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Investigation of vibrations of a multilayered polymeric film

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1. Introduction

The quality of printing production and packaging depends not only on appropriate application of printing and packaging technologies, but also on the properties of the materials used, their mechanical and other characteristics [1]. Every year the application of polymeric materials in packaging industry is growing, with increasing requirements for their mechanical resistance. Polymeric materials are being adjusted to higher requirements for print quality and increased productivity, and the properties of these materials are being upgraded [2].

During the operation of the printing machine, vibrations of the polymeric film band, of the transportation, printing and other devices occur [3], as well as deformations of printing materials. They have a negative effect on the quality of polymeric films and the graphic images on them, cause noise, reduce precision, durability and other characteristics of the printing machine.

In order to increase the quality of protective functions of the packaging materials from external mechanical, chemical and other effects and to be able to protect the packaged material the surface of the packaging material is often coated by other materials or by layers of several materials [4]. The most common combined material is paper coated by synthetic films. In such multi-layered materials the paper is located in internal layer, while the synthetic film protects the paper from humidity, decreases the transmissibility of gasses and vapours and also enables to perform thermal welding.

For the investigation of surface deformations of multilayered polymeric films the method of digital speckle photography can be used [5, 6]. This method is of very high precision.

After performing the analytical investigation of available research papers it was found that there are some investigations in which the application of various speckle and holographic methods for the investigation of deformations of various surfaces is performed [7, 8], also the application of the speckle methods for the investigation of displacements of materials and their stresses is performed [9]. In the research papers [10, 11] the effects of the types of polymeric films or of their constituent parts (if the films are multilayered) to mechanical, optical and barrier properties are analyzed. The authors of the paper consider that there are insufficient investigations in which mechanical characteristics and vibrations of multilayered polymeric materials are analyzed. Because of the mentioned reasons this research is considered important.

Currently market requirements among the producers of polymeric packages are increasing. Thus the requirements for mechanical, thermomechanical, qualitative and other parameters become higher. The quality of production processes of packages to a large amount depends on qualitative parameters of the polygraphic materials (polymeric films). So the purpose of this paper is to investigate vibrations of the multilayered polymeric films used for production of packages in food industry and to determine their mechanical parameters.

2. Model for the analysis of multilayered polymeric film vibrations

The multilayered polymeric film is assumed to have equal plate - type top and bottom layers (in such a layer, the displacements of the top and bottom planes in the direction of the z axis of the orthogonal Cartesian system of coordinates are equal), while in between them there is an elastic inner layer (its displacements of the top and bottom planes in the direction of the z axis are not necessarily equal) (Fig. 1, a). The parameters of the outer and inner layers – thickness, density, Young's modulus, Poisson's ratios – are essentially different.

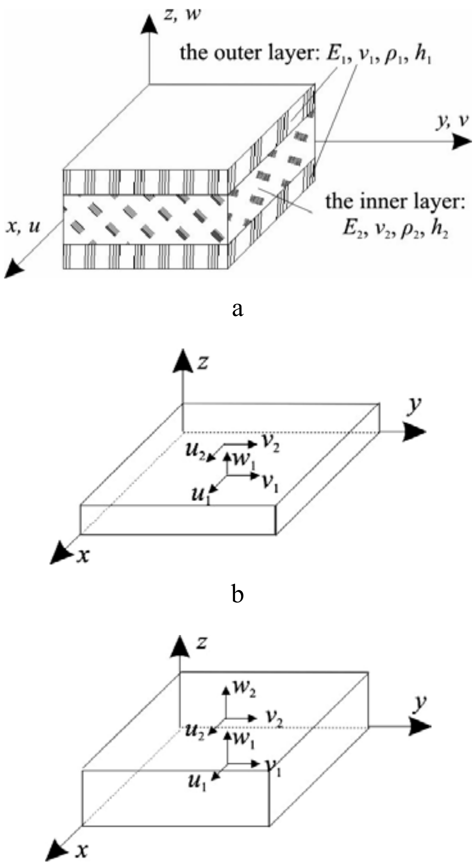
In the outer layer of a multilayered polymeric film, the subelement has five nodal degrees of freedom [12, 13]: the displacement of the layer w_1 in the direction of z axis, the displacements of the lower plane of the layer u_1 and v_1 in the directions of x and y axes, the displacements of the upper plane of the layer u_2 and v_2 in the directions of x and y axes (Fig. 1, b). The length and width of the polymeric film are chosen to be equal to 0.2 m. Poisson's ratio of the outer layer $\nu_1 = 0.3$, Young's modulus $E_1 = 8$ MPa, density $\rho_1 = 800$ kg/m^3, thickness $h_1 = 10$ μm.

