

## Mathematical simulation of operation of the weighing type filling machine for dry products

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### 1. Introduction

Filling machines are widely used in almost every industry. Construction industry employs filling machines for filling cement and various components. In agriculture filling machines are used for dosing feed and fertilizers. Food industry is exceptional for wide variety of applications of filling machines. Sugar, flour, spices and many other products and ingredients are being dosed and filled both manually and automatically during the production and especially packaging of foods. Packaging technology and technique is among the most powerful stimulus for development of filling machines [1-3]. It promotes the necessity for permanent increase of operation speed and dosing accuracy [4-7]. Dry powdery materials make a big part of products being dosed and packed through employment of various types of filling machines. Volumetric filling machines are among the most popular [6]. This is because the volumetric batchers feature high operating speed. However their dosing accuracy is dependent on properties of material and not always complies with the requirements. Therefore, besides the volumetric other types of filling machines are widely used too, first of all weighing type [6-9]. These can guarantee higher and more stable filling accuracy as they are much less dependent on the material properties. Furthermore, weighing machines allow for adjustment of accuracy through setting the right operating speed. Weighing machines are easy to control, easy to integrate into automation systems, including sophisticated ones, e.g. self-trained [5, 9, 10].

Working cycle of real weighing filling machine is quite complex. In this paper the results of theoretical research of weighing machine in operation shall be presented. Research out based on specially developed mathematical model presented in previous paper [5] was carried. The model covers all the dynamic processes, which may occur during the real operation of weighing machine for dry products. The sensitive element of the machine is regarded as an elastic system with one degree of freedom. It is presumed that a body having variable mass is operating in this elastic system [6-9]. This paper only presents some final equations of the model and the simulation results in a form of displacement curves of 10 different projects.

### 2. Structural scheme of the system containing weighing filling machine

Weighing type filling machines can achieve high dosing accuracy. However, investigation of the processes, which influence the accuracy is not simple as the structure of weighing filling machine is quite complex. All the components comprising the machine affect the dosing accuracy [2, 3, 5-7, 9] as well as their interaction due to dynamic processes, which take place during the operation. A typical structural scheme of the weighing filling machine is presented in [4, 5]. It should be noted that filling weighing machine itself is mainly used as an integral part of entire packaging system. Therefore its analysis should be carried out taking into account the complexity and features of whole packaging machine. Its especially important in case of development of self-learning control systems.

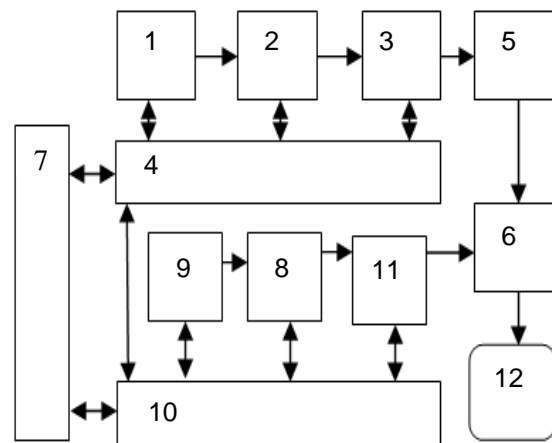


Fig. 1 Structural scheme of the system containing weighing filling machine

A typical structural scheme of packaging system with weighing filling machine (Fig. 1), contains: 1 - dry materials; 2 - feeder; 3 - weighing mechanism; 4 - control unit of weighing machine; 5 - dose of dry materials; 6 - packing of dose; 7 - control system; 8 - packing mechanism; 9 - packaging workpiece; 10 - control unit of packaging machine; 11 - transfer mechanism; 12 - package with dry product.

