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Investigation of adequacy of the analytical model of sound field in rectangular room

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1. Introduction

In free space the propagation of sound will radiate equally in all directions and should be described by particles or by waves. In closed rooms however the walls with their acoustic properties influence the character of the sound propagation. Additional difficulties can arise when sound travels through the screen in the room. The more reflective the surfaces of a room, the more a sound wave will be reflected and therefore the longer it will take before it is fully absorbed. There is a direct relationship between the reverberation time in a room and the absorption of the surfaces. A model for sound wave propagation in a room with heterogeneous medium, for example, a screen in the room leads to more or less efficient method for solving the wave equation, for example using the Boundary Element Method (BEM) and finds an approximate solution to the wave equation by solving the system of equations resulting from discretizing the surfaces into patches. The problem with this method is that the surface mesh must be fine enough to account for phase differences. Also, it is difficult to adapt the mesh of each surface to capture the irregularities and discontinuities of the sound field. Another possibility is to use the Finite Element Method (FEM), where the wave equation is being solved by dividing the enclosure into the elements [1]. Then the wave equation is expressed by the discrete set of linear equations for these elements. On the other hand, FEM also allows modeling energy transmission between the separate surfaces. The adequacy of the acoustic field in the room with heterogeneous medium to the real acoustic field was analyzed using FEM model and shows that the suggested theoretical model created on the basis of FEM is adequate to the real processes registered in the testing laboratory [2].

Another possibility is to describe the sound field in a room by sound particles moving around along sound rays. Such a geometrical model is using the simulation of sound in large rooms, for example the Ray tracing method and the Image Source Method. Ray tracing methods [3] find propagation paths between a source and receiver by generating rays emanating from the source (or receiver) position and following them individually as they propagate through the environment. Although this method is very general and simple to implement, it is subject to aliasing artifacts as the space of rays is sampled discretely. For instance, receiver position and diffracting edges are often approximated by volumes of space (in order to admit intersections with infinitely thin rays), which can lead to false hits and paths counted multiple times. More often, important propagation paths may be missed by all samples. Moreover, ray tracing is very compute intensive, usually taking minutes to hours to compute a receiver-dependent solution.

Image source methods compute specular reflection paths by considering virtual sources generated by mirroring the location of the sound source over each surface of the environment. The key idea is that a direct path from each virtual source has the same directionality and length as a specular reflection path. Thus, specular reflection paths can be modeled up to any order by recursive generation of virtual sources. This method is simple for rectangular rooms [4]. However, for every new receiver location, each of the virtual sources must be checked to see if it is visible to the receiver, since the specular reflection path might be blocked by a polygon or intersect a mirroring plane outside the polygon [5]. As a result, this method is practical only for computing very few specular reflections from stationary sources in simple environments. The beam tracing methods [6] find propagation paths from a source by tracing beams (i.e., bundles of rays) through a 3D polyhedral environment. In general, a set of beams is constructed that completely covers the space of rays from the source. For each beam, polygons are considered for intersection in order from front to back. The advantage of this approach is that it allows finding all propagation paths up to the termination criteria. The disadvantage is that the geometric operations required for beam tracing are more complex than for the individual paths.

This paper is intended to present a new possibility to describe the propagation of sound in an enclosed heterogeneous medium and using analytical method for the calculation of relative displacement [7, 8] of air points under the action of the sound source in a known place in a room. The created technique and obtained results of the theoretical calculation were compared to the results of practical experiment. The adequacy of the sound field model and the real room 's acoustic field was analyzed.

2. The analytical model

The analytical model of room acoustic field is based on the mathematical model derived by the calculation of relative displacements of particles of air under the action of sound source in the Cartesian system of axis. The series of calculation that are carried out are shown schematically in Fig. 1. Starting at the bottom of the figure a model of a room is created for example shown in Fig. 2. The system of axes *xy* are fixed and axes x_i, y_i can translate with respected *xy* with the speed of sound in air. So the motion of particles with respect to the frame x_i, y_i is rela-

