ISBN 1392 - 1207. MECHANIKA. 2010. Nr.6(86)

Viscoplasticity coupled with nonlocalized damage for incompatibilities due to strain softening

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1. Introduction

For continuum damage models, when the tangential stiffness matrix becomes negative definite, problems such as the uniqueness and stability of the solution must be investigated. In strain softening conditions, the tangential modulus becomes negative and the equation of motion changes from hyperbolic to elliptic form. This change, called Hadamard instability [1], indicates that softening region is unable to propagate and the localization zone stays confined in an infinitely small size with respect to elements size of the structure. Consequently, the problem suffers from loss of well-posedness and numerical solutions are obtained which are unacceptable from a physical point of view. In other words, classical continuum models are not capable to provide meaningful post-peak results and exhibit strong dependence on the size and orientation of the element mesh during softening [2].

Many researchers have used plasticity alone to characterize the concrete behavior [3-5]. Plasticity is based on an elastic unloading stiffness, which is in contradiction to the stiffness degradation observed in experiments. Therefore, these studies failed to address the degradation of the material stiffness due to micro-cracking. On the other hand, others have used the continuum damage theory alone to model the nonlinear behaviors of material such as progressive micro-cracking and strain softening, which are represented by a set of internal state variables causing the decrease of the stiffness [6, 7]. But, damage mechanics is not suitable for the description of the irreversible deformations alone.

Several researchers employed combination of damage and plasticity theories for modeling concrete behavior [8-12]. These investigators notified that the uniqueness of the solution for these models was not guaranteed such that mesh dependent results in the finite element analysis may be obtained.

Using classical plasticity theory or damage mechanics alone in constitutive formulation of the complex failure process of concrete, which is characterized by stiffness degradation and irreversible deformations, not only is not sufficient and make results inadequate, but also, they could not prevail on inconsistencies such as; mesh dependency, strain localization and solution instabilities due to these incompatibilities.

The aim of this work is to overcome the deficiencies of the previous models such as, mesh dependency, nonobjectivity of the numerical response and strain localization encountered by using general softening plasticity models [13-15].

These ill-posed problems require to be regularized using various methods, including visco-plasticity [16, 17], higher-order gradient models [18, 19] and integral type nonlocal models [20-22]. All these methods explicitly or implicitly incorporate a material characteristic length to control the width of the localization band, thus prevent strain from localizing into infinitely narrow zones and allow mesh-independent description of energy dissipation in a localized failure process. For concrete models, this length scale can be related to the maximum aggregate size [20].

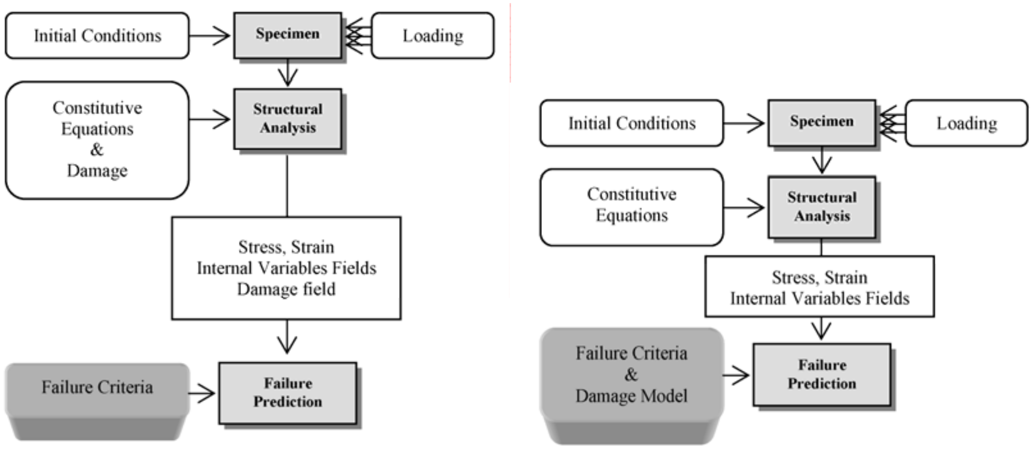
For these reasons, this paper focuses on the development of an approach for constitutive modelling of concrete materials, with emphasis on the use of combination between visco-plasticity and nonlocal damage models.