Further Θ_x and Θ_y denote the rotations about the axes of coordinates x and y. The displacements due to bending u and v in the directions of axes x and y are expressed as $u = z\,\Theta_y$ and $v = -z\,\Theta_x$.

Further u and v denote the displacements of the middle plane of the outer layer in the directions of x and y axes. Thus $u_1 = u - \frac{h_1}{2}\Theta_y$, $u_2 = u + \frac{h_1}{2}\Theta_y$, $v_1 = v + \frac{h_1}{2}\Theta_x$, $v_2 = v - \frac{h_1}{2}\Theta_x$, where h_1 is thickness of the outer layer.

This gives the following expressions:

$$u=\frac{u_1+u_2}{2},\quad v=\frac{v_1+v_2}{2},\quad \Theta_y=\frac{u_2-u_1}{h_1},\quad \Theta_x=\frac{v_1-v_2}{h_1}.$$



c

Fig. 1 Section of a multilayered polymeric film (a), displacements of subelements of its outer (b) and inner (c) layers

The mass matrix has the form

$$[M]=\int\left(\left[\hat{N}\right]^T\begin{bmatrix}\rho_1 h_1 & 0 & 0\\ 0 & \dfrac{\rho_1 h_1^3}{12} & 0\\ 0 & 0 & \dfrac{\rho_1 h_1^3}{12}\end{bmatrix}\left[\hat{N}\right]+ [N]^T\rho_1 h_1 [N]\right)dxdy \qquad (1)$$

where ρ_1 is the density of the material of the outer layer, and

$$\left[\hat{N}\right]=\begin{bmatrix}N_1 & 0 & 0 & 0 & 0 & \dots\\ 0 & 0 & \dfrac{N_1}{h_1} & 0 & -\dfrac{N_1}{h_1} & \dots\\ 0 & -\dfrac{N_1}{h_1} & 0 & \dfrac{N_1}{h_1} & 0 & \dots\end{bmatrix} \qquad (2)$$

$$[N]=\begin{bmatrix}0 & \dfrac{N_1}{2} & 0 & \dfrac{N_1}{2} & 0 & \dots\\ 0 & 0 & \dfrac{N_1}{2} & 0 & \dfrac{N_1}{2} & \dots\end{bmatrix} \qquad (3)$$

where N_i are shape functions of the finite element.

The stiffness matrix has the form

$$[K]=\int\left(\left[\hat{B}\right]^T\left[\hat{D}\right]\left[\hat{B}\right]+\left[\widetilde{B}\right]^T\left[\widetilde{D}\right]\left[\widetilde{B}\right]+[B]^T[D][B]\right)dxdy \quad (4)$$

where

$$\left[\hat{B}\right]=\begin{bmatrix}0 & -\dfrac{\partial N_1/\partial x}{h_1} & 0 & \dfrac{\partial N_1/\partial x}{h_1} & 0 & \dots\\ 0 & 0 & -\dfrac{\partial N_1/\partial y}{h_1} & 0 & \dfrac{\partial N_1/\partial y}{h_1} & \dots\\ 0 & -\dfrac{\partial N_1/\partial y}{h_1} & -\dfrac{\partial N_1/\partial x}{h_1} & \dfrac{\partial N_1/\partial y}{h_1} & \dfrac{\partial N_1/\partial x}{h_1} & \dots\end{bmatrix} \qquad (5)$$

$$\left[\hat{D}\right]=\frac{h_1^3}{12}\begin{bmatrix}\dfrac{E_1}{1-\nu_1^2} & \dfrac{E_1\nu_1}{1-\nu_1^2} & 0\\ \dfrac{E_1\nu_1}{1-\nu_1^2} & \dfrac{E_1}{1-\nu_1^2} & 0\\ 0 & 0 & \dfrac{E_1}{2(1+\nu_1)}\end{bmatrix} \qquad (6)$$