The system works as follows. Feeder 2 feeds the materials 1 to the weighing mechanism 3, signal from the weighing mechanism 3 is transferred to the control unit of weighing machine 4, which disables the feeder 2, when the quantity of the product in weighing mechanism 2 equals the asked value. At the same time the packing mechanism 8 takes a workpiece from the packaging stack 9 and produces the pack 6. The role of transfer mechanism 11 is to guarantee the right positioning of package 6 against the product 5 being fed. As a result we have a package filled with dry product. Control unit 10 controls operation of packaging machine. Control system 7 controls and adjusts operation of the whole system. Weighing filling machine combines several processes. These are: vibratory transport, weight measurement, preparation, assessment and control technology. All these processes need to be investigated individually including the interaction between them. Vibratory transport and vibratory processes are widely used [11, 12]. The elastic systems are paid a lot of attention too [13-15].

Working cycle of weighing filling machine comprises five stages [5]. First step is filling at increased productivity rate. Step two is filling at low productivity rate. Step three - suspension of filling (pause before discharge). The fourth step is discharge of the product. Step five - suspension of discharge (pause after discharge). Each of these phases is associated with the characteristics of the product, accuracy of dosing, dosing efficiency as well as with other parameters of the system. Those are dosing rate, the system sensitivity and working range, sensor type, the influence of technological regimes, and others.

### 3. Mathematical model for describing the operation of weighing filling machine for dry products

This article deals with a movement of sensitive element at all five stages. As already mentioned, in this case we deal with the system with one degree of freedom and the attached body of variable mass.

First step. Filling at high productivity rate. Mass of the product in the shovel increases by linear law  $m = \kappa_1 t$ . Where  $t$  is time,  $\kappa_1$  is constant coefficient (efficiency of feeder at increased rate). The duration of operation of the feeder depends on the dose size to be set up at this stage. The feeder working range in the first stage  $t \in [0; t_1]$ , where  $t_1 = \mu_1 / \kappa_1$  - duration of feeder work, related to the size of dose  $\mu_1$ .

In the first step the body moves according to the Eqs. (1) and (2) [4, 5, 7, 12, 16-18].

$$x = a_1 + gt/b_1 + (t + \tau_1)^{-n/2} \times \left\{ C_1 J_n \left[ 2\sqrt{b_1(t + \tau_1)} \right] + C_2 Y_n \left[ 2\sqrt{b_1(t + \tau_1)} \right] \right\} \quad (1)$$

and

$$\dot{x} = \frac{g}{b_1} - \sqrt{b_1} (t + \tau_1)^{-\frac{n+1}{2}} \times \left\{ C_1 J_{n+1} \left[ 2\sqrt{b_1(t + \tau_1)} \right] + C_2 Y_{n+1} \left[ 2\sqrt{b_1(t + \tau_1)} \right] \right\} \quad (2)$$

where

$$\left. \begin{aligned} a_1 &= (\kappa_1 u_1 + m_1 g) / k - (\kappa_1 + c) \kappa_1 g / k^2 \\ b_1 &= k / \kappa_1, \tau_1 = m_1 / \kappa_1, n = c / \kappa_1, u_1 = \sqrt{2hg} \end{aligned} \right\} \quad (3)$$

where  $u_1$  is the absolute velocity of joining particle [5],  $v$  is the absolute velocity of shovel,  $dv/dt$  is the absolute acceleration of the shovel,  $g$  is due to gravity acceleration. With this assess it is possible to write differential equation, which describes movement of the shovel.  $J_n$  is the first kind and  $Y_n$  second kind of Bessel functions [16-18].

$C_1$  and  $C_2$  are constants. The initial conditions of movement  $t = 0$ ,  $x = e = m_1 g / k$ ,  $\dot{x} = 0$ .

$$C_1 = A \tau_1^{1+\frac{n}{2}} Y_n(B) - \pi \sqrt{b_1} (e - a_1) \tau_1^{\frac{n+1}{2}} Y_{n+1}(B) \quad (4)$$

$$C_2 = -A \tau_1^{1+\frac{n}{2}} J_n(B) + \pi \sqrt{b_1} (e - a_1) \tau_1^{\frac{n+1}{2}} J_{n+1}(B_1) \quad (5)$$

where  $A = \pi g / b_1$ ;  $B = 2\sqrt{b_1} \tau_1$ .