We use rate-dependent plasticity, i.e. visco-plasticity instead of rate-independent plasticity and combine it with nonlocal damage to obtain a comprehensive constitutive model. A combination of visco-plasticity and damage mechanics can describe not only, most of the important features of the failure of cohesive-frictional materials but also, is so beneficial to overcome deficiencies of classical concrete model to have reliable results. Fig. 1 shows the differences between combined approach and not-combined one. As shown, in combined approach, the constitutive equations and damage are used together for structural analysis. But, in not-combined approach, damage model is used after structural analysis which makes results over-estimated.

At first, the overstress visco-plastic methods according to [23] will be investigated. These visco-plastic models are implemented by allowing the stress state to be outside the yield surface.

Next, the constitutive equations for the damaged material are written according to the principle of strain energy equivalence between the virgin material and the damaged material; that is, the damaged material is modelled using the constitutive laws of the effective undamaged material in which the nominal stresses are replaced by their effective ones.

The changing type of the governing partial differential equation from hyperbolic to elliptic is prevented by introducing visco-plasticity. In other words, visco-plasticity is introduced as an approach to regularize the behaviour of the concrete by employing viscosity term as a regularization parameter (computational point of view), or



a b

Fig. 1 a - combined approach; b - not-combined approach

as a sub-structural/micro-mechanical parameter, determined from observed shear band width [13], to prevent the field equation from being elliptic [14, 15].

Although the rate dependent formulation prevents the partial differential equation of motion from becoming elliptic and ensures that the problem is well posed, the results are not quite satisfactory. Despite the fact that viscosity prevents the strains from becoming infinite at localization, but the localization zone tends to an infinitely small size. So, a localization limiter or nonlocal formulation is used to overcome this problem.

Nonlocal formulation modifies damage models [24] by introducing nonlocal variables, which are weighted spatial averaging of local variables. So, the only required modification is to replace the usual local damage energy release rate with its spatial average over the representative volume of the material whose size is a characteristic of the material.

Finally, avoidance of spurious mesh sensitivity is demonstrated by Double Edge Notched (DEN) uniaxial test [25].

2. Theoretical framework for concrete model

2. 1. Viscoplastic behaviour

This model is similar to the viscoplastic model proposed by Perzyna [23] but the main difference is that the viscoplastic contribution is incorporated in the viscoplastic multiplier by using a parameter called characteristic length of material. The visco-plastic part is local and uses a drucker-prager yield condition formulated in the effective stress space. Softening is incorporated through the yield function of the model.

The strain rate $\dot{\varepsilon}_{ij}$ is decomposed into elastic and viscoplastic parts as

$$\dot{\varepsilon}_{ij} = \dot{\varepsilon}_{ij}^{e} + \dot{\varepsilon}_{ij}^{vp} \quad \text{and} \quad \dot{\varepsilon}^{vp} = \dot{\lambda}^{vp} \frac{\partial g}{\partial \tilde{\sigma}} \tag{1}$$

where, g is plastic potential, $\dot{\lambda}^{vp}$ is viscoplastic multiplier and $\tilde{\sigma}$ is effective stress. The effective stress ($\tilde{\sigma}_{ij}$) is defined as the stress in the micro level and related to the stress in the macro level (σ_{ij}) as

$$\tilde{\sigma}_{ij} = \frac{\langle\sigma\rangle_{ij}^{+}}{1-D} + \frac{\langle\sigma\rangle_{ij}^{-}}{1-hD} + \frac{\nu}{1-2\nu}\left(\frac{\delta_{kl}\langle\sigma\rangle_{kl}^{+} - \langle\sigma_{kk}\rangle}{1-D} + \frac{\delta_{kl}\langle\sigma\rangle_{kl}^{-} + \langle-\sigma_{kk}\rangle}{1-hD} \right)\delta_{ij} \tag{2}$$

where, $\langle\ \rangle_{ij}^{+}$ and $\langle\ \rangle_{ij}^{-}$ are the positive and negative parts of the stress tensor and obtained by decomposition of the stress tensor based on its principal values and principal directions, D is damage parameter and h is a microdefects closure parameter which is a material-dependent parameter and used to make different between compressive and tensional behaviours of material.