$$\left[\widetilde{B}\right]=\begin{bmatrix}\dfrac{\partial N_1}{\partial y} & 0 & -\dfrac{N_1}{h_1} & 0 & \dfrac{N_1}{h_1} & \dots\\ \dfrac{\partial N_1}{\partial x} & -\dfrac{N_1}{h_1} & 0 & \dfrac{N_1}{h_1} & 0 & \dots\end{bmatrix} \qquad (7)$$

$$\left[\widetilde{D}\right]=\frac{E_1 h_1}{2(1+\nu_1)1.2}\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix} \qquad (8)$$

$$[B]=\begin{bmatrix}0 & \dfrac{\partial N_1/\partial x}{2} & 0 & \dfrac{\partial N_1/\partial x}{2} & 0 & \dots\\ 0 & 0 & \dfrac{\partial N_1/\partial y}{2} & 0 & \dfrac{\partial N_1/\partial y}{2} & \dots\\ 0 & \dfrac{\partial N_1/\partial y}{2} & \dfrac{\partial N_1/\partial x}{2} & \dfrac{\partial N_1/\partial y}{2} & \dfrac{\partial N_1/\partial x}{2} & \dots\end{bmatrix} \qquad (9)$$

$$[D]=h_1\begin{bmatrix}\dfrac{E_1}{1-\nu_1^2} & \dfrac{E_1\nu_1}{1-\nu_1^2} & 0\\ \dfrac{E_1\nu_1}{1-\nu_1^2} & \dfrac{E_1}{1-\nu_1^2} & 0\\ 0 & 0 & \dfrac{E_1}{2(1+\nu_1)}\end{bmatrix} \qquad (10)$$

where E_1 is the Young's modulus and ν_1 is the Poisson's ratio of the outer layer.

In the inner layer of the multilayered polymeric film, the subelement has six nodal degrees of freedom: the displacements of the lower plane of the layer u_1, v_1, w_1 and the displacements of the upper plane of the layer u_2, v_2, w_2 in the directions of the axes of coordinates (Fig. 1, c).

Poisson's ratio of the inner layer $\nu_2 = 0.3$, Young's modulus $E_2 = 0.8$ MPa, density $\rho_2 = 80\ \mathrm{kg/m^3}$, thickness $h_2 = 100\ \mu\mathrm{m}$. Thickness of the inner layer of the polymeric film is 10 times larger, while density and Young modulus are 10 times smaller in comparison with the parameters of the outer layer of the polymeric film.

The displacements u, v, w in the directions of the axes of coordinates are represented in the following way

$$\begin{Bmatrix} u \\ v \\ w \end{Bmatrix} = \frac{h_2 - z}{h_2}\left[\overline{N}\right]\{\delta\} + \frac{z}{h_2}\left[\overline{\overline{N}}\right]\{\delta\} \tag{11}$$

where h_2 is thickness of the inner layer, $\{\delta\}$ is the vector of nodal displacements, and

$$\left[\overline{N}\right] = \begin{bmatrix} N_1 & 0 & 0 & 0 & 0 & 0 & \dots \\ 0 & N_1 & 0 & 0 & 0 & 0 & \dots \\ 0 & 0 & N_1 & 0 & 0 & 0 & \dots \end{bmatrix} \tag{12}$$

$$\left[\overline{\overline{N}}\right] = \begin{bmatrix} 0 & 0 & 0 & N_1 & 0 & 0 & \dots \\ 0 & 0 & 0 & 0 & N_1 & 0 & \dots \\ 0 & 0 & 0 & 0 & 0 & N_1 & \dots \end{bmatrix}. \tag{13}$$

The mass matrix has the form

$$\left[M\right] = \int \begin{pmatrix} \left[\overline{N}\right]^T \rho_2 \dfrac{h_2}{6}\left[\overline{\overline{N}}\right] + \left[\overline{\overline{N}}\right]^T \rho_2 \dfrac{h_2}{6}\left[\overline{N}\right] + \\ + \left[\overline{N}\right]^T \rho_2 \dfrac{h_2}{3}\left[\overline{N}\right] + \left[\overline{\overline{N}}\right]^T \rho_2 \dfrac{h_2}{3}\left[\overline{\overline{N}}\right] \end{pmatrix} dxdy \tag{14}$$

where ρ_2 is density of the material of the inner layer, and the following integrals have been taken into account

$$\int_0^{h_2} \frac{h_2 - z}{h_2}\frac{z}{h_2}dz = \frac{h_2}{6} \tag{15}$$

$$\int_0^{h_2} \left(\frac{h_2 - z}{h_2}\right)^2 dz = \int_0^{h_2} \left(\frac{z}{h_2}\right)^2 dz = \frac{h_2}{3}. \tag{16}$$