Second step. The second step is filling at low productivity rate. The mass of the dosing product in the shovel increases by linear law  $m = \kappa_2 t$ . Where  $t$  is time,  $\kappa_2$  is constant coefficient (efficiency of feeder at low rate). The duration of work of the feeder depends on the dose size to be set up at this stage. The feeder working range in the second stage  $t \in [0; t_2]$ , where  $t_2 = \mu_2 / \kappa_2$  - duration of the feeder work required to from a dose  $\mu_2$ . It is necessary assess the change mass of the shovel. At this stage reduced mass of the shovel [12]  $m_2 = M + \kappa_1 t_1 + \frac{33}{140} m'$  where  $m_2$

is reduced mass of the shovel in the second step. In the second step the body moves according to the Eqs. (6) and (7) [4, 5, 7, 12, 16-18].

$$x = a_2 + gt/b_2 + (t + \tau_2)^{-n/2} \times \left\{ C_1' J_n \left[ 2\sqrt{b_2(t + \tau_2)} \right] + C_2' Y_n \left[ 2\sqrt{b_2(t + \tau_2)} \right] \right\} \quad (6)$$

and

$$\dot{x} = g/b_2 - \sqrt{b_2} (t + \tau_2)^{-\frac{n+1}{2}} \times \left\{ C_1' J_{n+1} \left[ 2\sqrt{b_2(t + \tau_2)} \right] + C_2' Y_{n+1} \left[ 2\sqrt{b_2(t + \tau_2)} \right] \right\} \quad (7)$$

where  $u_2$  is the absolute velocity of joining particle in the second step, and

$$\left. \begin{aligned} a_2 &= (\kappa_2 u_2 + m_2 g) / k - (\kappa_2 + c) \kappa_2 g / k^2 \\ b_2 &= k / \kappa_2, \tau_2 = m_2 / \kappa_2, n = c / \kappa_2, u_2 = \sqrt{2h_2 g} \end{aligned} \right\} \quad (8)$$

$C_1'$  and  $C_2'$  are constants. The initial conditions of movement are:  $t = 0$ ,  $x = x_1$ ,  $\dot{x} = \dot{x}_1$ . Dimensions  $x_1$  and  $\dot{x}_1$  are displacement and speed of the shovel at the end of step one. These values are derived from Eqs. (1) and (2) when  $t = t_1$ , and

$$C_1' = \pi\tau_2^{1+\frac{n}{2}} (g/b_2 - \dot{x}_1) Y_n(2\sqrt{b_2\tau_2}) - \pi\sqrt{b_2} (x_1 - a_2) \tau_2^{\frac{n+1}{2}} Y_{n+1}(2\sqrt{b_2\tau_2}) \quad (9)$$

$$C_2' = -\pi\tau_2^{1+\frac{n}{2}} (g/b_2 - \dot{x}_1) J_n(2\sqrt{b_2\tau_2}) + \pi\sqrt{b_2} (x_1 - a_2) \tau_2^{\frac{n+1}{2}} J_{n+1}(2\sqrt{b_2\tau_2}) \quad (10)$$

Third step. The third step begins when filling is suspended (pause before discharge). The shovel mass makes  $\mu = \kappa_1 t_1 + \kappa_2 t_2$ . In third step  $t \in [0: t_3]$ , here  $t_3$  is pause time.

In the third step the movement of the body is described by Eqs. (11), (12) or (14), (15) [4, 5, 12, 16-18]:

When  $\omega_0 > \mathcal{G}$ :

$$x = g/k (m_1 + \mu) + M e^{-\mathcal{G}t} \sin(\omega t + N) \quad (11)$$

$$\dot{x} = -M \mathcal{G} e^{-\mathcal{G}t} \sin(\omega t + N) + M \omega e^{-\mathcal{G}t} \cos(\omega t + N) \quad (12)$$

where

$$\left. \begin{aligned} 2\mathcal{G} &= c/(m_1 + \mu), \omega_0^2 = k/(m_1 + \mu), \omega^2 = \omega_0^2 + \mathcal{G}^2 \\ M^2 &= [x_2 - (m_1 + \mu)g/k]^2 + \\ &+ \left\{ \dot{x}_2/\omega + [x_2 - (m_1 + \mu)g/k] \mathcal{G}/\omega \right\}^2 \\ t\mathcal{G}N &= \frac{x_2 - (m_1 + \mu)g/k}{\dot{x}_2/\omega + [x_2 - (m_1 + \mu)g/k] \mathcal{G}/\omega} \end{aligned} \right\} \quad (13)$$

