ISSN 1392 - 1207. MECHANIKA. 2009. Nr.2(76)

Mechanics of initial dot contact

M. Matlin*, E. Kazankina, V. Kazankin*****
**Volgograd State Technical University, Lenin Ave 28, 400131 Volgograd, Russia, E-mail: matlin@vstu.ru*
***Volgograd State Technical University, Lenin Ave 28, 400131 Volgograd, Russia, E-mail: detmash@vstu.ru*
****Volgograd State Technical University, Lenin Ave 28, 400131 Volgograd, Russia, E-mail: detmash@vstu.ru*

1. Introduction

Operational properties of many units of machines depend on parameters of the connected details contact. Modes of superficial plastic deformation, processes of friction and deterioration, results of the control of hardness, and also other physicomechanical properties of materials depend on these parameters [1-3]. So, for example, in work [4] results of research of durability on cave-in of a metal covering are resulted, in work [5] is shown, that by results of cave-in of a ball it is possible to measure elasto-plastic properties of the tempered samples, and in work [6] by nanoindentation with the spherical indenter defined mechanical properties of amorphous alloys. In work [7] the opportunity of development of techniques of not destroying definition of limits of yielding and elasticity is shown with use of a method of continuous cave-in of spherical indenter.

It is necessary to note, that for a case of elastic contact of bodies parameters of their contact define according to the decision of a spatial contact problem of the theory of elasticity, which belongs to German physics Henry Hertz [8]. For a case of elastoplastic contact there are separate particular decisions (basically deciding problems of hardness definition); however the opportunity to use the indicated methods is limited to that they are fair only at rather small depths of spherical indenter (sphere) introducnion when the depth of residual print does not exceed 0.2 from the sphere radius.

In this work rated definition method of residual print diameter in contact of an elastic sphere with riders on known physicomechanical properties of contacting bodies, fair in a wide range loadings and depths of introduction is offered.

Calculation is based on the laws of plasticity deformation theory [9]. A basis of this theory is the assumption that under constant external conditions (constant speed of deformation at atmospheric pressure and room temperature) and irrespective of a kind tension conditions for the given material is fair a uniform curve of deformation. The given curve describes the relation of stress from intensity of its deformed condition ε_i

$$\sigma_i = \varphi(\varepsilon_i) \tag{1}$$

Thus, functional dependence σ_i on ε_i can be received both at tension of the samples made from rider material, and at cave-in spherical indenter in the surface of rider.

Expression for stress intensity σ_i looks like [10]

$$\sigma_i = \frac{1}{\sqrt{2}}\sqrt{(\sigma_1-\sigma_2)^2+(\sigma_2-\sigma_3)^2+(\sigma_3-\sigma_1)^2} \tag{2}$$

and intensity of strain ε_i

$$\varepsilon_i = \frac{\sqrt{2}}{3}\sqrt{(\varepsilon_1-\varepsilon_2)^2+(\varepsilon_2-\varepsilon_3)^2+(\varepsilon_3-\varepsilon_1)^2} \tag{3}$$

At tension of the sample [11]

$$\sigma_i = \sigma_1 \tag{4}$$

$$\varepsilon_i = \frac{2(1+\nu_2^*)}{3}\varepsilon_1 \tag{5}$$

where ν_2^* is Poisson's ratio of a material rider in the area of elastoplastic strain. Influence of Poisson's ratio on stress-strain state is usual insignificant and it is possible to accept equal [11]

$$\nu_2^* = 0.5\nu_2 + 0.25 \tag{6}$$

or to consider on elastic area $\nu_2^* = \nu_2$, and at presence of plastic strain $\nu_2^* = 0.5$, and then at tension $\varepsilon_i = \varepsilon_1$, that is the Eq. (1) describes also the diagram of a sample tension.

2. Stress state analysis

We shall consider contact of an elastic sphere and a flat surface as elastic riders (Fig. 1).