The viscoplastic multiplier ($\dot{\lambda}^{vp}$) is defined as; $\dot{\lambda}^{vp} = \langle f \rangle / \eta$, where, f is the yield function. Bracket $\langle a \rangle = a/2 + |a|/2$ means that $\langle a \rangle$ has value only if $a > 0$. When $f < 0, \langle f \rangle = 0$, meaning that the point of loading is inside of the yield surface and thus no viscoplastic strain occurs. η is viscosity coefficient defined as; $\eta = \frac{1}{2} l \sqrt{E\rho}$, where, E is the Young's modulus, ρ is the material density and l is the characteristic length [20]. A good approximation of the characteristic length for concrete specimen could be defined as at least triple size of the biggest aggregate dimension.

Since concrete shows sensitivity to mean (hydrostatic) stress, the appropriate yield criterion was chosen as Drucker- Prager type

$$\left.\begin{aligned} f(\tilde{\sigma}) &= c_{\phi} I_1(\tilde{\sigma}) + \sqrt{J_2(\tilde{\sigma})} - Y_H \\ g(\tilde{\sigma}) &= c_{\psi} I_1(\tilde{\sigma}) + \sqrt{J_2(\tilde{\sigma})} \end{aligned}\right\} \tag{3}$$

where, I_1 is the first invariant of the stress tensor, J_2 is the second invariant of the deviatoric stress tensor, Y_H is the yield stress, C_ϕ is the coefficient of friction and C_ψ is the dilatation coefficient. C_ϕ and C_ψ are material parameters and obtain from experiment. Note that $f(\tilde{\sigma})$ and $g(\tilde{\sigma})$ are in the effective stress space. In the internal friction materials like concrete and other geo-materials, the plastic potential g is different from the yield function f (i.e. nonassociated) and, therefore, the direction of the plastic strain increment is not normal to the yield surface.

To account the softening process, which is characterized by loss of the material cohesion, it was supposed that yield stress varies linearly with the accumulated plastic strain as below:

$$Y_H = \langle Y_0 - \beta p \rangle = \left\langle Y_0 - \beta \sqrt{\frac{2}{3} \varepsilon_{ij}^p \varepsilon_{ij}^p} \right\rangle \tag{4}$$

where, Y_0 is initial cohesion, β is a constant, and p is the accumulated plastic strain.

2. 2. Local Damage model

The thermodynamics of irreversible processes allows for the modeling of materials' behavior in three steps:

1. Definition of state variables, the actual value of each defining the present state of the corresponding mechanism involved.
2. Definition of a state potential from which derive the state laws and the definition of the variables associated with the internal state variables.
3. Definition of a dissipation potential from which derive the laws of evolution of the state variables associated with the dissipative mechanisms.

These three steps offer several choices for the definitions, each chosen in accordance with experimental results and purpose of use. Then, the second principle of the thermodynamics must be checked for any evolution.

According to the thermodynamic framework, the evolution law for damage derives from the potential of dissipation and particularly from the function F_D (dissipative damage potential function) as

$$\dot{D}_{ij} = \dot{\lambda} \frac{\partial F_D}{\partial Y_{ij}} \quad \text{or} \quad \dot{D} = \dot{\lambda} \frac{\partial F_D}{\partial Y} \tag{5}$$

where Y_{ij} is the associated variable with the damage parameter. Y is the damage driving force which characterizes damage evolution and interpreted here as the energy release rate.

$$Y = \frac{1+\nu}{2E} \left[\frac{\langle \sigma \rangle_{ij}^{+} \langle \sigma \rangle_{ij}^{+}}{(1-D)^2} + h \frac{\langle \sigma \rangle_{ij}^{-} \langle \sigma \rangle_{ij}^{-}}{(1-hD)^2} \right] - \frac{\nu}{2E} \left[\frac{\langle \sigma_{kk} \rangle^2}{(1-D)^2} + h \frac{\langle -\sigma_{kk} \rangle^2}{(1-hD)^2} \right] \tag{6}$$

There are many possible choices for the analytical form of the function F_D, depending on the experimental results and the purpose of use. The best is the simplest with the domain of validity required, where the simplest means the smallest possible number of material parameters.