The strains are expressed as

$$\{\varepsilon\} = \frac{1}{h_2}\left[B\right]\{\delta\} + \frac{h_2 - z}{h_2}\left[\overline{B}\right]\{\delta\} + \frac{z}{h_2}\left[\overline{\overline{B}}\right]\{\delta\} \tag{17}$$

where

$$\left[B\right] = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 0 & 0 & -N_1 & 0 & 0 & N_1 & \dots \\ 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 0 & -N_1 & 0 & 0 & N_1 & 0 & \dots \\ -N_1 & 0 & 0 & N_1 & 0 & 0 & \dots \end{bmatrix} \tag{18}$$

$$\left[\overline{B}\right] = \begin{bmatrix} \dfrac{\partial N_1}{\partial x} & 0 & 0 & 0 & 0 & 0 & \dots \\ 0 & \dfrac{\partial N_1}{\partial y} & 0 & 0 & 0 & 0 & \dots \\ 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ \dfrac{\partial N_1}{\partial y} & \dfrac{\partial N_1}{\partial x} & 0 & 0 & 0 & 0 & \dots \\ 0 & 0 & \dfrac{\partial N_1}{\partial y} & 0 & 0 & 0 & \dots \\ 0 & 0 & \dfrac{\partial N_1}{\partial x} & 0 & 0 & 0 & \dots \end{bmatrix} \tag{19}$$

$$\left[\overline{\overline{B}}\right] = \begin{bmatrix} 0 & 0 & 0 & \dfrac{\partial N_1}{\partial x} & 0 & 0 & \dots \\ 0 & 0 & 0 & 0 & \dfrac{\partial N_1}{\partial y} & 0 & \dots \\ 0 & 0 & 0 & 0 & 0 & 0 & \dots \\ 0 & 0 & 0 & \dfrac{\partial N_1}{\partial y} & \dfrac{\partial N_1}{\partial x} & 0 & \dots \\ 0 & 0 & 0 & 0 & 0 & \dfrac{\partial N_1}{\partial y} & \dots \\ 0 & 0 & 0 & 0 & 0 & \dfrac{\partial N_1}{\partial x} & \dots \end{bmatrix}. \tag{20}$$

The stiffness matrix has the form

$$\left[K\right] = \int \begin{pmatrix} \left[B\right]^T\left[D\right]\dfrac{1}{2}\left[\overline{B}\right] + \left[\overline{B}\right]^T\left[D\right]\dfrac{1}{2}\left[B\right] + \\ + \left[B\right]^T\left[D\right]\dfrac{1}{2}\left[\overline{\overline{B}}\right] + \left[\overline{\overline{B}}\right]^T\left[D\right]\dfrac{1}{2}\left[B\right] + \\ + \left[\overline{B}\right]^T\left[D\right]\dfrac{h_2}{6}\left[\overline{\overline{B}}\right] + \left[\overline{\overline{B}}\right]^T\left[D\right]\dfrac{h_2}{6}\left[\overline{B}\right] + \\ + \left[\overline{B}\right]^T\left[D\right]\dfrac{h_2}{3}\left[\overline{B}\right] + \left[\overline{\overline{B}}\right]^T\left[D\right]\dfrac{h_2}{3}\left[\overline{\overline{B}}\right] + \\ + \left[B\right]^T\left[D\right]\dfrac{1}{h_2}\left[B\right] \end{pmatrix} dxdy \tag{21}$$

where the following integrals have been taken into account

$$\int_0^{h_2} \frac{1}{h_2}\frac{h_2 - z}{h_2}dz = \int_0^{h_2} \frac{1}{h_2}\frac{z}{h_2}dz = \frac{1}{2} \tag{22}$$