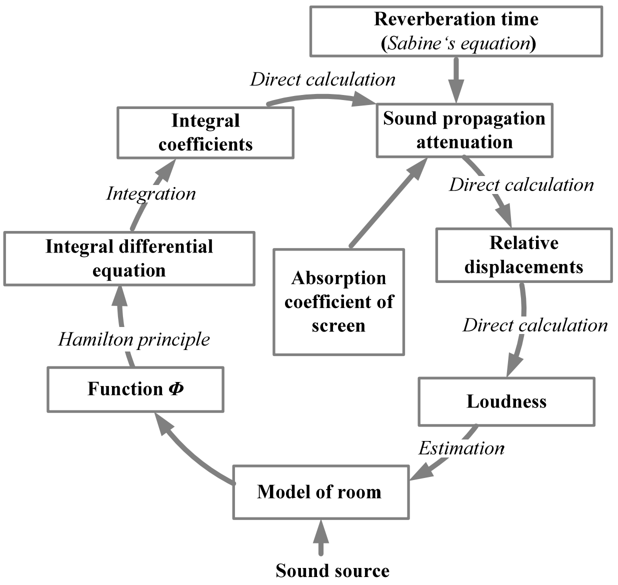

Fig. 1 Cycle of analytical model

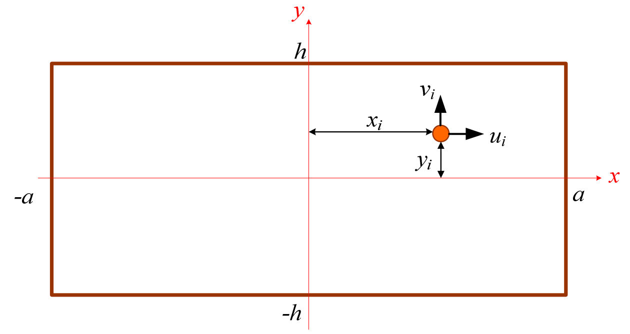

Fig. 2 The model of a room for displacement analysis

tive.

The structural model was set up in which walls of room are absolutely rigid and in equilibrium; the external volume (air mass) forces were neglected; the sound propagation is adiabatic and sound source parameters are known; the number and location particles depend on the known frequency and speed of sound in air. The aim is to calculate the relative displacement of air points under the action of the sound source in a known place in a room.

3. Mathematical model

The mathematical model is based on analytical calculation of relative displacements due to sound impact action, resulting from Hamilton principle [7, 8].

$$m_o\frac{dq}{dt} = -k_o\int_0^T q\,dt + F_L I \tag{1}$$

$$m_o = \rho_o\int_{-a}^{a}\int_{-h}^{h} F^2\,dxdy \tag{2}$$

$$k_o = \frac{\rho_o}{c^2}\int_{-a}^{a}\int_{-h}^{h}\left[\left(F_x\right)^2+\left(F_y\right)^2\right]dxdy \tag{3}$$

$$F_L = F_{LX} + F_{LY} \tag{4}$$

$$F_{LX} = \int_{-a}^{a} F\,dx \tag{5}$$

$$F_{LY} = \int_{-h}^{h} F\,dy \tag{6}$$

$$F_x = \frac{\partial F}{\partial x} \tag{7}$$

$$F_y = \frac{\partial F}{\partial y} \tag{8}$$

$$I = \int_0^T \overline{X}dt = \int_0^T \overline{Y}dt \tag{9}$$

where ρ_o is density of air; c is speed of sound in air; $\overline{X},\overline{Y}$ is projections of external surface force (line unit is subjected to that force) on coordinate axes. In the case of sound source, taking into account that the pressure of sound source is the same in all directions $(\overline{X}=\overline{Y})$; $F = F(x,y)$; $q = q(t)$.

The function F is selected on the basis of the boundary conditions, i. e. it should fit for the room presented in Fig. 2. The duration of the sound source equal reverberation time T by Sabine's reverberation formula [9] is

$$T = 0.161\frac{V}{A} \tag{10}$$

where V is the room volume in cubic meters; A is the total absorption in square meters.