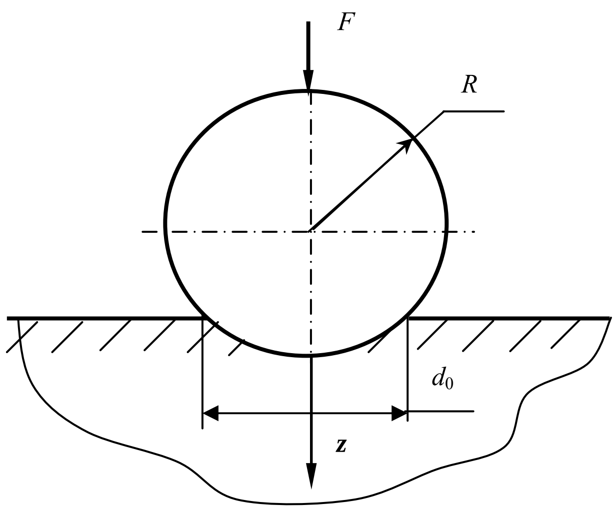

Fig. 1 The diagram of contact of a sphere with half-space

Pressure on the platform along axis z [12]

$$\sigma_Z = -q_0 \frac{1}{1+\left(\frac{z}{0.5d_0}\right)^2} \tag{7}$$

where d_0 is diameter of the contact zone; $q_0 = 1.5F / \pi(0.5d_0)^2$ is pressure in the center of the contact zone.

Due to axial symmetry of a stress state in case of a circular plate of contact, normal pressures on any plate, are equal

$$\sigma_x = \sigma_y = -q_0 \left[(1+\nu_2) - \frac{0.5}{1+\left(\frac{z}{0.5d_0}\right)^2} - (1+\nu_2)\frac{z}{0.5d_0} arctg \frac{0.5d_0}{z} \right] \tag{8}$$

As is known [9, 11, 12], function (1) does not depend on the character of a stress state, only in conditions of simple loading (that is when all components of pressure are proportional to the same parameter) or complex stress, close to simple. It is possible to show [13], that the condition of simple loading is strictly carried out for a unique point in the center of contact (i.e. at $z = 0$).

Thus, for the center of the plate contact from (7) and (8) we shall receive

$$\sigma_{z,0} = -q_0 \tag{9}$$

$$\sigma_{x,0} = \sigma_{y,0} = -q_0 \frac{1+2\nu_2}{2} \tag{10}$$

Intensity of pressures $\sigma_{i,0}$ in this point we shall determine from the Eq. (2) with the account of Eqs. (9) and (10) as

$$\sigma_{i,0} = q_0 \frac{1-2\nu_2}{2} \tag{11}$$

Thus means, that $\sigma_1 = \sigma_2 = \sigma_{x,0} = \sigma_{y,0}$; $\sigma_3 = \sigma_{z,0.}$

According to [12]

$$q_0 = 0.27 \sqrt[3]{\frac{F}{(k_1+k_2)^2 R^2}} \tag{12}$$

where $k_{1,2} = \left(1-\nu_{1,2}^2\right)/\pi E_{1,2}$; ν is Poisson's ratio; E is module elasticity (indexes 1 and 2 concern to materials of a sphere and plate).

For a conclusion of contact laws of an elastic sphere with plate we shall take advantage of variable elasticity parameters method [11]. This method is based on the representation of dependences for elastoplastically deforming material of a rider in the form of the equations of elasticity in which the parameters of elasticity E_2^* and ν_2^* depend on an intense condition. Thus secant module of the tension diagram described (conterminous, as it is shown above, with the tension diagram of a sample) by the formula (1) and is equal

$$E_2^* = \sigma_i / \varepsilon_i \tag{13}$$

Approximating stress-strain curve by power law [9]

$$\sigma_i = A\varepsilon_i^n \tag{14}$$

instead of the Eq. (13) we receive

$$E_2^* = A\varepsilon_i^{n-1} \tag{15}$$

where A and n are parameters of the diagram of deformation (curve of hardening) of the riders materials are equal

$$A = S_b / \varepsilon_p^n \tag{16}$$

$$n = \frac{lg(S_b / \sigma_{0.2})}{lg(\varepsilon_p / \varepsilon_T)} = \frac{lg(S_b / \sigma_{0.2})}{lg(500\varepsilon_p)} \tag{17}$$

where $S_b, \sigma_{0.2}$ are accordingly fracture stress and a conditional yield stress of the rider material; $\varepsilon_p, \varepsilon_T$ are accordingly limiting uniform strain of the rider material and the admission on residual deformation (equal 0.002), adequate to conditional yield stress.

