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Elastic analysis of multilayered thick-walled spheres under external load

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1. Introduction

Working on issues of walking of anthropomorphic mechanisms, the author had to meet a problem of simulation of mobile articulations of kinematic links of devices, in the form of multilayer elements [1-4]. This research was conducted by a trial-and-error way. However, in the process of models complication the need of their thoretical description arose. The stuff set forth in the given article, is a theoretical generalization of models of a hip joint modelled with a different degree of accuracy, with allowance for its structure being multilayer and multicomponent [5-11]. Thus the solution of the given problem has an independent nature and can be applied to the description of any multilayer structures in compliance with the suppositions, used in the given model. Further on, being set the concrete values of the number of the spheres, it is possible to receive particular solutions. Each solution for a definite model is complete and they can be directly used performing calculations in actual situations arising in practice.

The object of the work is the study of stress and deformations distribution in a system of thick-walled spheres under external pressure.

The analytical solution, as presented, is obtained for the first time and is complete.

2. Main equations and dependences

Let us create a universal mathematical model for different arbitrary finite number of layers, of which the system of spheres consists.

Let us then consider, as the first approaching, a model – a hollow sphere, with the outer radius R_1 and internal radius R_2, under external pressure p_1 and internal pressure p_2 (Fig. 1).

L.D. Landau and E.M. Livschitz [12] defining deformation of a hollow sphere give the solution of the problem for a single-layer shell. They obtained components of strain tensor in spatial spherical coordinates

$$u_{rr} = a - \frac{2b}{r^3} \qquad u_{\theta\theta} = u_{\varphi\varphi} = a + \frac{b}{r^3} \tag{1}$$

and radial pressure (voltage, stress)

$$\sigma_{rr} = \frac{E}{(1+\sigma)(1-2\sigma)}\left[(1-\sigma)u_{rr} + 2\sigma u_{\theta\theta}\right] = \\ = \frac{E}{1-2\sigma}a - \frac{2E}{1+\sigma}\frac{b}{r^3} \tag{2}$$

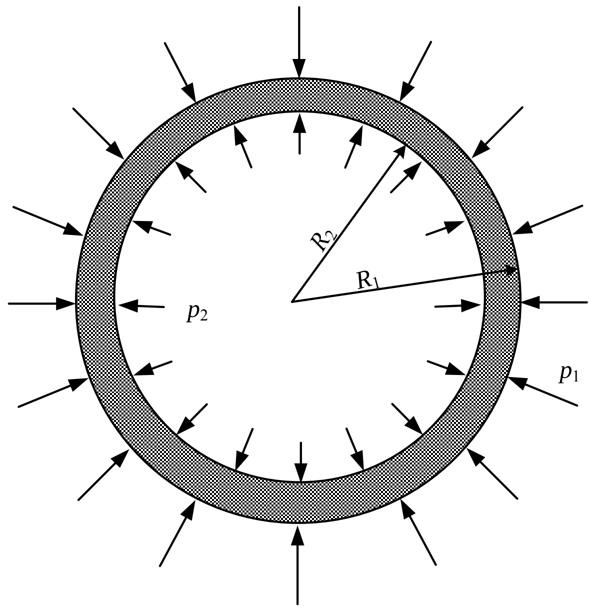

Fig. 1 Model of a single-layer sphere under external and internal pressure

The constants a and b are determined by them from boundary conditions: $\sigma_{rr} = -p_2$ at $r = R_2$ and $\sigma_{rr} = -p_1$ at $r = R_1$:

$$a = \frac{p_1R_1^3 - p_2R_2^3}{R_2^3 - R_1^3}\frac{1-2\sigma}{E}, \; b = \frac{R_1^3R_2^3(p_1 - p_2)}{R_2^3 - R_1^3}\frac{1+\sigma}{2E} \tag{3}$$

The authors limited themselves to this solution. But it opens a way to the solution of more complex problems, and includes the possibility to describe a model, analytically offered by us.