In addition to determine the influence of the screen in the room for sound radiation we can use the absorption coefficient α. Sound waves more or less are absorbed by a screen. The absorption coefficients express the absorption factor of materials at given frequencies [10]. We can suggest using the multiplier ζ and following calculation technique, for example in x direction where the screen is located at $x = x_w$

$$\zeta = 1 - \alpha \tag{11}$$

$$F^*_{LX} = \int_{-a}^{x_w} F\,dx + \zeta\int_{x_w}^{a} F\,dx \tag{12}$$

In this case, the integral differential equation (1) solved approximately by means of the iteration method, for example in x direction got the fifth approximation

$$q_{ui}^{(5)} = \frac{F^*_{LX} IT}{m_o}\left(1-\frac{k_oT^2}{6m_o}+\frac{k_o^2T^4}{120m_o^2}-\frac{k_o^3T^6}{5040m_o^3}+\frac{k_o^4T^8}{362880m_o^4}\right)-\frac{k_o^5T^{11}}{39916800m_o^5} \tag{13}$$

Then, having applied equation (13), we can calculate approximate relative displacements of air particles

$$u_i = F_i q_{ui}\,;\ v_i = F_i q_{vi} \tag{14}$$

Finally, taking into account relationships of acoustic quantities associated with a plane progressive acoustic sound wave [11], loudness at any point of the room is calculated.

4. Numerical examples and experimental results

In order to determine the adequacy of the created analytical model, the experimental test was done and the obtained numerical results of the theoretical modeling can be comparing to the experimental ones. All geometrical and sound source parameters of the theoretical experiment were selected through the imitation of the real experiment where the sound pressure measurements were done in the different points around the screen using device Investigator 2260 and applying to analyze the modular precise vibration and noise analyzer PULSE 3560 [12]. In order to reduce the acoustic noise the screen was used. Fig. 3 shows the general view of this screen and sound sources.

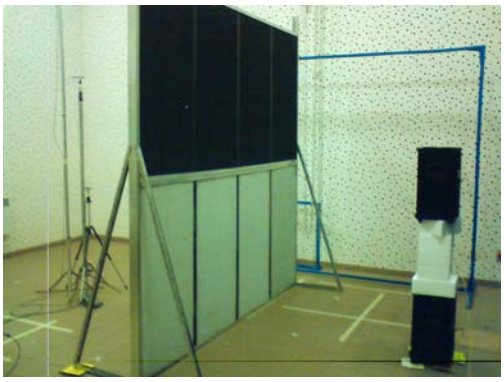

Fig. 3 General view of the screen and sound sources in the room

The problem simulated numerically and the principal scheme of the measurement experiment of sound pressure is sketched in Fig. 4.

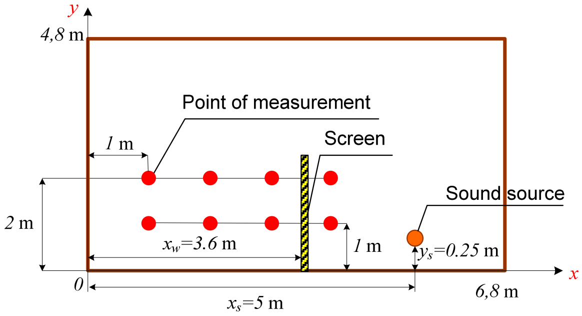

Fig. 4 The principal scheme of sound pressure measurement

For this case taking into account the model of a room for displacement analysis (Fig. 2), the geometrical values $a = 3.4$ m and $h = 2.4$ m. Let's suppose that density of air $\rho_o = 1.224$ kg/m^3, speed of sound in air $c = 343$ m/s, frequency of sound wave $\nu = 1000$ Hz and function F is selected on the basis of the boundary conditions (Fig. 2)

$$F = \frac{1}{a^{11}}\left(x^4 - a^4\right)\left(y^2 - h^2\right)\times$$
$$\times\left(k_1 a^5 + k_2 a^4 x + k_3 h^3 y^2 + a^4 y + h^5\right) \qquad (15)$$

All parameters for the calculation of Eqs. (15), (2) and (3) obtained in accordance with earlier created technique [10] are shown in Table.

Table

Parameters

Parameters	Value
k_1	-1.35661
k_2	0.678137
k_3	2.86242
m_o	5.52053
k_o	0.0000373902

The numerical examples of analytical model were done with different absorption coefficient α. Taking into account principal scheme of the sound pressure measurement (Fig. 4), the obtained results of the theoretical and experimental tests in different points of measurement are presented below in Figs. 5 and 6.