Experimental results [10, 26, 27] have shown that F_D must be a nonlinear function of Y. A good and simple choice is

$$F_D = \frac{S}{(1+s)(1-D)} \left(\frac{Y_{ij}}{S} \right)^{1+s} \rightarrow \dot{D}^{Local} = \dot{\lambda} \frac{\partial F_D}{\partial Y_{ij}} = \dot{\lambda} \frac{\partial}{\partial Y_{ij}} \left(\frac{S}{(1+s)(1-D)} \left(\frac{Y_{ij}}{S} \right)^{s+1} \right) = \frac{\dot{\lambda}}{(1-D)} \left(\frac{Y_{ij}}{S} \right)^{s} \tag{7}$$

where S and s are material parameters and calibrated from regression analysis of experiments. For material parameters; $f_c' = 50\text{MPa}$, $Y_0 = 15\text{MPa}$, $\rho = 2300\,\text{kg/m}^3$, $C_\phi = 0.44$ and $C_\psi = 0.40$, s and S is calibrated as $s = 2.8210e-4$, $S = 4.3510e+3$.

2. 3. Discretization and derivation

The local residual is defined as

$$\{R_{loc}\} = \{R_{\varepsilon^e} \quad R_r \quad R_D\}^T \tag{8}$$

where R_{ε^e} is residual of elastic strain, R_r is the residual of yield function, and R_D is the residual of damage constitutive function. The set of nonlinear equations is discretized in time considering the resolution variables at the intermediate time, $t_{n+\theta} = t_n + \theta \Delta t$, with θ as the numerical parameter of the θ method. Then, based on the Newton iterative scheme for the local residual, the residuals must be derivated respect to selected variables $\{\Delta\varepsilon^e, \Delta\lambda, \Delta D\}$ to form Jacobian matrix. The Jacobian matrix obtained in this approach, not only has terms of the damage parameter, but also has term of viscosity. This results that damage process and visco-plasticity state are considered together or in the other word, in a coupled manner.

According to (1), the residual of elastic strain is obtained as

$$R_{\varepsilon^e} = \Delta\varepsilon^e - \Delta\varepsilon + \Delta\lambda \frac{\partial g}{\partial \tilde{\sigma}} = \Delta\varepsilon^e - \Delta\varepsilon + \Delta\lambda q \tag{9}$$

where $\frac{\partial g}{\partial \tilde{\sigma}} = q$.

Then, according to (9) and (2), the residual of elastic strain is derivated respect to the selected variables as

$$\frac{\partial R_{\varepsilon^e}}{\partial \Delta\varepsilon^e} = I + \theta\Delta\lambda\left(\frac{\partial q}{\partial \tilde{\sigma}}\right)\left(\frac{\partial \tilde{\sigma}}{\partial \varepsilon^e}\right) = I + \theta\Delta\lambda\left(\frac{\partial q}{\partial \tilde{\sigma}}\right)\tilde{E}\ ,\ \frac{\partial R_{\varepsilon^e}}{\partial \Delta\lambda} = q\ ,\ \frac{\partial R_{\varepsilon^e}}{\partial \Delta D} = \theta\Delta\lambda\frac{\partial q}{\partial D} = 0 \tag{10}$$

where $\frac{\partial \tilde{\sigma}}{\partial \varepsilon^e} = \tilde{E}$.

Since $q = \partial g/\partial\tilde{\sigma}$ is in effective stress space, its derivation respect to damage parameter is zero.