Thus, with the account of Eqs. (12) and (15) will become

$$q_0 = 0.27 \sqrt[3]{\frac{F}{(k_1+k_2^*)^2 R^2}} \tag{18}$$

where

$$k_2^* = \frac{1-(\nu_2^*)^2}{\pi E_2^*} = \frac{1-(\nu_2^*)^2}{\pi A\varepsilon_i^{n-1}} \tag{19}$$

In the same way, having taken advantage of the dependence determining diameter d_0 of the plate contact elastic sphere and a rider [12] for quasi-elastic material of the rider, we shall receive the expression determining the diameter of the print in contact of an elastic sphere with elastoplastic material of rider

$$d_0 = 2\sqrt[3]{\frac{3}{4}\pi RF(k_1+k_2^*)} \tag{20}$$

From the simultaneous solution of the Eqs. (11), (18) and (19) we shall receive a dependence for the definition of intensity of stress in the center of contact of an elastic sphere with elastoplastic half-space

$$\sigma_{i,0} = 0.955(1-2\nu_2)\frac{F}{d_0^2} \tag{21}$$

Intensity of strains $\varepsilon_{i,0}$ in the center of contact we shall define, having solved the Eq. (20) with the account Eq. (13)

$$\varepsilon_{i,0} = (0.222\frac{d_0^2}{FR} - 4.189k_1)\sigma_{i,0} \quad (22)$$

Thus, Eqs. (21) and (22) define intensity of stress and strains in the center of contact elastoplastic print.

3. Experimental investigations

We in addition execute a special experimental research in which the values of intensity of stress $\sigma_{i,0}$ in the center of the contact have been compared (Eq. (21)), with true pressures $S = \sigma_i$ determined by results of sample tension test. The samples made from constructional steels of a various level of strength, and also of some nonferrous metals and alloys have been tested. The research has shown, that at identical ε_i the values of $\sigma_{i,0}$ calculated under the Eq. (21), are a little lower than values S. This difference is reduced with the growth of strength and hardness of a material and makes, for example, at HB 1000 MPa about 20%, and at HB 4000 MPa - 5%. Such position arises, apparently, for the lack of the account of friction forces in the contact by Eq. (21). For the investigated materials by us it is experimentally established, that for concurrence of values S and $\sigma_{i,0}$ the last it is necessary to increase by the correction function determined by expression $exp(\varepsilon_p)$, that is

$$\sigma'_{i,0} = \sigma_{i,0} exp(\varepsilon_p) = 0.955(1-2\nu_2)\frac{F}{d_0^2} exp(\varepsilon_p) \quad (23)$$

or (see the Eq. 14)

$$\sigma_{i,0} = \frac{S}{exp(\varepsilon_p)} = \frac{A}{exp(\varepsilon_p)}\varepsilon_{i,0}^n = A'\varepsilon_{i,0}^n \quad (24)$$

Calculation of diameter d_0 of a residual print can be executed by Eq. (20) which with the account of (15), (19), (24) will be transformed to

$$d_0 = 2.66\sqrt[3]{FR\left(k_1 + \frac{0.239}{A'\varepsilon_{i,0}^{n-1}}\right)} \quad (25)$$

Experimental evaluation of settlement definition of a residual print diameter method determined by Eq. (25) is executed on the specimens (bars) made from constructional steel of various level of strength, and also some nonferrous metals and alloys. Mechanical properties of the material of specimens defined by results of tension test agrees GOST 1497-84 "Metals. A test method on a tension"; the results are presented in the Table.

