# Determination of indicial response for an adaptive hydraulic system by means of the trapeze method

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## 1. Introduction

The necessity to decrease energy consumption and the growing requirements regarding technical performances of hydraulically driven systems implies finding new building solutions and applying them in the adjustment process. Both static and dynamic performances and, as well, stability of the hydraulically driven systems, are emphasized to their maximum thanks to electronic command systems that equip them. In terms of energy, the AHS hydraulic drive concept with secondary control in a constant pressure circuit, ensures substantial efficiency improvability, a fact which places the solution in front of other similar types. It is well known that, regarding hydraulic drive systems, the concept aims developing a connection to the same source of pressure that is shared by multiple consumers working independently. Thanks to this method, the required amount of energy requested by the consumers is adjusted directly by them, thus avoiding their reciprocal influence. Using hydrostatic machines with variable capacity in a system with secondary control and forced pressure allows the consumers to benefit from the added power without important losses caused by regulation elements that work based on the fluid flow stranglehold fundament [1]. Regarding rotation movements specific for hydraulic rotary motors, only the necessary power is transferred from the system in order to overtake a specific momentum at a given speed. This is accomplished by changing the capacity of the hydrostatic units that have zero-type varying capacities.

## 2. Description of the hydraulic drive diagram

In Fig. 1 you can see the diagram of a drive system with two consumers, both motors with adjustable unitary volumes integrated in an AHS *p~ct* [2]. The drive diagram was subjected to Matlab simulation [3]. Both hydraulic motors with variable capacity (*8* and *8'*) have distinct loading systems and secondary control circuits. Their charging is provided at the pressure delivered by pumps *1* and *1'* on the discharge pipe indicated with symbol *P*. At the same time, they are isolated by means of the hydraulic isolators *5* and *5'*. The system benefits from excess pressure protection systems, described in the schematic as *2* and *2'*. The tachometers *7* and *7'* can have either hydraulic or electric drive. Pressure valves *6* and *6'* mounted on the servo cylinder command circuit, work in a two-way system: firstly as relief valves and secondary as restraint valves.

Proportional controllers *10* and *10'* provide drive for the servo-motors *9* and *9'* only when the tachometers signal changes from the zero position of the rotary motors. Presence of the three key components of the secondary control system ensures equal pressure in all three components. The coefficients of these pressures are dictated by the load state of the accumulators *4* and *4'*.
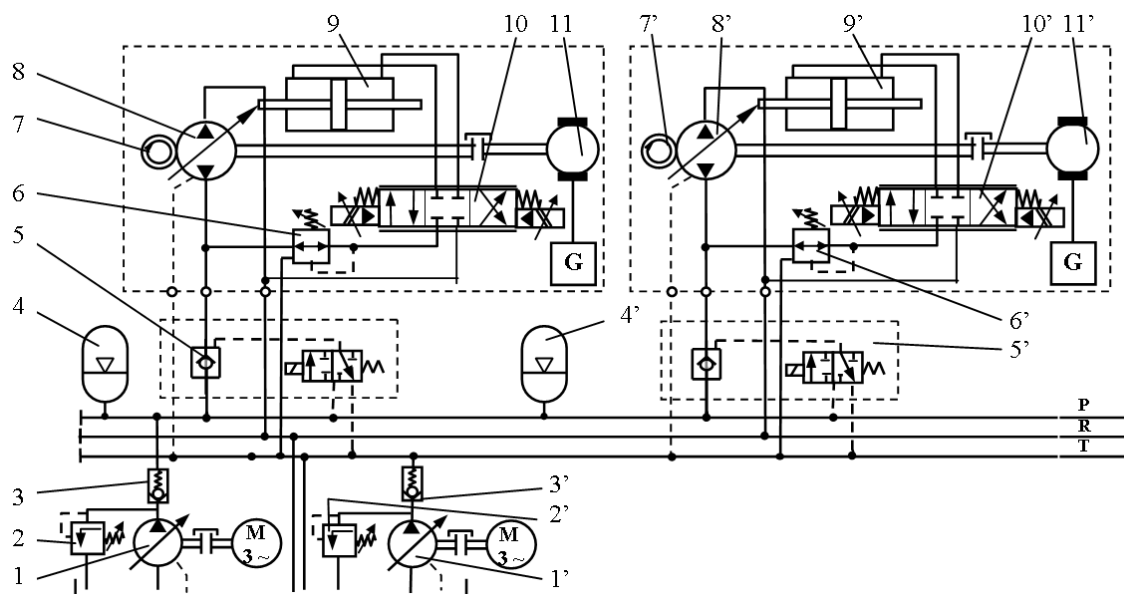