On the basis of the resuls offered by L. D. Landau and E.M. Livschitz it is possible to present the formulas of stress distribution on the depth of spherical layer for the considered single-layer sphere.

$$\left.\begin{aligned} \sigma_{rr} &= \frac{1}{R_2^3 - R_1^3}\left(p_1R_1^3 - p_2R_2^3 - \frac{R_1^3R_2^3}{r^3}(p_1 - p_2)\right) \\ \sigma_{\theta\theta} = \sigma_{\phi\phi} &= \frac{1}{R_2^3 - R_1^3}\left(p_1R_1^3 - p_2R_2^3 + \frac{R_1^3R_2^3}{2r^3}(p_1 - p_2)\right) \end{aligned}\right\} \tag{4}$$

where $\sigma_{\theta\theta}$ and $\sigma_{\varphi\varphi}$ are angular components of stress tensor.

Similarly, substituting Eq. (3) in Eq. (1) we receive components of the strain tensor.

$$\left.\begin{aligned}
&u_{rr}=\frac{p_1R_1^3-p_2R_2^3}{R_2^3-R_1^3}\frac{1-2\sigma}{E}-\frac{1}{r^3}\frac{R_1^3R_2^3\left(p_1-p_2\right)}{R_2^3-R_1^3}\frac{1+\sigma}{E}\\
&u_{\theta\theta}=u_{\phi\phi}=\frac{p_1R_1^3-p_2R_2^3}{R_2^3-R_1^3}\frac{1-2\sigma}{E}+\\
&+\frac{1}{r^3}\frac{R_1^3R_2^3\left(p_1-p_2\right)}{R_2^3-R_1^3}\frac{1+\sigma}{2E}
\end{aligned}\right\}\quad(5)$$

Thus, the first approaching for one sphere under the load is reviewed.

Further on there is a problem of extention of the given consideration on the greater, arbitrary, finite number of shells (envelopes) n.

3. Method and construction of the solution

Let us consider n shells and a sphere inside (Fig. 2). The outer shell is under external pressure p_1. Under pressure there is a deformation of the outer shell, it changes its shape, contracts, taking on an inner shell p_2. The inner shell takes on the outer shell with same pressure modulus p_2. In its turn, the inner shell, under pressure p_2 takes on the following shell and so on until the pressure is transmitted from the last shell to the sphere. It is located inside under pressure p_n. The sphere takes on the last shell with the same pressure.

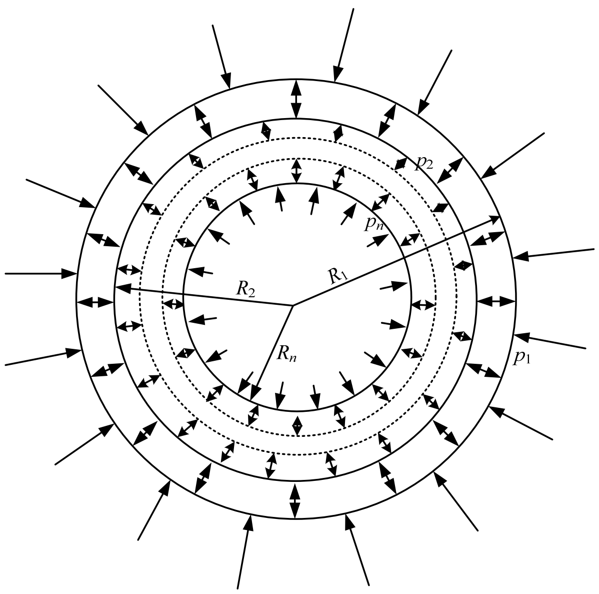

Fig. 2 Model of a *n*-layer sphere under external and internal pressure

Let us copy the Eqs. (4) and (5) of stress distribution on the depth of each spherical layer for the considered multilayer sphere

$$\left.\begin{aligned}
&\sigma_{rri}=\frac{1}{R_{i+1}^3-R_i^3}\left(p_iR_i^3-p_{i+1}R_{i+1}^3-\frac{R_i^3R_{i+1}^3}{r^3}\left(p_i-p_{i+1}\right)\right)\\
&\sigma_{\theta\theta i}=\sigma_{\phi\phi i}=\\
&=\frac{1}{R_{i+1}^3-R_i^3}\left(p_iR_i^3-p_{i+1}R_{i+1}^3+\frac{R_i^3R_{i+1}^3}{2r^3}\left(p_i-p_{i+1}\right)\right)
\end{aligned}\right\}\quad(6)$$

where σ_{rri} is radial component of the stress tensor of *i*-sphere, $\sigma_{\theta\theta i}$ and $\sigma_{\varphi\varphi i}$ are angular components of the stress tensor of *i*-sphere.