Unlike elasto-plasticity theory, according to the visco-plasticity theory, the yield function can be greater than zero and can have a positive amount which is viscoplastic stress, so the yield function was modified as below to account viscosity effects in the proposed model

$$\dot{\varepsilon}_{vp} = \dot{\lambda}\frac{\partial g}{\partial \tilde{\sigma}} = \frac{\langle f\rangle}{\eta}\frac{\partial g}{\partial \tilde{\sigma}} \rightarrow \langle f\rangle = \eta\dot{\lambda} \rightarrow \langle f\rangle - \eta\dot{\lambda} = 0 \tag{11}$$

According to Kuhn-Tucker relation, $\dot{\lambda} \geq 0$, so the bracket can be eliminated. Thus the modified yield function and its residual can be determined as

$$f = C_\phi I_1 + \sqrt{J_2} - Y_H = \sigma_{vp} \rightarrow \tag{12}$$

$$R_r = C_\phi I_1 + \sqrt{J_2} - \left\langle Y_0 - \beta\sqrt{\frac{2}{3}\varepsilon_{ij}^p\varepsilon_{ij}^p}\right\rangle - \sigma_{vp} = C_\phi I_1 + \sqrt{J_2} - \left\langle Y_0 - \beta\sqrt{\frac{2}{3}\varepsilon_{ij}^p\varepsilon_{ij}^p}\right\rangle - \eta\dot{\lambda} \tag{13}$$

According to (13), the residual of modified yield function has term of $\dot{\lambda}$, which is a rate form of λ respect to time.

Then, according to (13) and (2), the residual of yield function is derivated respect to the selected variables as

$$\frac{\partial R_r}{\partial \Delta\varepsilon^e} = \theta\left(\frac{\partial R_r}{\partial \tilde{\sigma}}\right)\left(\frac{\partial \tilde{\sigma}}{\partial \varepsilon^e}\right) = \theta\left(C_\phi I + \frac{S_{ij}}{2\sqrt{J_2}}\right)\tilde{E}\ ,\ \frac{\partial R_r}{\partial \Delta\lambda} = -\underbrace{\left\langle -\beta\sqrt{\frac{2}{3}q_{ij}q_{ij}}\right\rangle}_{=0} - \frac{\eta}{\Delta t} = -\frac{\eta}{\Delta t}\ ,\ \frac{\partial R_r}{\partial \Delta D} = 0 \tag{14}$$

where $\frac{\partial \tilde{\sigma}}{\partial \varepsilon^e} = \tilde{E}$.

According to (7), the residual of damage function is obtained as

$$R_D = \Delta D - \frac{\Delta\lambda}{(1-D)}\left(\frac{Y}{S}\right)^s \tag{15}$$

Then, according to (7) and (6), the residual of damage function is derivated respect to the selected variables as

$$\frac{\partial R_D}{\partial \Delta\varepsilon^e} = -\frac{sY^{s-1}\theta\Delta\lambda}{S^s(1-D)}\frac{\partial Y}{\partial \varepsilon^e} = -\frac{sY^{s-1}\theta\Delta\lambda}{S^s(1-D)}\frac{\partial Y}{\partial \tilde{\sigma}}\tilde{E}\ ,\ \frac{\partial R_D}{\partial \Delta\lambda} = -\left(\frac{Y}{S}\right)^s\frac{1}{1-D}\ ,\ \frac{\partial R_D}{\partial \Delta D} = I - \left(\frac{Y}{S}\right)^s\frac{\theta\Delta\lambda}{(1-D)^2} \tag{16}$$

where $\frac{\partial Y}{\partial \varepsilon^e} = \frac{\partial Y}{\partial \tilde{\sigma}}\tilde{E}$.

3. Numerical results

3. 1. Mesh sensitivity

In the previous sections, combination of the visco-plasticity theory and local damage was explained. Although the viscosity is introduced as a regularization parameter (computational point of view), or as a sub-structural/micro-mechanical parameter determined from observed shear-band widths (physical point of view) and prevents the strains from becoming infinite at localization, but the localization zone tends to an infinitely small size and the results are not quite satisfactory. This inconsistency is investigated here by using a Double Edge Notched (DEN) uniaxial test [25].

