{cu} = \varepsilon_{cu1}$ when parabola diagram with descending branch is used, $\varepsilon_{cu} = \varepsilon_{cu2}$ when parabola-rectangle diagram is used, ε_{yd} is strain of tensile reinforcement at yield stress or strain at conventional yield stress, f_{yd} is design strength of tensile reinforcement

$$f_{yd} = f_{yk}/1.1 \tag{37}$$

It should be remarked that the hypothesis of plane section for the section through the crack is not entirely correct since reinforcement in concrete slips under the action of great internal force. Therefore when yield stress is reached average reinforcement strain at the crack is greater than f_y/E_s. Here f_y is tensile reinforcement strength. Otherwise the hypothesis of plane sections is not entirely correct and due to the fact that in compression zone of the section through the crack warping of the cross-section takes place. This warping greatly depends on the bond between reinforcement and concrete [27]. The poorer the reinforcement bond the greater warping of the cross-section. It is known that the reinforcement bond depends on concrete shear strength, reinforcement diameter, stress in reinforcement at failure and other factors. Therefore actual relative concrete compression zone depth is smaller in comparison with the calculated using formula (36).

According to STR [1] limit value of relative compression zone depth $\xi_{lim,eff}$ for rectangular stress diagram is determined by

$$\xi_{lim,eff,STR} = \frac{\lambda_{STR}}{1+\dfrac{\sigma_{s,lim}}{\sigma_{sc,lim}}\left(1-\dfrac{\lambda_{STR}}{1.1}\right)} \tag{38}$$

here $\sigma_{s,lim}$ is reinforcement stress with allowance for reinforcement yield limit, $\sigma_{sc,lim}$ is limit stress in compression reinforcement. For reinforced concrete (without prestress) when $f_{yk} \le 400$ MPa, $\sigma_{s,lim} = f_{yd}$ is taken; when $f_{yk} > 400$ MPa, $\sigma_{s,lim} = f_{yd} + 400$ MPa is taken. For structures from normal weight, small grain lightweight concrete $\sigma_{sc,lim} = 500$ MPa. Factor λ_{STR} in formula (38) is determined according to (14).

Direct comparison of ξ_{lim} and $\xi_{lim,eff,STR}$ is impossible, since, as it was already mentioned, ξ_{lim} is the depth of nonlinear stress diagram compression zone, while $\xi_{lim,eff,STR}$ the depth of rectangular stress diagram. If the depths of nonlinear diagram $\xi_{lim,parab}$ and $\xi_{lim,parab\text{-}rect}$ are multiplied by corresponding coefficients η_{parab} and $\eta_{parab\text{-}rect}$

$$\xi_{lim,eff} = \lambda\xi_{lim,EC2} \tag{39}$$

here $\xi_{lim,eff} \in \{\xi_{lim,eff,parab}, \xi_{lim,eff,parab\text{-}rect}\}$, $\xi_{lim,EC2} \in \{\xi_{lim,parab}, \xi_{lim,parab\text{-}rect}\}$ according to (36) and $\lambda \in \{\lambda_{parab}, \lambda_{parab\text{-}rect}\}$ from Table, then the depths of rectangular stress diagrams $\xi_{lim,eff,parab}$ and $\xi_{lim,eff,parab\text{-}rect}$ equivalent to parabola and parabola-rectangle stress diagrams are obtained. Then the values of $\xi_{lim,eff,STR}$ and $\xi_{lim,eff,parab}$ and $\xi_{lim,eff,parab\text{-}rect}$ can be compared.

Mentioned above limit depths of compression zone $\xi_{lim,eff,parab}$, $\xi_{lim,eff,parab\text{-}rect}$ $\xi_{lim,eff,STR}$ determined by formulae (38) and (39) for different values of characteristic reinforcement strengths, 400, 500, 600 and 800 MPa, are shown in Fig. 4. One can see in Fig. 4 that $\xi_{lim,eff,parab} > \xi_{lim,eff,parab\text{-}rect}$ when $f_{ck} < 36$ MPa and when $f_{ck} \ge 36$ MPa $\xi_{lim,eff,parab}$ practically is equal to $\xi_{lim,eff,parab\text{-}rect}$.