As an elastic sphere tempered (HRC_e 63 - 64) steel ball with radius of curvature R = 2.5 mm was used. Loading was made with the help Brinelle's press (up to F= 29.4 kN), and at high loadings with the help of a program-technical complex for test of metals IR 5143-200. Diameter of a residual print was measured with the help of tool microscope MMI-2. Each experiment was repeated 4 - 5 times and average value d_0 was calculated. Apparently from Fig. 2, coincidence of experimental and calculated results by Eq. (25) d_0 is quite satisfactory. In the most cases the difference does not exceed (4 - 6)%. Thus the formula (25) allows to determine values of diameter d_0 of a residual print in all the range of the depths of spherical indenter introduction, including and those cases when the depth of a residual print is close to radius R of the sphere, and the ratio $d_0 / 2R$ is close to one.

Table

Mechanical properties of materials

No	Materials	$\sigma_{0.2}$, MPa	σ_u, MPa	ε_p
1	Armko-iron	256	410	0.191
2	Steel 45	480	725	0.102
3	Steel 30 ХГСА	677	942	0.073
4	Steel 30 ХГСА	1207	1344	0.045
5	Copper M2	69	196	0.581
6	Duralumi-nium	265	392	0.157
7	Titanium BT6	687	883	0.071

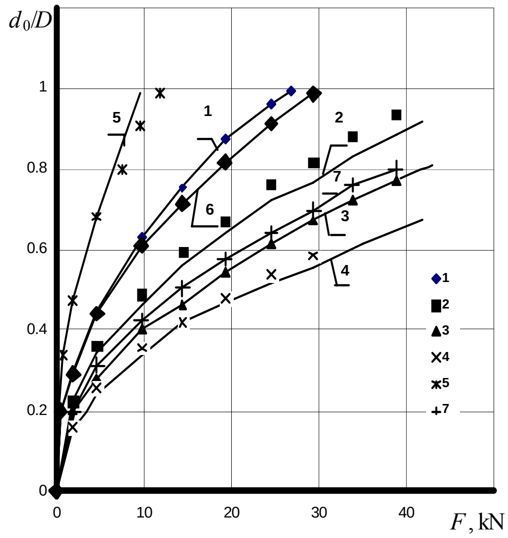

Fig. 2 Relative diameter d_0 / D of a residual print depending on loading F in contact of elastic sphere (with diameter D = 5 mm) and a flat surface half-space; lines - calculation by the formula (25); symbols - experiment; *1-7* - numbers of samples in the Table

From the formula (25) follows, that in case of only elastic deformation when $k_2^* = k_2$ (when plastic deformation in contact is absent), the formula (25) determining diameter d_0 of a print, becomes, adequate to elastic decision of Hertz.

Thus, the problem of definition of the diameter of a print is analytically solved at originally dot contact of an elastic sphere with elastoplastic half-space.

4. Conclusions

1. On the basis of the strain theory of plasticity and the method of variable parameters of elasticity the analytical decision of a problem of diameter definition of a residual print is received at introduction of an elastic sphere (or bodies of double curvature) in elastic-plastic half-space, in a wide range of depths of introduction, down to the depths close to radius of a sphere.

2. The settlement dependences determining intensity of stress and strain in the center of elasto-plastic contact are received, and also correction function is experimentally established (dependent on mechanical properties of a half-space material), allowing to proceed from intensity of stress in the center of contact to true stress at monoaxial tension.

3. Experimental evaluation has shown, that the error of an analytical method of a residual print diameter definition does not exceed (4 - 6) %.

Thus, the problem of print diameter definition is analytically solved at originally dot contact of an elastic sphere with elastic-plastic half-space.