$$\int_0^{h_2} \left(\frac{1}{h_2}\right)^2 dz = \frac{1}{h_2} \tag{23}$$

and

$$\left[D\right] = \begin{bmatrix} K + \frac{4}{3}G & K - \frac{2}{3}G & K - \frac{2}{3}G & 0 & 0 & 0 \\ K - \frac{2}{3}G & K + \frac{4}{3}G & K - \frac{2}{3}G & 0 & 0 & 0 \\ K - \frac{2}{3}G & K - \frac{2}{3}G & K + \frac{4}{3}G & 0 & 0 & 0 \\ 0 & 0 & 0 & G & 0 & 0 \\ 0 & 0 & 0 & 0 & \dfrac{G}{1.2} & 0 \\ 0 & 0 & 0 & 0 & 0 & \dfrac{G}{1.2} \end{bmatrix} \tag{24}$$

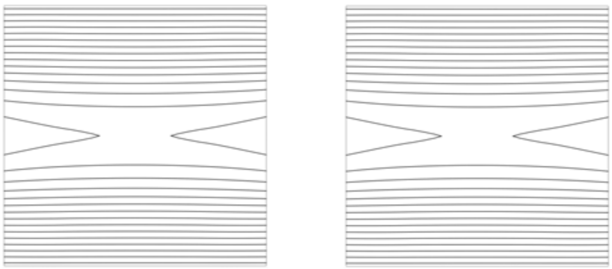
where $K = \dfrac{E_2}{3(1-2\nu_2)}$ is the bulk modulus, $G = \dfrac{E_2}{2(1+\nu_2)}$ is the shear modulus, E_2 is Young's modulus and ν_2 is Poisson's ratio of the inner layer.

3. Results of the analysis of vibrations of the multilayered polymeric film

The finite element consists of three subelements: the lower and upper plates (outer layers), and an elastic (inner) layer between them. The square piece of a polymeric film is analyzed. On the lower and upper boundaries, all the displacements are assumed as equal to zero.

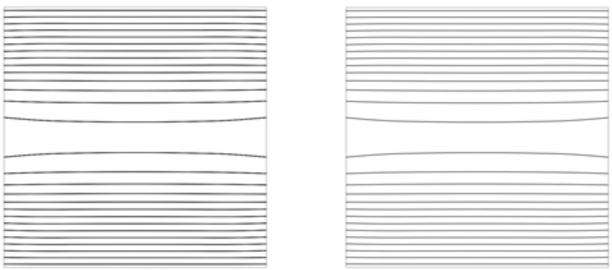
Contour plots of the transverse displacement for the lower and upper planes for the first eigenmode are presented in Fig. 2, for the second eigenmode in Fig. 3, for the fourth eigenmode in Fig. 5.

In the first eigenmode both surfaces move in the same direction, while in the second eigenmode they move in opposite directions. In the third eigenmode both surfaces move in the same direction, while in the fourth eigenmode they move in the opposite directions.