When  $\omega_0 < \mathcal{G}$ :

$$x = (m_1 + \mu)g/k + e^{-\mathcal{G}t} (M'e^{\omega t} + N'e^{-\omega t}) \quad (14)$$

$$\dot{x} = M'(\omega - \mathcal{G})e^{(\omega - \mathcal{G})t} - N'(\omega + \mathcal{G})e^{-(\omega + \mathcal{G})t} \quad (15)$$

In this case

$$\left. \begin{aligned} \omega^2 &= \mathcal{G}^2 - \omega_0^2, M' = \frac{\dot{x}_2}{2\omega} + \frac{\omega + \mathcal{G}}{2\omega} \left[ x_2 - \frac{g}{k} (m_1 + \mu) \right] \\ N' &= -\frac{\dot{x}_2}{2\omega} + \frac{\omega - \mathcal{G}}{2\omega} \left[ x_2 - \frac{g}{k} (m_1 + \mu) \right] \end{aligned} \right\} \quad (16)$$

Dimensions  $x_2$  and  $\dot{x}_2$  are accordingly displacement and speed of the shovel at the end of the second stage filing. These values are derived from Eqs. (6) and (7) when  $t = t_2$ .

Fourth step. The fourth step is discharge of the product. Mass of the dosing product in the shovel is falling by linear law  $m = -\kappa_3 t$ . Here  $\kappa_3$  is constant coefficient (efficiency of discharge). The duration of discharge depends on the size of the dose  $\mu = \mu_1 + \mu_2$  and is expressed  $t \in [0: t_4]$ , where  $t_4 = \mu/\kappa_3$  is time needed to empty the dose  $\mu$ .

In the fourth step the body moves according to the Eqs. (17) and (18) [4, 5, 7, 12, 16-18].

$$x = a_3 - gt/b_3 + (\sigma - t)^{\frac{1+i}{2}} \times \left\{ C_1'' J_{1+i} \left[ 2\sqrt{b_3(\sigma - t)} \right] + C_2'' Y_{1+i} \left[ 2\sqrt{b_3(\sigma - t)} \right] \right\} \quad (17)$$

and

$$\dot{x} = -g/b_3 - (1+i)(\sigma - t)^{\frac{i+1}{2}} \times Z_{1+i} \left[ 2\sqrt{b_3(\sigma - t)} \right] - (\sigma - t)^{\frac{i}{2}} \sqrt{b_3} Z_{2+i} \left[ 2\sqrt{b_3(\sigma - t)} \right] \quad (18)$$

$$\text{here } Z_n(x) = C_1'' J_n(x) + C_2'' Y_n(x) \quad (19)$$

where  $C_1''$  and  $C_2''$  are constants,  $i = \frac{c}{\kappa_3}$ .

When

$$\left. \begin{aligned} a_3 &= g c \kappa_3 / k^2 + (m_1 + \mu) g / k \\ b_3 &= k / \kappa_3, \quad \sigma = (m_1 + \mu) / \kappa_3 \end{aligned} \right\} \quad (20)$$

At the time, the movement of shovel in third phase is described by the equation at  $t = t_3$ . When  $\omega_0 > \mathcal{G}$ , from formulas (11) and (12) follows that

$$x_3 = (m_1 + \mu)g/k + M e^{-\mathcal{G}t_3} \sin(\omega t_3 + N) \quad (21)$$

$$\dot{x}_3 = -M \mathcal{G} e^{-\mathcal{G}t_3} \sin(\omega t_3 + N) + M \omega e^{-\mathcal{G}t_3} \cos(\omega t_3 + N) \quad (22)$$