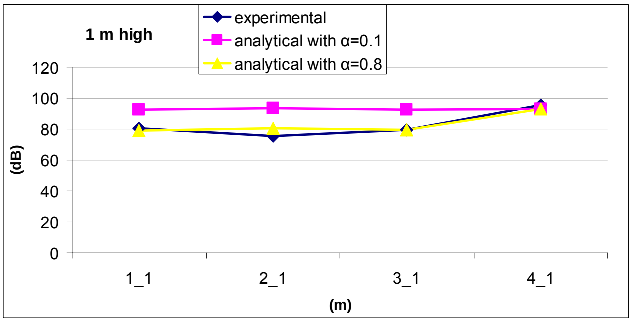

Fig. 5 Distributions values of sound pressure (dB) in separate measurement points at 1 m high

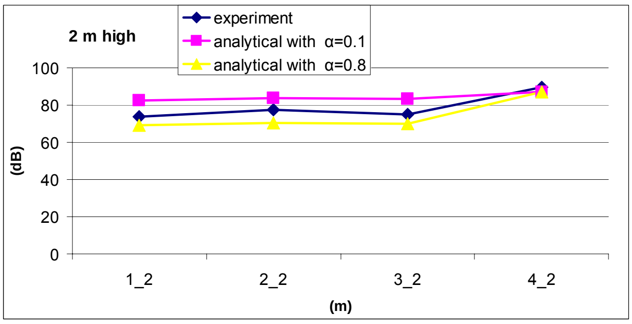

Fig. 6 Distributions values of sound pressure (dB) in separate measurement points at 2 m high

5. Conclusions

The proposed and developed analytical method allows the analysis of sound field in rectangular room. The analytical method enables:

1. to calculate approximately the displacements of air particles appeared under sound impact at a specific place of the room;
2. to create precondition for operation of acoustic field in enclosure taking into account screen in specific place of the room.

6. Acknowledgement

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AKUSTINIO LAUKO STAČIAKAMPIAME KAMBARYJE ANALIZINIO MODELIO ADEKVATUMO TYRIMAS

Reziumė

Straipsnyje, naudojantis 2D analiziniu modeliu, ištirta galimybė modeliuoti garso šaltinio, veikiančio tam tikru dažniu stačiakampiame kambaryje, kurio tam tikroje vietoje pastatytas ekranas, sukurtą akustinį lauką. Gauti teorinio eksperimento rezultatai buvo lyginami su praktinio eksperimento rezultatais. Išnagrinėtas analizinio modelio akustinio lauko adekvatumas realiam akustiniam laukui. Modeliavimo rezultatai parodė, kad, naudojant sukurtąjį analizinį modelį, galima modeliuoti akustinį lauką, žadinamą vieno dažnio šaltinio stačiakampėje uždaroje erdvėje –tam tikrų savybių turinčiame kambaryje ir vertinti garso slėgio lygį bet kuriame taške.

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INVESTIGATION OF ADEQUACY OF THE ANALYTICAL MODEL OF SOUND FIELD IN RECTANGULAR ROOM

Summary

This paper used the 2D analytical model to analyze the possibility to model sound field of the sound sources acting at certain frequency. The obtained results of the theoretical experiment were compared to the results of practical experiment. The adequacy of the acoustic field's analytical model to the real acoustic field was analyzed. Results showed that using the created analytical model it is possible to model the acoustic field, produced by restricted rectangular space – the room with certain acoustical features and estimate the sound level at any point of the room.

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ИССЛЕДОВАНИЕ АДЕКВАТНОСТИ АНАЛИТИЧЕСКОЙ МОДЕЛИ АКУСТИЧЕСКОГО ПОЛЯ В ПРЯМОУГОЛЬНОМ ПОМЕЩЕНИИ

Резюме

В статье, на базе 2D аналитической модели акустического поля, исследуется поле, создаваемое одночастотным источником в прямоугольном помещении, имеющем в определенном месте акустический экран. Полученные результаты математического моделирования сопоставлены с данными физического эксперимента и исследована адекватность аналитической модели акустического поля реальному физическому акустическому полю. Полученные результаты моделирования показали, что используя предложенную аналитическую модель, можно моделировать акустические поля, создаваемые одночастотным акустическим источником в ограниченном прямоугольном пространстве - помещении с определенными акустическими свойствами и оценивать уровень звукового давления в любой его точке.

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