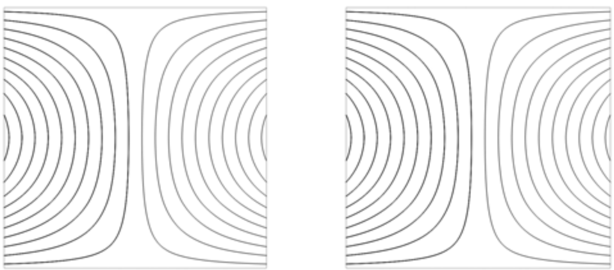
a b

Fig. 2 The first eigenmode: a) the lower surface, b) the upper surface



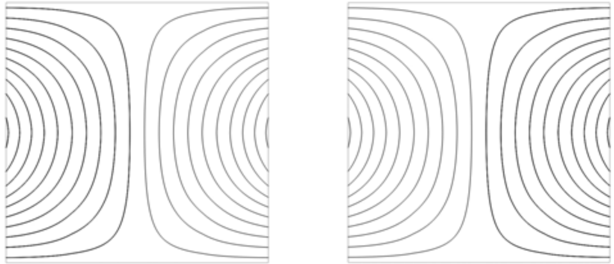
a b

Fig. 3 The second eigenmode: a) the lower surface, b) the upper surface



a b

Fig. 4 The third eigenmode: a) the lower surface, b) the upper surface



a b

Fig. 5 The fourth eigenmode: a) the lower surface, b) the upper surface

From the presented contour plots of the eigenmodes of vibrations obtained during the numerical investigation one can note that the open horizontal lines take place (Figs. 2 and 3) and lines of the shape of the part of a circle take place (Figs. 4 and 5). In the first eigenmode of vibrations (Fig. 2) in the transverse direction to the side boundaries of the analyzed multilayered polymeric film open horizontal continuous lines are seen, which bend when approaching the central part of the film. In the second eigenmode of vibrations (Fig. 3) in the transverse direction to the side boundaries of the analyzed multilayered polymeric film open horizontal continuous lines are seen. In the third (Fig. 4) and fourth (Fig. 5) eigenmodes of vibrations the lines are of the same shape: along the side boundaries the lines of the shape of the part of a circle take place. For each of the eigenmodes the contour plots of the upper and lower surfaces look similar, though in some eigenmodes both surfaces move in the same direction, while in other eigenmodes they move in the opposite directions.

4. Method of experimental investigations

In order to investigate the effect of vibrations of multilayered polymeric film when the band of the polymeric film is transported in printing machine, the experimental setup for the digital speckle photography investigations was designed and produced (Fig. 6).

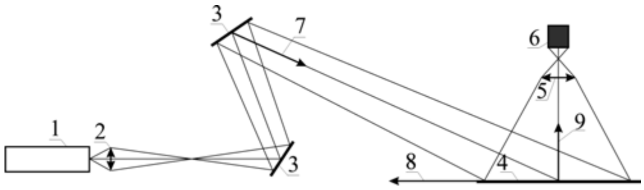
The surface of the multi-layered polymeric film was illuminated with coherent monohromatic light using the Helium–Neon laser (HN-40, corresponding to IEC 825-1:1993). This laser generates the light beam with the wavelength λ = 632.8 nm (part of the spectrum of the red colour seen by an eye). The main parameters and characteristics of the laser and of the power source are presented in Table 1.

The laser beam was expanded by using expansion lens and directed to the side of the investigated tape of polymeric material by using the system of mirrors. By

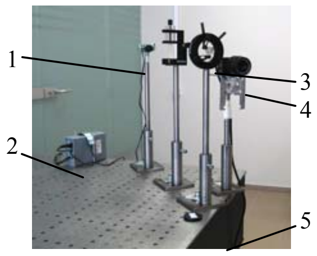
Table 1

Main parameters of the laser HN-40

	Parameter	Value
Parameters of the laser	Radiated wavelength, nm	632.8
	Maximum power, mW	39
	Polarisation	1000:1
	Diameter of the beam, mm	2.1 (±0.1)
	Expansion, mrad	2.1



a



b

Fig. 6 Experimental setup for the digital speckle photography investigations: (a) *1* – He-Ne laser, *2* – expanding lens, *3* – mirrors, *4* – the investigated polymeric material, *5* – focusing lens of the digital image camera, *6* – digital image camera, *7* – direction of illumination, *8* – direction of measurement, *9* – direction of observation; (b) *1* – He-Ne laser, *2* – power supply, *3* – divergent lenses, *4* – digital camera, *5* – antivibration table

changing frequency and amplitude the shapes of the obtained speckle images varied. At definite values of frequencies and amplitudes the images of the eigenmodes occurred.

Speckle images were registered with high resolution colour digital image camera Edmund Optics EO-1312C USB Camera. Each photo was registered with the time interval of 40 ms. Quality of the photo did not change when changing the time interval between the registrations. The main characteristics of the image camera are presented in Table 2.

Table 2

Main technical characteristics of the image camera Edmund Optics EO-1312C USB Camera

Model	EO-1312C USB Camera
Sensor Type	1/2" Progressive Scan CMOS
Pixels ($H\times V$)	1280×1024
Pixel Size ($H\times V$)	5.2 µm× 5.2 µm
Sensing Area ($H\times V$)	6.6 mm× 5.3 mm
Pixel Depth	8-bitm
Frame Rate	25 fps
Resolution	>100 lp/mm image space resolution 50% @ F4, 400 mm WD (18 mm FL meets 100 lp/mm with max 2/3" CCD)
Distortion	<0.3% @ 400mm WD <1% @ 400mm WD for 18 mm FL
Dimensions ($W\times H\times L$)	34×32×27.4 mm

Focusing lens Edmund Industrial Optics 55326 Double Gauss Focus 25 mm was used with the video-camera, which has the lens of wide viewing field and variable aperture. The main characteristics of the focusing lens are presented in Table 3.