Formula (36) shows that limit value of relative compression zone depth depends on ultimate concrete strain ε_{cu}. In design codes EC2 and STR [1] this strain depends only on concrete class and character of σ_c-ε_c diagram. However, theoretically it has been determined [28] that in the case of short-time load, when a beam is destroyed during 1 hour, and $f_{cm} = 20.68$ MPa, for tee cross-section members $\varepsilon_{cu} \approx 0.22\%$, for rectangular cross-section members $(0.3 \le \varepsilon_{cu} \le 0.35)\%$, for triangular cross-section members $(0.38 \le \varepsilon_{cu} \le 0.48)\%$. Value of ε_{cu} may vary within wide limits depending on load action duration. According to [29] compressive concrete ultimate strain for long term loading is about 2 - 3 times greater in comparison with that for the short term loading. According to [7] ε_{cu} varies within the limits of $(0.42 \le \varepsilon_{cu} \le 0.56)\%$ depending on relative air moisture for the case of long term loading. In general the value of ε_{cu} can vary from 0.18 to 1% [4, 15, 16, 18, 28, 30]. Thus actuall compression zone depth varies within quite wide limits. Strain values presented by codes for diagrams of parabola with descending branch, parabola-rectangle and bi-linear diagrams are conditional. All earlier mentioned factors affect load carrying capacities of structures determined by tests. Therefore comparison of experimental data with theoretical results is possible only in the case when conditions of tests comply with the conditions of validity for diagram σ_c–ε_c. When $f_{ck} = 90$ MPa the ratio of $\xi_{lim,eff,parab}/\xi_{lim,eff,STR}$ varies from 1.5 to 1.8 depending on f_{yk}. Such great difference between $\xi_{lim,eff,parab}$ and $\xi_{lim,eff,STR}$ also $\xi_{lim,eff,parab\text{-}rect}$ and $\xi_{lim,eff,STR}$ emerges due to very small values of λ_{STR} for higher concrete classes. Therefore it is possible to conclude that for higher concrete classes greater values of $\xi_{lim,eff,STR}$ would be applied. When $\lambda_{STR} = 0.667$ is taken then the smallest values of $\xi_{lim,eff,STR}$ should be equal to 0.518, 0.491, 0.467, 0.444 for $f_{yk} \in \{400, 500, 600, 800\}$ MPa. Also it should be noted that the difference between $\xi_{lim,eff,STR}$ and $\xi_{lim,eff,parab\text{-}rect}$ and $\xi_{lim,eff,parab}$ emerges and due to the fact that the value of $\varepsilon_{cu} = 2.5\cdot10^{-3}$ is taken in formula (38) which is substantially less than the values of ε_{cu} specified in EC2. It should be stressed that calculation method of limit value of relative compression zone depth according to STR is analogous to that of SNiP [31]. But according to SNiP [31] the maximal value of characteristic compressive cube strength of concrete is 60 MPa.

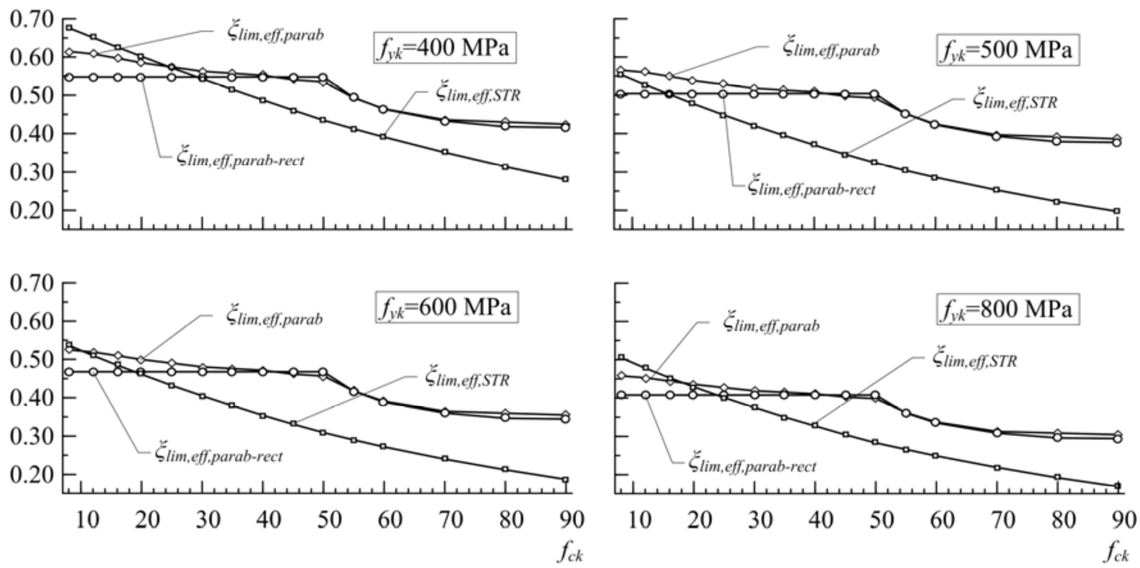

Fig. 4 Limits values of relative depths of compression zone rectangular stress diagrams