When  $\omega_0 < \mathcal{G}$  from Eqs. (14) and (15) follows that

$$x_3 = (m_1 + \mu)g/k + e^{-\mathcal{G}t_3} (M'e^{\omega t_3} + N'e^{-\omega t_3}) \quad (23)$$

$$\dot{x}_3 = M'(\omega - \mathcal{G})e^{(\omega - \mathcal{G})t_3} - N'(\omega + \mathcal{G})e^{-(\omega + \mathcal{G})t_3} \quad (24)$$

Finally we get

$$C_1'' = -\pi\sigma^{\frac{1-i}{2}} \left[ \dot{x}_3 + g/b_3 + (x_3 - a_3)(1+i)1/\sigma \right] \times Y_{1+i} \left( 2\sqrt{b_3\sigma} \right) - \pi\sqrt{b_3} (x_3 - a_3) \sigma^{-i/2} Y_{2+i} \left( 2\sqrt{b_3\sigma} \right) \quad (25)$$

$$C_2'' = \pi\sigma^{\frac{1-i}{2}} \left[ \dot{x}_3 + g/b_3 + (x_3 - a_3)(1+i)1/\sigma \right] \times J_{1+i} \left( 2\sqrt{b_3\sigma} \right) + \pi\sqrt{b_3} (x_3 - a_3) \sigma^{-i/2} J_{2+i} \left( 2\sqrt{b_3\sigma} \right) \quad (26)$$

Fifth step. In the fifth step the discharge is suspended (pause after discharge). In this case the body moves according to the Eqs. (27) and (28) or (30) and (31) [4, 5, 12, 16-18].

If condition is that  $\Omega > \Theta$ , the body moves according to Eqs. (27) and (28).

$$x = m_1 g/k + A e^{-\Theta t} \sin(pt + B) \quad (27)$$

$$\dot{x} = -A\Theta e^{-\Theta t} \sin(pt + B) + A p e^{-\Theta t} \cos(pt + B) \quad (28)$$

where  $\Omega = \sqrt{k/m_1}$ ;  $\Theta = c/2m$ ;  $p^2 = \Omega^2 - \Theta^2$ ,  $A$  and  $B$  is

constant. When  $t = 0$ ,  $x = x_4$ ,  $\dot{x} = \dot{x}_4$ , receive

$$\left. \begin{aligned} A^2 &= (x_4 - m_1 g/k)^2 + [\dot{x}_4 + \Theta(x_4 - m_1 g/k)]^2 / p^2 \\ B &= \arctg \frac{x_4 p - m_1 g p/k}{\dot{x}_4 + \Theta(x_4 - m_1 p/k)} \end{aligned} \right\} \quad (29)$$

When we have condition  $\Omega < \Theta$ , the body moves according to the Eqs. (30) and (31).

$$x = m_1 g/k + A'e^{(p-\Theta)t} + B'e^{-(p+\Theta)t} \quad (30)$$

$$\dot{x} = A'(p-\Theta)e^{(p-\Theta)t} - B'(p+\Theta)e^{-(p+\Theta)t} \quad (31)$$

where  $p^2 = \Theta^2 - \Omega^2$ ,  $A'$  and  $B'$  are constants.

When  $t = 0$ ,  $x = x_4$ ,  $\dot{x} = \dot{x}_4$ , it is obtained

$$\left. \begin{aligned} A' &= \dot{x}_4/2p + (1/2 + \Theta/2p)(x_4 - m_1 g/k) \\ B' &= -\dot{x}_4/2p + (1/2 - \Theta/2p)(x_4 - m_1 g/k) \end{aligned} \right\} \quad (32)$$

Dimensions  $x_4$  and  $\dot{x}_4$  are the displacement and speed of the shovel at the moment when step four ends. These values are derived from Eqs. (17) and (18) when  $t_4 = \mu/\kappa_3$ .