Similarly we receive the formulas for components of the strain tensor.

$$\left.\begin{aligned}
&u_{rri}=\frac{p_iR_i^3-p_{i+1}R_{i+1}^3}{R_{i+1}^3-R_i^3}\frac{1-2\sigma_i}{E_i}-\\
&-\frac{R_i^3R_{i+1}^3\left(p_i-p_{i+1}\right)}{r^3\left(R_{i+1}^3-R_i^3\right)}\frac{1+\sigma_i}{E_i}\\
&u_{\theta\theta i}=u_{\phi\phi i}=\frac{p_iR_i^3-p_{i+1}R_{i+1}^3}{R_{i+1}^3-R_i^3}\frac{1-2\sigma_i}{E_i}+\\
&+\frac{R_i^3R_{i+1}^3\left(p_i-p_{i+1}\right)}{r^3\left(R_{i+1}^3-R_i^3\right)}\frac{1+\sigma_i}{2E_i}
\end{aligned}\right\}\quad(7)$$

where u_{rri} is radial component of the strain tensor of *i*-sphere, $u_{\theta\theta i}$ and $u_{\varphi\varphi i}$ are angular components of the strain tensor of *i*-sphere.

Supposing in formula (7) $r=R_i$, (i = 2, 3, …, n) where R_i is internal radius of the outer and the inner spheres, from radial components of the strain tensor we derive a system of n equations, linear in relation to the unknowns of pressure p_i (i = 2, 3, …, n). We receive the system of equations which was written in supposition that all the spheres have identical elastic modules.

$$\left\{\begin{aligned}
&\frac{p_1R_1^3-p_2R_2^3}{R_2^3-R_1^3}\frac{1-2\sigma}{E}-\frac{R_1^3\left(p_1-p_2\right)}{R_2^3-R_1^3}\frac{1+\sigma}{E}=\\
&=\frac{p_2R_2^3-p_3R_3^3}{R_3^3-R_2^3}\frac{1-2\sigma}{E}-\frac{R_3^3\left(p_2-p_3\right)}{R_3^3-R_2^3}\frac{1+\sigma}{E},\\
&\frac{p_2R_2^3-p_3R_3^3}{R_3^3-R_2^3}\frac{1-2\sigma}{E}-\frac{R_2^3\left(p_2-p_3\right)}{R_3^3-R_2^3}\frac{1+\sigma}{E}=\\
&=\frac{p_3R_3^3-p_4R_4^3}{R_4^3-R_3^3}\frac{1-2\sigma}{E}-\frac{R_4^3\left(p_3-p_4\right)}{R_4^3-R_3^3}\frac{1+\sigma}{E},\\
&\frac{p_3R_3^3-p_4R_4^3}{R_4^3-R_3^3}\frac{1-2\sigma}{E}-\frac{R_3^3\left(p_3-p_4\right)}{R_4^3-R_3^3}\frac{1+\sigma}{E}=\\
&=\frac{p_4R_4^3-p_5R_5^3}{R_5^3-R_4^3}\frac{1-2\sigma}{E}-\frac{R_5^3\left(p_4-p_5\right)}{R_5^3-R_4^3}\frac{1+\sigma}{E},\\
&\ldots\\
&\frac{p_{n-1}R_{n-1}^3-p_nR_n^3}{R_n^3-R_{n-1}^3}\frac{1-2\sigma}{E}-\frac{R_{n-1}^3\left(p_{n-1}-p_n\right)}{R_n^3-R_{n-1}^3}\frac{1+\sigma}{E}=\\
&=-p_n\frac{1-2\sigma}{E}
\end{aligned}\right.\quad(8)$$

Solving this system of equations we find (as σ and E are identical for all the spheres), that $p_2=p_3=\ldots=p_n=p_1$, i.e. for a homogeneous sphere. It is a natural outcome that proofs the mathematical description of the model. In reality, elastic modules of materials E and σ are different. It will be taken into further consideration of system (8) of equations with the help of indexes. For the outer shell there is index 1, for inner - 2, and so on, and for the sphere - n.