Fig. 1 Hydraulic drive with secondary regulation in an open circuit with individual regulation and hydraulic isolator

Changing the hydraulic motor torque will not produce any effects directly on the circuit's pressures system. By accelerating or decelerating the driven shaft, the flow of the hydraulic generator or motor is modified in a certain limit. The indicated flows are provided by means of pressure accumulators and can take values independent of

each other. The difference between the two flows passes to, or from the accumulators, thus modifying their load state and, respectively, pressure. This is achieved thanks to the regulation circuit that quickly intervenes and, according to the command delivered by the tachometer, alters the flow, thus ensuring operation at the rated pressure. As a result, adjusting the system to the new torque is achieved by changing the hydraulic motor capacity depending on the momentum that develops at its shaft. Also, the secondary unit has, for a given speed and depending on the torque value, a $V_2$ capacity inside the range $[-V_{2max} \div +V_{2max}]$. The system described in the above figure can be supplied by a single pump or by multiple units depending on the amount of power that the system requires.

## 3. Describing the simulation diagram in Simulink

In Fig. 2 we describe the simulation diagram that was accomplished using Matlab [3, 4]. Also, the secondary control loop was prepared by [5]. It can be noticed, from the schematic under simulation, the existence of two blocks enclosing the variable capacity hydraulic motors and the existence of the secondary control loops that work independently. The diagram also contains two variable pumps connected in parallel and included in subsystems *1* and *2*. For simulation purposes, the parameters of the two pumps can be regarded as identical or different.
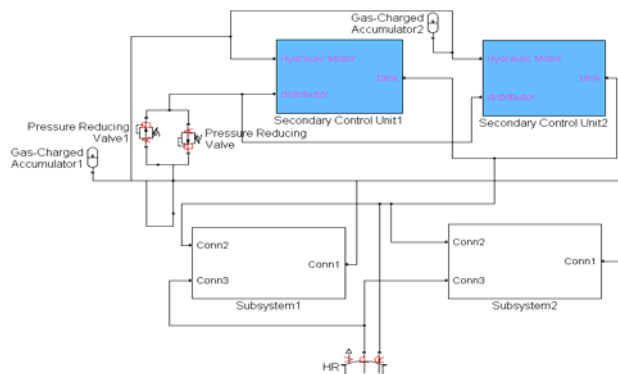


Fig. 2 Block diagram describing simulation of the SHARS

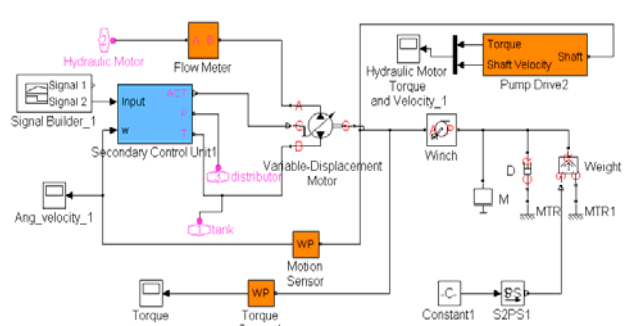In Fig. 3 we present the diagram of subsystems *1* and *2*.