As shown in Fig. 2, when the same material parameters are used for different element mesh, numerical results are not meaningful.

As shown in Fig. 3, not only the damage tends to localize in an infinitely small size, i.e., in a volume which is much smaller than that of the microstructural elements, but also its distribution strongly depends on the size and orientation of the meshing elements of the structure. This damage distribution conflicts with the assumed smoothness of the damage variable. The deformation contours are also of similar contours. This observation has no consistent physical meaning and must be obviated with an appropriate approach. Thus, a localization limiter or in other word, a nonlocal formulation is used to overcome this problem.

Nonlocal formulation modifies damage models by introducing nonlocal variables, which are weighted spatial averaging of local variables [24].

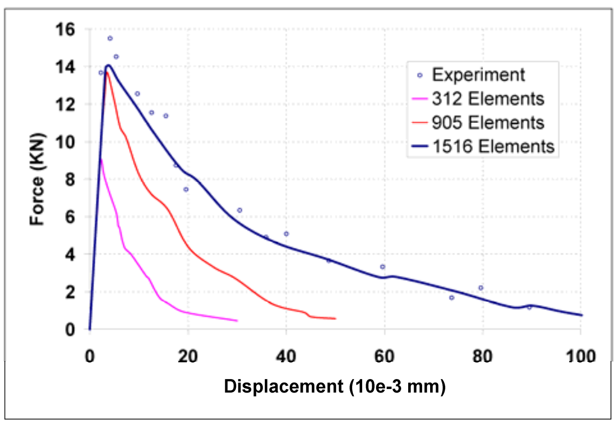


Fig. 2 Load-displacement curves for different number of elements

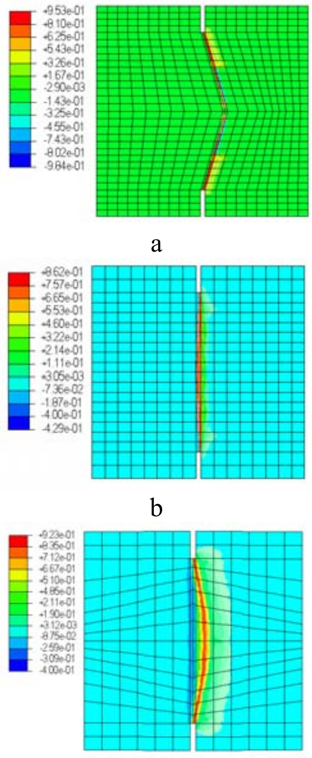


Fig. 3 Damage distributions in DEN specimen for different mesh sizes and orientations

3. 2. Nonlocal damage model

Continuum damage theory introduces a set of field variables (damage variables) which explicitly characterize the local loss of material cohesiveness. A continuous damage variable which describes microdefects in a continuum medium indicates that this variable varies smoothly at the microscale of the structure.

In this section, the necessary smoothness of the damage field is ensured by relating the damage growth in a material point to a weighted average of the corresponding field in a neighbourhood of the point.

The growth of damage (D^{Local}) in a point is delocalized as follow

$$D^{Non\,local} = \frac{1}{V_r}\int_V \phi(y;x)\,D^{Local}\,dV$$

$$\text{and} \quad V_r = \int_V \phi(y;x)\,dV \tag{17}$$

where $\phi(y;x)$ is a weight function and determines the influence of the local damage in the infinitesimal volume V . The nonlocal weight function is a new parameter of the model, chosen as the Gauss distribution function

$$\phi(|x-y|) = \frac{1}{(2\pi)^{3/2}\,l^3}\,exp\left(-\frac{|x-y|^2}{2l^2}\right) \tag{18}$$

where l is the length parameter which represents the scale of the microstructure. For concrete models, this length parameter is related to the maximum aggregate size [20].