This mathematical model describes the weight batcher operation at various working conditions and can be used for theoretical considerations at various development stages of dosing process and the batcher itself in order to eliminate its critical elements and find the right setup of the system.

#### 4. Results

The developed mathematical model was used to analyse and compare the behaviour and efficiency of several projects, which are basically of the same setup, but differ in dimensions, mass as well as operational parameters: travel, speed, acceleration, productivity, etc. The sensitive element of weight batcher is regarded as elastic system with one degree of freedom. Performance of the system has been analysed based on displacement curves of the shovel. Project details can be found in Table. Results of mathematical modelling are shown in Figs. 2-11.

Table

Projects details of weighing filling machine

details project	$k$ , N/m	$m_1$ , kg	$\mu_1$ , kg	$\mu_2$ , kg	$c$ , Ns/m	$\kappa_1$ , kg/s	$\kappa_2$ , kg/s	$\kappa_3$ , kg/s	$h$ , m
project 1	2000	0.1	0.126	0.024	0	0.2	0.1	0.08	0.01
project 2	2000	0.145	0.126	0.024	0	0.2	0.1	0.08	0.01
project 3	2000	0.1	0.126	0.012	0	0.2	0.05	0.08	0.01
project 4	2000	0.1	0.126	0.024	0	0.2	0.1	0.08	0.05
project 5	2000	0.1	0.126	0.024	0.1	0.2	0.1	0.08	0.01
project 6	4000	0.1	0.126	0.024	0	0.2	0.1	0.08	0.01
project 7	2000	0.1	0.126	0.024	0	0.2	0.1	0.2	0.01
project 8	2000	0.063	0.126	0.024	0	0.2	0.1	0.3	0.01
project 9	2000	0.1	0.222	0.018	0	0.35	0.075	0.3	0.01
project 10	2000	0.1	0.222	0.018	0.2	0.35	0.075	0.3	0.1

The results of mathematical modelling of the projects have been compared and influence from various design and process parameters investigated without deep quantitative analysis. Presented curves show that both design and process parameters change the nature and size of movement. The obtained results clearly show the need for

further research. Detailed analysis should be carried out in order to assess the affect each of the parameters could make and how this would influence the process of dosing, productivity and accuracy, etc. This information is essential for establishing optimal filling procedure setup and a proper design of weighing filling machine.