Fig. 3 Diagram of the subsystems that include the secondary control units

In the schematic we depict the blocks that allow measuring angular velocity of the hydraulic motor, the flow provided to the hydraulic motor, torque, shaft velocity and the block of secondary control unit [4]. In order to

make adjustments, by means of secondary control, we used the diagram represented in Fig. 4.
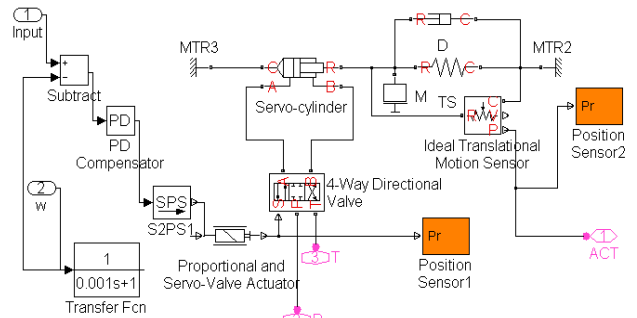


Fig. 4 Simulink Block diagram for secondary control

For the transitory regime of the angular velocity developing at a step signal inside the drive schematic that has two consumers type motors with unitary adjustable volumes integrated in SHARS, $p \sim ct$., we obtained [4], as a result of Matlab simulation, a response like the one described in Fig. 5.
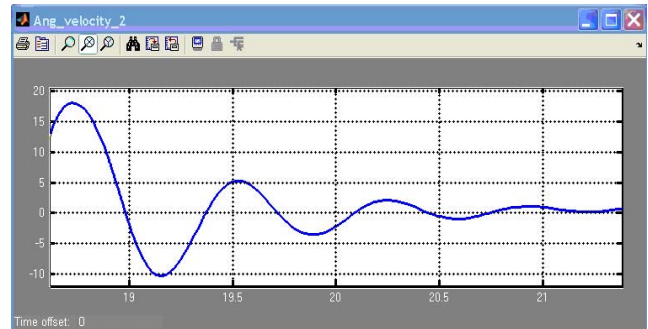


Fig. 5 Transitory indicial response of the angular speed

## 4. Discovering the indicial response using the trapezes method

The trapezes method consists in approximating the representative curve for the $P(\omega)$ variation – the real part of the transfer function of the automated system – together with line segments [6].
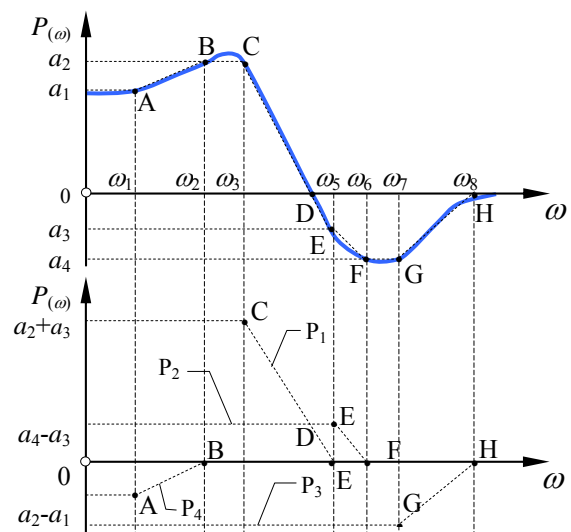


Fig. 6 Approximating the $P(\omega)$ variation using the trapezes method

These segments (Fig. 6) are chosen in a manner that, together with axes or verticals parallel to the abscissa, provides separation for a series of rectangular trapezes.

Areas of the trapezes can be regarded as positive or negative. We refer to an area as being positive when we describe trapezes that have the $P(\omega)$ value decreasing on the inclined segment in comparison to the $\omega$ value. Also, we refer to an area as being negative when $P(\omega)$ increases on the inclined segment compared to $\omega$. Positive trapezes are graphically represented above the axis $\omega$ and negative trapezes below it. Using this method [6], the curve $P(\omega)$ can be replaced with a series of line segments that have $n$ trapezes leading to the following approximation

$$P(\omega) \approx \sum_{i=1}^{n} P_i(\omega) \qquad (1)$$

Regarding the example described in Fig. 6, the $y(t)$ response for a lead-in considered as being an unitary step type [6], will be

$$y(t)\Big|_1 = \frac{2}{\pi} \sum_{i=1}^{n} \int_0^{\infty} P_i(\omega) \sin(\omega t) \frac{d\omega}{\omega} \qquad (2)$$