The camera transferred the obtained images to personal computer through USB 2.0 connection where the obtained images were processed with the specialised software uc480viewer (version 2.40.0005) and their correlation analysis was performed.

Table 3

Main characteristics of the focusing lens Edmund Industrial Optics 55326 Double Gauss Focus 25 mm

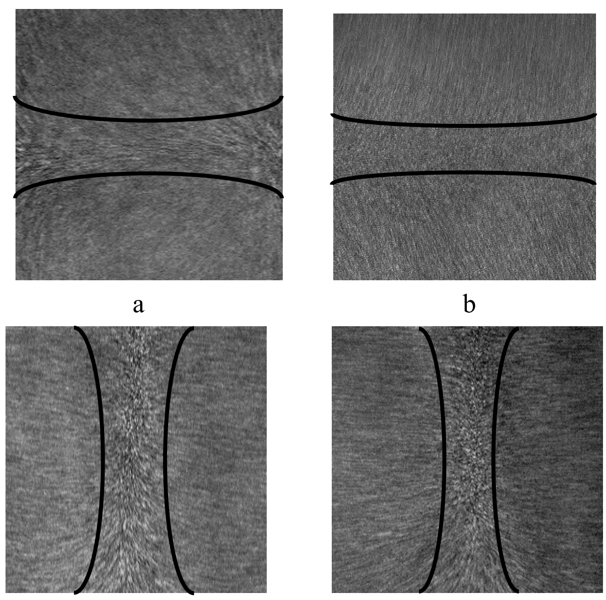
	Min.	Max.
Primary Magnification	0.106 X	∞
FOV (2/3" CCD Hor)	87.5 mm	20.7°
FOV (1/2" CCD Hor)	63.6 mm	15.1°
Resolution in Object Space (1/2" CCD)	11 lp/mm	N/A
Working Distance	240 mm	∞
Focal Length	25 mm	
Aperture (f/#)	F4 – closed	
Distance to First Lens	21.4~24.1 mm	
Filter Thread	M 30.5×0.5 mm	

In the experimental tests, samples of multilayer polymeric film of square geometrical shape (0.2×0.2 m) were produced.

The tests were carried out at the ambient temperature of 20 ± 2ºC and air humidity 65 ± 2%.

5. Results of experimental investigations and their analysis

The obtained results of the experimental investigations are presented in Fig. 7: in Fig. 7, a the image of the first eigenmode, in Fig. 7, b the image of the second eigenmode, in Fig. 7, d the image of the fourth eigenmode of the multilayered polymeric film PET+PAP+LDPE are presented.



a b c d

Fig. 7 The upper surface mode of a multilayered film PET+PAP+LDPE obtained during experimental study: a - first eigenmode – excitation frequency 171 Hz, amplitude 3.8×10^{-6}m; b - second eigenmode – excitation frequency 178 Hz, amplitude 3.6×10^{-6}m; c - third eigenmode – excitation frequency 187 Hz, amplitude 2.0×10^{-6}m; (d) fourth eigenmode – excitation frequency 199 Hz, amplitude 1.8×10^{-6}m

The analysis of the obtained results, generated by vibrations of multilayered polymeric film PET+PAP+ +LDPE, shows that overall four shapes of eigenmodes were obtained (Figs. 7, a-d), which differ in their geometrical shape, generating frequency and amplitude: at a higher number of the eigenmode the generating frequency is higher, and the amplitude decreases with increasing frequency. After the four eigenmodes are formed, higher modes are not formed at increased excitation amplitude and frequency, because the polymeric film vibrates not evenly, speckle images are located non-uniformly and it is difficult to identify the eigenmodes.

When comparing the results obtained by experimental method (Figs. 7, a-d) with the results of digital study (Figs. 2-5), it may be noted that the results according to the sequence of eigenmode shapes correlate with the digital model of the multilayered polymeric film. Obvious correspondence can be noted in terms of the geometric configuration of the shapes and the shape sequence.