Also, in addition to above formulation, for sufficiently smooth damage field, the integral relation (17) can be rewritten in gradient terms of $D^{Non\,local}$ by expanding D^{Local} into a Taylor series [24,28]. The integral relation (17) can then be approximated by the relation

$$D^{Non\,local} = \frac{1}{V_r}\int_V \phi(y;x)D^{Local}dV + \frac{1}{V_r}\int_V \phi(y;x)\nabla D^{Local}(|x-y|)dV + \frac{1}{2!V_r}\int_V \phi(y;x)\nabla^2 D^{Local}(|x-y|)^2 dV + \frac{1}{3!V_r}\int_V \phi(y;x)\nabla^3 D^{Local}(|x-y|)^3 dV + \frac{1}{4!V_r}\int_V \phi(y;x)\nabla^4 D^{Local}(|x-y|)^4 dV + \cdots \tag{19}$$

With the assumption of an isotropic influence of the averaging equation, the integral of the odd terms vanish. Thus, the nonlocal damage can be written as

$$D^{Non\,local} = D^{Local} + c_1\nabla^2 D^{Local} + c_2\nabla^4 D^{Local} + c_3\nabla^6 D^{Local} + \cdots \tag{20}$$

where the Laplacian operator ∇^n is defined by $\nabla^n = \sum_i \partial^n/\partial x_i$. If the nonlocal damage defined by (20) is used instead of the growth law (7), a gradient-enhanced damage model is obtained. Since the nonlocal damage is given by (20) explicitly in terms of the local damage, this gradient model will be referred to as explicit. The characteristic length of the nonlocal model is preserved in the gradient coefficients c_i , which is of the dimension length squared. For instance, the Gaussian weight function (18) $c = l^2/2$.

Employing of (20) together with the set of constitutive equations (stated previous), results a new set of modified partial differential equations which are able to prevent damage distribution from being localized and consequently, make it independent from size and orientation of meshing elements. As shown in Figs. 4 and 5, the numerical results are now objective, i.e., global responses converged upon mesh refinements and the damage distributions are consistent for different element sizes and orientations.

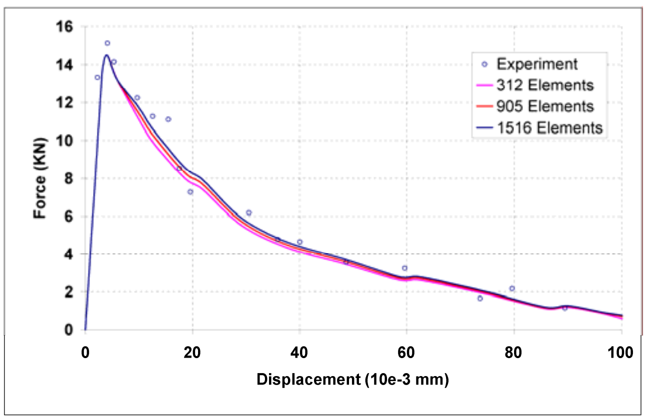
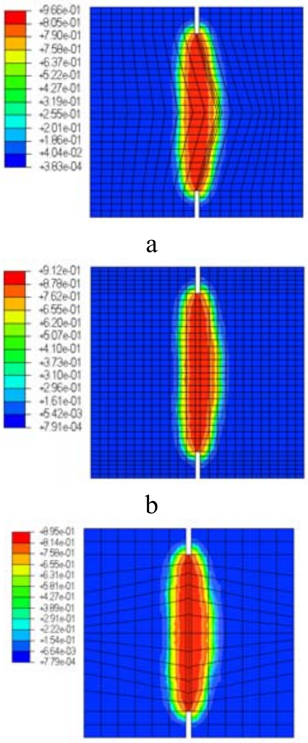


Fig. 4 Load-displacement curves for different number of elements



a

b

c

Fig. 5 Damage distributions in DEN specimen for different mesh sizes and orientations.

4. Conclusion

The ill-posedness of the set of partial differential equations, nonobjectivity of the numerical response, mesh dependency and strain localization are the basic deficiencies encountered by using general softening plasticity models. Overcoming these deficiencies requires two categories. The first involves the use of rate dependent formulation, i.e. visco-plasticity to regularize constitutive equations. The second involves the introduction of nonlocal damage based on spatial averaging to achieve a realistic description of the damage localization instability.