The above relation allows summing simple integrals determined by standard trapezes that have the same shape as an elementary trapeze [6], like the one referred hereunder (Fig. 7).
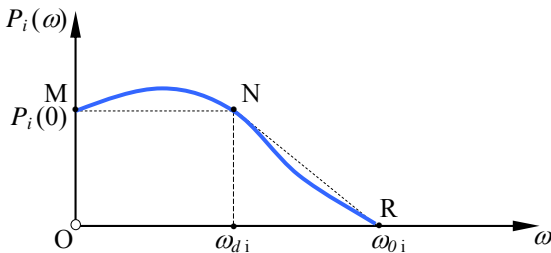


Fig. 7 Adjusting $P(\omega)$ according to the elementary trapeze OMNR

The $P_i(\omega)$ variation that will be in accordance with the OMNR elementary trapeze described in Fig. 7, will be a discontinuous function defined as referred hereunder:

$$P_i(\omega) = \begin{cases} P_i(0), & \omega \in \left[0, \omega_{di}\right] \\ P_i(0)\dfrac{\omega_{0i} - \omega}{\omega_{0i} - \omega_{di}}, & \omega \in \left[\omega_{di}, \omega_{0i}\right] \\ 0, & \omega \in \left[\omega_{0i}, \infty\right] \end{cases} \qquad (3)$$

The indicial response for the elementary trapeze, which was previously obtained by means of Eq. 3, is

$$y(t)\Big|_1 = \frac{2}{\pi} \int_0^{\omega_{di}} P_i(0) \sin(\omega t) \frac{d\omega}{\omega} +$$

$$+ \frac{2}{\pi} \int_{\omega_{di}}^{\omega_{0i}} P_i(0) \frac{\omega_{0i} - \omega}{\omega_{0i} - \omega_{di}} \sin(\omega t) \frac{d\omega}{\omega} \qquad (4)$$

The first part of Eq. 4 was integrated in Mathcad inside the integration limits specific for the $\omega$ value. For the second part of the same equation, we used the following relations, described by [6]

$$P_\omega := \begin{vmatrix} \text{for } i \in 0..lenght(P)-1 \\ \quad P_{\omega i} \leftarrow \dfrac{2}{\pi} \int_0^i \dfrac{P_i \sin(\omega t)}{\omega} d\omega \\ P_\omega \end{vmatrix} \qquad (5)$$

$$P := \begin{vmatrix} \text{for } i \in 8, ..., 15 \\ \quad P_i \leftarrow \left(1 - \dfrac{i - \omega_{di}}{\omega_{oi} - \omega_{di}}\right) P_{io} \\ P \end{vmatrix} \qquad (6)$$

$$data := \begin{pmatrix} 0 & 16.136 \\ 1 & 12.531 \\ ... & 2.431 \\ n & -2.779 \end{pmatrix} \quad X := data^{\langle 0 \rangle} \quad Y := data^{\langle 1 \rangle} \qquad (7)$$

Bellow is defined a vector of guesses

$$Guess := \begin{pmatrix} 2 \\ 2 \\ 0 \end{pmatrix} \qquad (8)$$

The following *pwrfit* is used to find the parameters in the model function

$$cpw := pwrfit(X, Y, Guess) \quad cpw = \begin{pmatrix} -5.198 \\ 0.551 \\ 16.881 \end{pmatrix} \qquad (9)$$

It is necessary to define a function using these coefficients, and then define a range variable over which to graph the function

$$h_2(x) := cpw_0 \, x^{cpw_1} + cpw_2 \quad x := min(X), ..., max(X) \quad (10)$$

Due to the fact that the indicial response has geometrical symmetry, we mathematically created models, in Mathcad, on different segments. The selected areas for creating models are similar to the ones described in Fig. 7. Solving the integrals has led, for the first two areas subjected to mathematical modelling, to the charts described in Fig. 8.
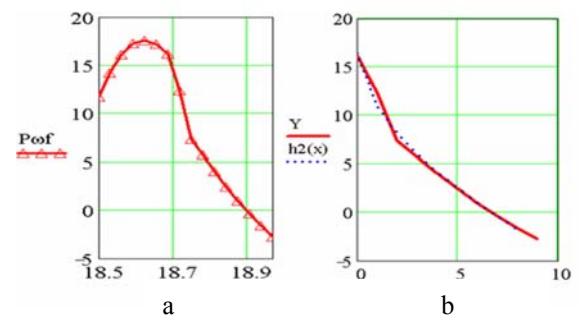