Then the system of Eqs. (8) becomes

$$\begin{cases} \frac{p_1R_1^3 - p_2R_2^3}{R_2^3 - R_1^3}\frac{1-2\sigma_1}{E_1} - \frac{R_1^3\left(p_1 - p_2\right)}{R_2^3 - R_1^3}\frac{1+\sigma_1}{E_1} = \\ = \frac{p_2R_2^3 - p_3R_3^3}{R_3^3 - R_2^3}\frac{1-2\sigma_2}{E_2} - \frac{R_3^3\left(p_2 - p_3\right)}{R_3^3 - R_2^3}\frac{1+\sigma_2}{E_2}, \\ \frac{p_2R_2^3 - p_3R_3^3}{R_3^3 - R_2^3}\frac{1-2\sigma_2}{E_2} - \frac{R_2^3\left(p_2 - p_3\right)}{R_3^3 - R_2^3}\frac{1+\sigma_2}{E_2} = \\ = \frac{p_3R_3^3 - p_4R_4^3}{R_4^3 - R_3^3}\frac{1-2\sigma_3}{E_3} - \frac{R_4^3\left(p_3 - p_4\right)}{R_4^3 - R_3^3}\frac{1+\sigma_3}{E_3}, \\ \frac{p_3R_3^3 - p_4R_4^3}{R_4^3 - R_3^3}\frac{1-2\sigma_3}{E_3} - \frac{R_3^3\left(p_3 - p_4\right)}{R_4^3 - R_3^3}\frac{1+\sigma_3}{E_3} = \\ = \frac{p_4R_4^3 - p_5R_5^3}{R_5^3 - R_4^3}\frac{1-2\sigma_4}{E_4} - \frac{R_5^3\left(p_4 - p_5\right)}{R_5^3 - R_4^3}\frac{1+\sigma_4}{E_4}, \\ \dots \\ \frac{p_{n-1}R_{n-1}^3 - p_nR_n^3}{R_n^3 - R_{n-1}^3}\frac{1-2\sigma_{n-1}}{E_{n-1}} - \\ - \frac{R_{n-1}^3\left(p_{n-1} - p_n\right)}{R_n^3 - R_{n-1}^3}\frac{1+\sigma_{n-1}}{E_{n-1}} = -p_n\frac{1-2\sigma_n}{E_n} \end{cases} \quad (9)$$

Let us introduce the following notations:

$$\frac{1-2\sigma_i}{E_i} = a_i, \quad \frac{1+\sigma_i}{E_i} = b_i, \quad R_i^3 = d_i, \quad \frac{1}{R_{i+1}^3 - R_i^3} = c_i$$

($i = 1, 2, 3, \dots, n$).

The system of equations (9) becomes

$$\begin{cases} c_1a_1\left(p_1d_1 - p_2d_2\right) - \\ - d_1c_1b_1\left(p_1 - p_2\right) = \\ = c_2a_2\left(p_2d_2 - p_3d_3\right) - \\ - d_3c_2b_2\left(p_2 - p_3\right), \\ c_2a_2\left(p_2d_2 - p_3d_3\right) - \\ - d_2c_2b_2\left(p_2 - p_3\right) = \\ = c_3a_3\left(p_3d_3 - p_4d_4\right) - \\ - d_4c_3b_3\left(p_3 - p_4\right), \\ c_3a_3\left(p_3d_3 - p_4d_4\right) - \\ - d_3c_3b_3\left(p_3 - p_4\right) = \\ = c_4a_4\left(p_4d_4 - p_5d_5\right) - \\ - d_5c_4b_4\left(p_4 - p_5\right), \\ \dots \\ c_{n-1}a_{n-1}\left(p_{n-1}d_{n-1} - p_nd_n\right) - \\ - d_{n-1}c_{n-1}b_{n-1}\left(p_{n-1} - p_n\right) = -p_na_n. \end{cases} \quad (10)$$

As pressure applied to the outer sphere p_1 is known, we shall copy the system of Eqs. (10), linear in relation to the unknowns of pressure $p_2, p_3, \dots, p_n$ in the standard form.