5. Conclusions

1. It was determined that for high strength concrete some values of coefficients for rectangular stress diagram according to STR 2.05.05:2005 and these given according to other methods determined experimentally and defined theoretically are less than the maximum coefficient values obtained according hypotheses of plane section and of full bond between reinforcement and concrete.

2. It was determined that the ratio between depths of rectangular and nonlinear diagrams experimentally obtained is very close to theoretically obtained ratio between the depths of equivalent rectangular and nonlinear with descending branch stress diagrams according to EC2. However experimentally determined ratios between widths of the said diagrams differ substantially from the ratio between widths of theoretically determined equivalent rectangular and nonlinear stress diagrams according to EC2.

3. It was determined that limit depths of concrete compression zone depths calculated according to the method of STR 2.05.05:2005 and these according to hypothesis of plane sections taking ultimate concrete strain and reinforcement yield strain or conventional yield strain in general differ quite substantially.

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D. Zabulionis, E. Dulinskas

LENKIAMŲ GELŽBETONINIŲ ELEMENTŲ NORMALINIŲ PJŪVIŲ GNIUŽDOMOSIOS ZONOS PARAMETRŲ PAGAL EC2 IR STR 2.05.05:2005 ANALIZĖ

R e z i u m ė

Straipsnyje analizuojami lenkiamų gelžbetoninių elementų normalinio pjūvio stačiakampės įtempių diagramos koeficientai pagal skirtingas normas ir metodikas. Šie koeficientai palyginti su eksperimentiniais kitų autorių duomenimis. Nustatyta, kad stačiakampės įtempių diagramos kai kurių koeficientų teorinės ir eksperimentinės reikšmės esant aukštoms betono klasėms yra mažesnės už mažiausias galimas reikšmes, gautas imant trikampę įtempių diagramą. Taip pat buvo analizuotas ribinis gniuždomos zonos aukštis pagal STR 2.05.05:2005 ir galiojant plokščiųjų pjūvių hipotezei. Nustatyta, kad šių diagramų aukštis skiriasi gana žymiai.

D. Zabulionis, E. Dulinskas

ANALYSIS OF COMPRESSION ZONE PARAMETERS OF CROSS-SECTION IN FLEXURAL REINFORCED CONCRETE MEMBERS ACCORDING TO EC2 AND STR 2.05.05:2005

S u m m a r y

Coefficients for rectangular stress diagram used in various regulations and methods are analyzed. These coefficients are compared with experimental data presented by other authors. It was revealed that theoretical and experimental values of some coefficients for rectangular stress diagram when concrete classes are high are less than possible minimum values obtained using triangular stress diagram. The limit compression zone depth value according to STR 2.05.05:2005 and according to hypothesis of plane sections was analyzed as well. It was determined that the depth of these diagrams differs quite substantially.

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АНАЛИЗ ПАРАМЕТРОВ СЖАТОЙ ЗОНЫ БЕТОНА НОРМАЛЬНЫХ СЕЧЕНИИ ЖЕЛЕЗОБЕТОННЫХ ЕЛЕМЕНТОВ СОГЛАСНО EC2 И STR 2.05.05:2005

Р е з ю м е

В статье анализируются коэффициенты прямоугольной диаграммы напряжений нормальных сечении изгибаемых железобетонных элементов согласно различным нормам и методикам. Анализируются коэффициенты прямоугольной диаграммы напряжений, представленные в различных нормах и методиках. Эти коэффициенты сравнены с опытными коэффициентами, полученными другими авторами. Установлено, что некоторые значения коэффициентов сжатой зоны бетона при высоких его классах меньше чем минимальные, установленные принимая треугольную диаграмму напряжений. Также анализируется граничная высота сжатой зоны бетона согласно STR 2.05.05:2005 при гипотезе плоских сечений. Установлено, что граничная высота этих диаграмм различается значительно.

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