Fig. 8 First part of the curve $P(\omega)$: a - calculated, b - the descending portion of the curve interpolated

The Mathcad results obtained with Eqs. 5 and 6 (left of Figs. 8 and 9) can be interpolated using Eqs. 7-10, and the resulted graphs are found on the right of the same figure.
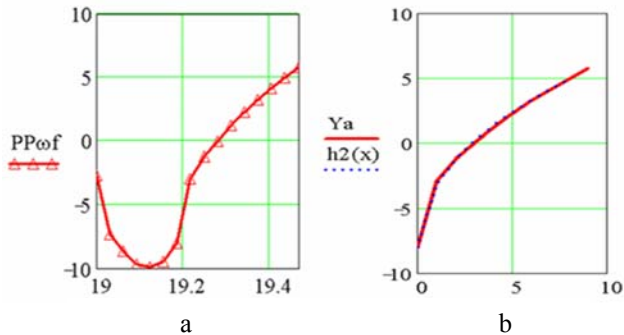


Fig. 9 Second part of the curve $P(\omega)$: a - calculated, b - the ascending portion of the curve interpolated

## 5. Conclusions

In this paper we established that the mathematical modelling to determine the indicial response, although apparently simple and, as well, convenient (given that it is a graph-analytic method) can induce inaccuracies due to successive interpolations. However, Inaccuracies can be considered acceptable in order to determine the indicial response of an automated drive system. As an application, the described method allows the analytical determination of both the indicial response, by means of the trapeze method applied to an Adaptive Hydraulic System with Secondary Control, and the response quality of the transient regimes.

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ADAPTYVIOS HIDRAULINĖS SISTEMOS ATSKIRO SPRENDINIO NUSTATYMAS TRAPECIJOS METODU

R e z i u m ė

Straipsnio pradžioje aprašoma, kokiu pagrindu kuriama sistemos antrinė kontrolė. Tai leidžia suprasti hidraulinės pavaros atviros grandinės su antrinės kontrolės bei individualaus reguliavimo sistema ir hidrauline izoliacija diagramą. Aprašius struktūrinės diagramos imitavimą „Matlab" aplinkoje ir antrinės subsistemos kontrolės detales, parodoma, kaip atskiras sprendinys gali būti nustatytas trapecijos metodu. Praktiniam pritaikymui, naudojant Mathcad aplinką sudarytas skaičiavimo modelis ir palyginti imitavimo ir trapecijos metodais gauti rezultatai.

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DETERMINATION OF INDICIAL RESPONSE FOR AN ADAPTIVE HYDRAULIC SYSTEM BY MEANS OF THE TRAPEZE METHOD

S u m m a r y

At the beginning of the paper, we have described the fundaments that lead to developing secondary control systems, after which we had an in depth presentation of the diagram for the Hydraulic drive with secondary control in an open circuit with individual regulation and hydraulic isolator. After describing the block diagram for Matlab simulation and the details regarding secondary control subsystems, we showed how the indicial response can be determined by means of the trapeze method. As an application, we assembled a calculus model using Mathcad and we used it to compare the indicial response to the one obtained by simulation.

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ОПРЕДЕЛЕНИЕ ЕДИНИЧНОГО РЕШЕНИЯ АДАПТИВНОЙ ГИДРАВЛИЧЕСКОЙ СИСТЕМОЙ МЕТОДОМ ТРАПЕЦИИ

Р е з ю м е

В данной статье представлены основы разработки вторичного контроля системы. Это дает ясное представление о диаграмме гидравлической передачи с вторичным контролем в открытой цепи, индивидуальным регулированием и гидравлической изоляцией. После описания имитирования блочной диаграммы в Matlab системе и деталей вторичного контроля субсистемы показано, что единичное решение может быть определено методом трапеции. Для применения на практике составлена расчетная модель в Mathcad системе и сопоставлены результаты единичного решения и имитирования.