$$\begin{cases} p_2\left(d_1c_1b_1 + d_3c_2b_2 - d_2c_1a_1 - d_2c_2a_2\right) + \\ + p_3\left(d_3c_2a_2 - d_3c_2b_2\right) = \\ = p_1\left(d_1c_1b_1 - d_1c_1a_1\right), \\ p_2\left(d_2c_2a_2 - d_2c_2b_2\right) + \\ + p_3\left(d_2c_2b_2 + d_4c_3b_3 - d_3c_2a_2 - d_3c_3a_3\right) + \\ + p_4\left(d_4c_3a_3 - d_4c_3b_3\right) = 0, \\ p_3\left(d_3c_3a_3 - d_3c_3b_3\right) + \\ + p_4\left(d_3c_3b_3 + d_5c_4b_4 - d_4c_3a_3 - d_4c_4a_4\right) + \\ + p_5\left(d_5c_4a_4 - d_5c_4b_4\right) = 0, \\ \dots \\ p_{n-1}\left(d_{n-1}c_{n-1}a_{n-1} - d_{n-1}c_{n-1}b_{n-1}\right) + \\ + p_n\left(d_{n-1}c_{n-1}b_{n-1} - d_nc_{n-1}a_{n-1} + a_n\right) = 0 \end{cases} \quad (11)$$

The system of Eqs. (11) can be presented in a more compact view if we introduce the following notations

$$e_i = d_{i-1}c_{i-1}b_{i-1} + d_{i+1}c_ib_i - d_ic_{i-1}a_{i-1} - d_ic_ia_i$$

$$f_i = d_ic_{i-1}a_{i-1} - d_ic_{i-1}b_{i-1}$$

$$g_i = d_ic_ia_i - d_ic_ib_i$$

$$h_i = d_{i-1}c_{i-1}b_{i-1} - d_ic_{i-1}a_{i-1} + a_i$$

where $i = 1, 2, \dots, n$.

The system (11) becomes

$$\begin{cases} p_2e_2 + p_3f_3 = -p_1g_1, \\ p_2g_2 + p_3e_3 + p_4f_4 = 0, \\ p_3g_3 + p_4e_4 + p_5f_5 = 0, \\ \dots \\ p_{n-1}g_{n-1} + p_nh_n = 0. \end{cases} \quad (12)$$

Let us record determinant of the system (12)

$$\Delta_1 = \begin{vmatrix} e_2 & f_3 & 0 & 0 & \dots & 0 & 0 \\ g_2 & e_3 & f_4 & 0 & \dots & 0 & 0 \\ 0 & g_3 & e_4 & f_5 & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & 0 & \dots & g_{n-1} & h_n \end{vmatrix} \quad (13)$$

Let us set down the determinants received from Δ_1 by replacing of the column drawn up with factors at the corresponding unknown, with the column drawn up with the free members of the system of equations

$$\Delta_2 = \begin{vmatrix} -p_1g_1 & f_3 & 0 & 0 & \dots & 0 & 0 \\ 0 & e_3 & f_4 & 0 & \dots & 0 & 0 \\ 0 & g_3 & e_4 & f_5 & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & 0 & \dots & g_{n-1} & h_n \end{vmatrix}$$

$$\Delta_3 = \begin{vmatrix} e_2 & -p_1 g_1 & 0 & 0 & \dots & 0 & 0 \\ g_2 & 0 & f_4 & 0 & \dots & 0 & 0 \\ 0 & 0 & e_4 & f_5 & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & 0 & \dots & g_{n-1} & h_n \end{vmatrix}$$

$$\Delta_4 = \begin{vmatrix} e_2 & f_3 & -p_1 g_1 & 0 & \dots & 0 & 0 \\ g_2 & e_3 & 0 & 0 & \dots & 0 & 0 \\ 0 & g_3 & 0 & f_5 & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & 0 & \dots & g_{n-1} & h_n \end{vmatrix} \quad (14)$$

$$\Delta_5 = \begin{vmatrix} e_2 & f_3 & 0 & -p_1 g_1 & \dots & 0 & 0 \\ g_2 & e_3 & f_4 & 0 & \dots & 0 & 0 \\ 0 & g_3 & e_4 & 0 & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & 0 & \dots & g_{n-1} & h_n \end{vmatrix}$$

$$\dots$$

$$\Delta_n = \begin{vmatrix} e_2 & f_3 & 0 & 0 & \dots & 0 & -p_1 g_1 \\ g_2 & e_3 & f_4 & 0 & \dots & 0 & 0 \\ 0 & g_3 & e_4 & f_5 & \dots & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & 0 & \dots & g_{n-1} & 0 \end{vmatrix}$$