This paper illustrates both the regularizing capabilities of the rate dependent formulation via visco-plasticity theory and nonlocalizing of the damage distribution via gradient approach. This combination approach satisfies expected requirements and the numerical results show the desired behaviour, i.e. a realistic damage distribution and a satisfactory mesh independency.

The proposed model could be coupled with predictive models [29, 30] and used for simulation of concrete behaviour.

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TĄSUS PLASTIŠKUMAS IR LOKALIZUOTO PAŽEIDIMO NESUDERINAMUMAS SU DEFORMACINIU SILPNĖJIMU

Reziumė

Šio darbo tikslas – pašalinti ankstesnių modelių trūkumus, t.y. priklausomybę nuo tinklelio, skaitmeninės reakcijos ir deformacijos lokalizacijos įvertinimo naudojant bendrus plastiškumo silpnėjimo modelius, neobjektyvumą. Pagrindinių dalinių diferencinalinių lygčių tipo pakeitimo galima išvengti naudojant greičio priklausomybės modelius, pavyzdžiui, tąsaus plastiškumo, nepaisant to, kad tąsumas neleidžia deformacijoms neribotai didėti lokalizacijos metu, nes jos vyksta labai mažuose tūriuose. Lokalizacijos metu pažeidimas yra be galo mažas, ir jo pasiskirstymo nevienalytiškumas prieštarauja tolygaus pažeidimo principui. Reikalingas pažeidimo lauko tolygumas gali būti pasiektas lokalizacijos ribojimu ir tąsumo reguliavimu. Šiame straipsnyje siūlomas kombinuotas požiūris: atliekant betono elgsenos struktūrinį modeliavimą naudoti tąsaus plastiškumo teoriją ir nelokalų pažeidimo modelį.

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VISCOPLASTICITY COUPLED WITH NONLOCALIZED DAMAGE FOR INCOMPATIBILITIES DUE TO STRAIN SOFTENING

Summary

The aim of this work is to overcome the deficiencies of the previous models such as, mesh dependency, nonobjectivity of the numerical response and strain localization encountered by using general softening plasticity models. The change of type in the governing partial differential equation can be prevented by introducing rate-dependent models such as visco-plasticity. Despite the fact that viscosity prevents the strains from becoming infinite at localization, but the localization zone tends to an infinitely small size. When localization occurs, damage is confined in an infinitely small zone and its discontinuous distribution conflicts with the supposed smoothness of the damage variable. The necessary smoothness of the damage field can be ensured by using localization limiter in addition to viscous regularization. This paper focuses on the construction of a combining approach with emphasis on the use of visco-plasticity theory and nonlocal damage model to constitutive modelling of concrete behaviour.

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НЕСОВМЕСТИМОСТЬ ВЯЗКОПЛАСТИЧНОСТИ И НЕЛОКАЛИЗИРОВАННОГО ПОВРЕЖДЕНИЯ С ДЕФОРМАЦИОННЫМ РАЗУПРОЧНЕНИЕМ

Резюме

Цель настоящей работы – устранение недостатков прежних моделей, например, зависимости от сетки, необъективности численной реакции и оценки локализации деформации при использовании общих моделей снижения пластичности. Изменения типа основных частных дифференциальных уравнений можно предотвратить вводом моделей зависимости скорости, например, вязкопластичности, не учитывая того, что вязкость предохраняет деформации от бесконечного увеличения при локализации, потому что она происходит в очень малых объемах. При локализации поврежденность происходит в бесконечно малых объемах и неоднородность ее распределения противоречит ожидаемому равномерному повреждению. Необходимая равномерность поля повреждения может быть достигнута используя ограничитель локализации совместно с регулированием вязкости. Эта статья предлагает комбинированный подход, акцентируя теорию вязкопластичности и нелокальную модель повреждения при оценке поведения бетона.

Received June 21, 2010
Accepted December 07, 2010