Comparing the obtained experimental and numerical results of contour plots of the field of transverse displacements of the first – fourth eigenmodes of vibrations one can see that the experimental results obtained using the method of speckle photography correspond to the data from numerical investigations.

6. Conclusions

It is assumed that a layered plate has a lower layer and an upper layer of a plate type and between them there is an elastic layer. The finite element consists of three subelements: the lower and upper plates and an elastic layer between them. The square piece of polymeric film is analyzed. On the lower and upper boundaries all the displacements are assumed equal to zero.

Contour plots of the transverse displacement for the lower and upper planes for the first eigenmodes are presented. From the obtained results it is seen that there are eigenmodes in which both surfaces move in the same direction and also there are similar eigenmodes in which both surfaces move in the opposite directions.

The setup for experimental investigations was designed and created which enabled to determine the images of eigenmodes of the multilayered polymeric film PET+PAP+LDPE by using the method of digital speckle photography.

When comparing the configurations of the images of eigenmodes of the multilayered polymeric film PET+PAP+LDPE obtained by using the numerical study and the method of digital speckle photography, it may be noted that the results according to the sequence of eigenmode shapes correlate with the digital model of the multilayered polymeric film.

The obtained results are used in the process of design of construction of packaging elements.

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DAUGIASLUOKSNĖS POLIMERINĖS PLĖVELĖS VIRPESIŲ TYRIMAS

R e z i u m ė

Skaitmeniniu metodu ištirti daugiasluoksnės polimerinės plėvelės virpesiai, laikant, kad plėvelė sudaryta iš trijų sluoksnių. Viršutinis ir apatinis sluoksniai yra standūs ir nesideformuoja skersine kryptimi, o vidinis sluoksnis šia kryptimi gali deformuotis. Nustatytos pirmos keturios tikrinės formos, gauti šių formų skaitmeninio tyrimo grafiniai rezultatai.

Atlikti daugiasluoksnės polimerinės medžiagos PET+PAP+LDPE eksperimentiniai tyrimai, žadinant šią medžiagą priverstiniais virpesiais. Skaitmenine raibumų fotografija gautos pirmosios keturios tikrinės formos. Parodyta, kad, didėjant žadinimo dažniui, tikrinės formos keičiasi. Gauti skaitmeninio tyrimo rezultatai palyginti su eksperimentiniais. Jie taikomi pakavimo medžiagų charakteristikoms, savybėms ir kokybei nustatyti.

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INVESTIGATION OF VIBRATIONS OF A MULTILAYERED POLYMERIC FILM

S u m m a r y

Vibrations of a multi – layered polymeric film are investigated numerically by assuming that the film consists from three layers. The upper and lower layers are stiff and do not deform in the transverse direction, while the internal layer can deform in the transverse direction. The first four eigenmodes are analysed, graphical results of their investigation are presented.

Experimental investigations of the multi – layered polymeric material PET+PAP+LDPE are performed exciting this material by forced vibrations. By using the method of digital speckle photography the first four eigenmodes are obtained. It is shown that with the increase of excitation frequency the eigenmodes change. The obtained results of numerical investigations are compared with the experimental ones. The obtained results of investigations are applied for the determination of characteristics and qualities of packaging materials.

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ИССЛЕДОВАНИЕ ВИБРАЦИЙ МНОГОСЛОЙНОЙ ПОЛИМЕРНОЙ ПЛЕНКИ

Р е з ю м е

Колебания многослойной полимерной плёнки исследуются численно, принимая, что плёнка состоит из трёх слоёв. Верхний и нижний слои являются жёсткими и не деформируются в поперечном направлении, а внутренний слой может деформироваться в этом направлении. Определены первые четыре собственные формы, численные результаты исследований которых представлены в графической форме.

Экспериментальные исследования многослойного полимерного материала PET+PAP+LDPE выполнены возбуждая этот материал вынужденными колебаниями. Используя метод цифровой спекл фотографии получены первые четыре собственные формы. Показано, что с увеличением частоты возбуждения собственные формы изменяются. Полученные результаты численных исследований сравниваются с экспериментальными. Результаты исследований применяются при определении характеристик, свойств и качества упаковочных материалов.

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