Further on, solving the given linear system of Eqs. (12), consisting of $n-1$ equations with $n-1$ unknowns, using the written out determinants (13) and (14), under the formulas of Cramer, we discover unknowns of pressure $p_2, p_3, \dots, p_n$.

$$\begin{cases} p_2 = \dfrac{\Delta_2}{\Delta_1} \\ p_3 = \dfrac{\Delta_3}{\Delta_1} \\ p_4 = \dfrac{\Delta_4}{\Delta_1} \\ p_5 = \dfrac{\Delta_5}{\Delta_1} \\ \dots \\ p_n = \dfrac{\Delta_n}{\Delta_1} \end{cases} \quad (15)$$

Substituting the found values of pressure on intermediate shells and the sphere in Eqs. (6) and (7), we receive stress and deformations distributions inside the outer shell, the inner shell, the following shell and so on up to the sphere accordingly. In view of alwardness of received expressions, they are not given here.

Thus, the stress and pressure tensors for the offered system of shells and a sphere inside are obtained. Therefore, the considered problem is solved.

4. Conclusions

1. In the given work a method of solution of a problem of defining the component of stress and deformation tensor for the system of spheres with different properties of materials is offered for the first time.

2. The given approach allows to receive analytically the values of pressure on the limits and inside the spheres, and also stress and pressure tensors.

3. Theoretical findings obtained in the given study may be used in practice.

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DAUGIASLUOKSNIŲ STORASIENIŲ SFERŲ VEIKIAMŲ IŠORINE APKROVA TAMPRUS TYRIMAS

R e z i u m ė

Praktikoje dažnai tenka analizuoti sistemas, kuriose naudojami daugiasluoksniai objektai su skirtingų savybių sluoksniais. Straipsnyje tiriamos deformacijos ir įtempiai sistemoje, sudarytoje iš bet kurio skaičiaus strorasienių sferų. Kiekviena sfera apibūdinama savo tamprumo moduliu, be to, kiekviena sfera su gretima sfera liečiasi visu paviršiumi. Šis uždavinys sprendžiamas analitiškai. Nustatytos slėgio pasiskirstymo sferų paviršiuose priklausomybės bei deformacijų ir įtempių tenzoriai sferų viduje. Gauti teoriniai rezultatai turi svarbią praktinę reikšmę.

A.V. Borisov

ELASTIC ANALYSIS OF MULTILAYERED THICK-WALLED SPHERES UNDER EXTERNAL LOAD

S u m m a r y

In practice it is often necessary to conduct analysis of systems, with multilayer objects, each layer of which has individual properties. Deformations and stresses inside a system consisting of an arbitrary finite number of thick-walled spheres are investigated in the present article. Each sphere is characterized by its elastic modules. The zone of contact between each of the spheres is continuous on the surface. This problem can be completely solved analyticaly. The relations of pressure on the limits of the sphres, stress tensors and deformations inside the spheres are obtained. The findings have the relevant practical value.

А.В. Борисов

УПРУГИЙ АНАЛИЗ МНОГОСЛОЙНЫХ ТОЛСТОСТЕННЫХ СФЕР ПОД ВНЕШНЕЙ НАГРУЗКОЙ

Р е з ю м е

В практике достаточно часто приходится проводить анализ систем, в которых присутствуют многослойные объекты, каждый слой, которого имеет свои индивидуальные свойства. В статье исследуются деформации и напряжения внутри системы, состоящей из произвольного конечного количества толстостенных сфер. Каждая сфера характеризуется своими упругими модулями. Зона контакта между каждой из сфер является непрерывной по поверхности. Эта задача полностью решается аналитическим путем. Получены зависимости давления на границах сфер, тензоров напряжений и деформаций внутри сфер. Полученные теоретическим путем результаты имеют важное практическое значение.

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