References

1. **Atkočiūnas, J., Rimkus, L., Skaržauskas, V., Jarmolajeva, E.** Optimal shakedown design of plates.-Mechanika.-Kaunas: Technologija, 2007, Nr.5(67), p.14-23.
2. **Baron, A., Bakhracheva, J.** A method for impact strength estimation.-Mechanika.-Kaunas: Technologija, 2007, Nr.4(66), p.31-35.
3. **Matlin, M., Lebsky, S., Mozgunova, A., Frolova, A.** Features of hardening of the case parts made of aluminium alloys. -Mechanika. -Kaunas: Technologija, 2007, Nr.6(68), p.19-25.
4. **Lima, M.M., Godoy, C., Avelar-Batista, J.C., Modenesi, P.J.** Toughness evaluation of HVOF WC-Co coatings using nonlinear regression analysis.-Research Centre in Surface Engineering University of Hull, UK//Mater. Sci. and Eng/ A.N1-2, 2003, p.337-345.
5. **Zhao, Manhong, Ogasawara, Nagahisa, Chiba, Norimasa, Chen, Xi.** A new approach to measure the elastic-plastic properties of bulk materials using spherical indentation.-Acta Mater., 2006, No1, p.23-32.
6. **Bei, H., Lu, Z.P., George, E.P.** Theoretical strenght and the onset of plasticity in bulk metallic glasses investigated by nanoindentation with a spherical indenter. -Phys. Rev. Lett., 2004, No12, p.45-47.
7. **Shabanov, V.M.** Resistance of metals of initial plastic deformation at cave-in spherical indenter.-Factory laboratory. Diagnostics of materials, 2008, No6, p.63-69 (in Russian).
8. **Malinin, N.N**. The Applied Theory of Plasticity and Creep. -Moscow: Mashinostrojenije, 1975. -399p. (in Russian).
9. **Timoshenko, S.P., Gudjer, Dz**. The Theory of Elasticity.-Moscow: Nauka, 1975.-576p.
10. **Smirnov-Alyaev, G.A**. Resistance of Materials to Plastic Deformation. -Leningrad: Mashinostrojenije, 1978. -368p. (in Russian).
11. **Birger, I.A., Shorr, B.F., Josilevich, G.B.** Calculation on Durability of Details of Machines: Reference-Book. -Moscow: Mashinostrojenije, 1993.-640p. (in Russian).
12. **Ponomarev, S.D., Biderman, V.L., Liharev, K.K. and others.** Calculations on Durability in Mechanical Engineering. In 3 vol.-Moscow: Mashgiz, v.1, 1956. -884p.; v.2, 1958.-974p.; v.3, 1959.-1118p. (in Russian).
13. **Drozd, M.S., Polonskiy, A.Y.** About distribution of intensity of deformation in power contact of elastic sphere with elastoplastic half-space.-Metallurgical science and durability of materials.-Mezhvuz. Sb. Proceedings.-Volgograd: VolgPI, 1989, p.9-18 (in Russian).

M. Matlin, E. Kazankina, V. Kazankin

PIRMINIO TAŠKINIO KONTAKTO MECHANIKA

R e z i u m ė

Aprašyta skaitmeninė liekamojo įspaudo skersmens nustatymo visame sferinio indentoriaus įspaudo gylių diapazone metodika, įskaitant tuos atvejus, kai liekamojo įspaudo gylis artimas rutulio spinduliui.

M. Matlin, E. Kazankina, V. Kazankin

MECHANICS OF INITIAL DOT CONTACT

S u m m a r y

The technique of calculated definition of the diameter of a residual print in all the range of depths of spherical indenter introduction is described, including and those cases when the depth of residual print is close to the radius of sphere.

М. Матлин, Е. Казанкина, В. Казанкин

МЕХАНИКА ПЕРВОНАЧАЛЬНО ТОЧЕЧНОГО КОНТАКТА

Р е з ю м е

Описана методика расчетного определения диаметра остаточного отпечатка во всем диапазоне глубин внедрения сферического индентора, включая и те случаи, когда глубина остаточного отпечатка близка к радиусу шара.

Received April 17, 2008
Accepted February 27, 2009