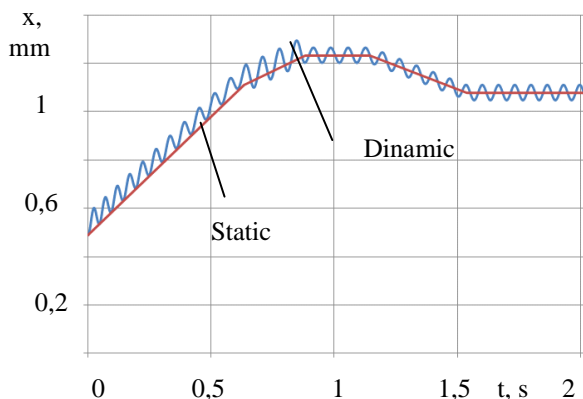


Fig. 2 Displacement curve, project 1

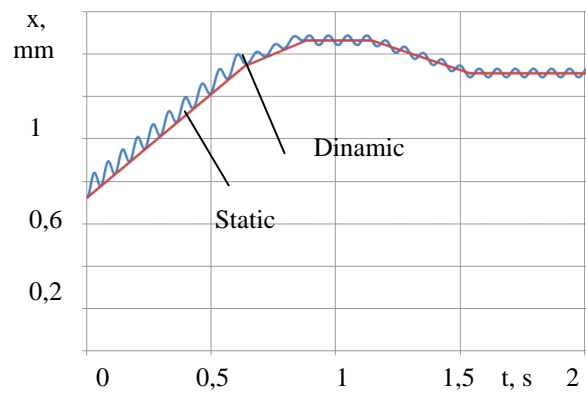


Fig. 3 Displacement curve, project 2

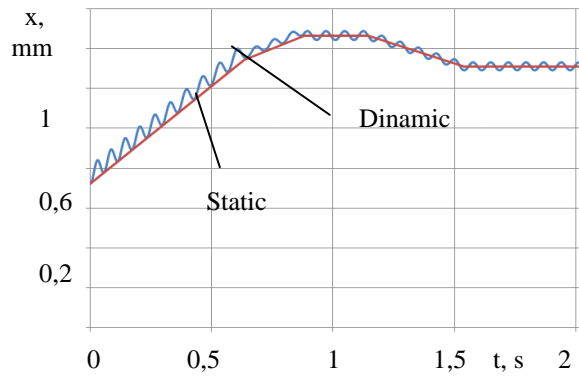


Fig. 4 Displacement curve, project 3

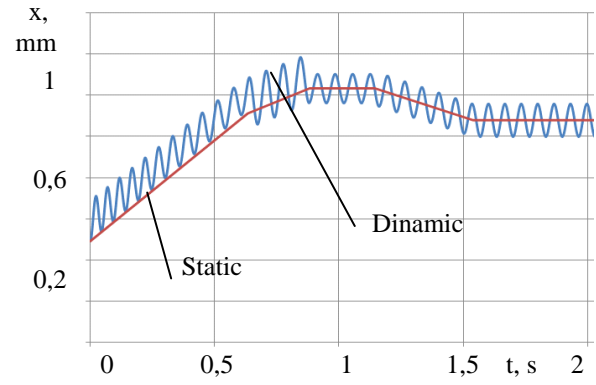


Fig. 5 Displacement curve, project 4

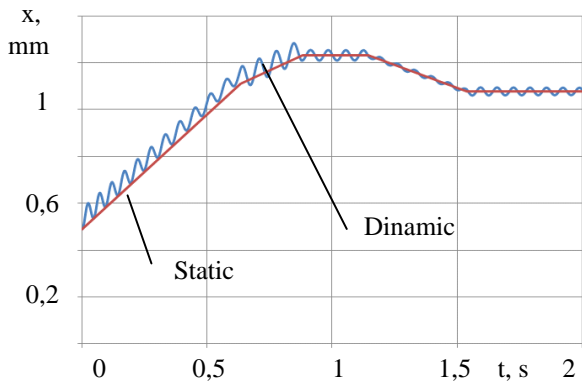


Fig. 6 Displacement curve, project 5

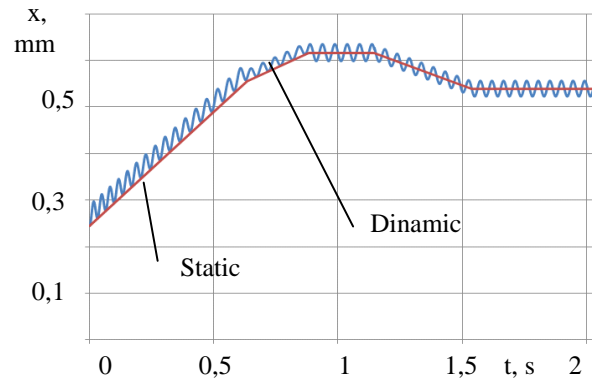


Fig. 7 Displacement curve, project 6

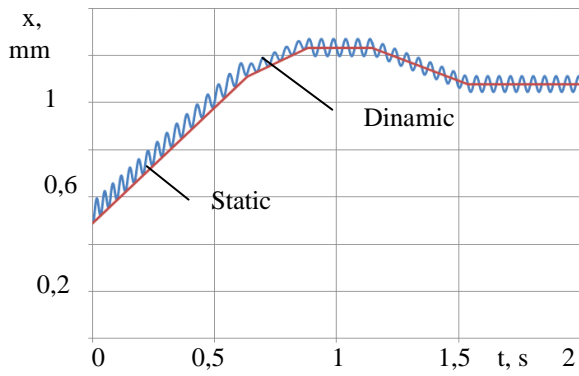


Fig. 8 Displacement curve, project 7

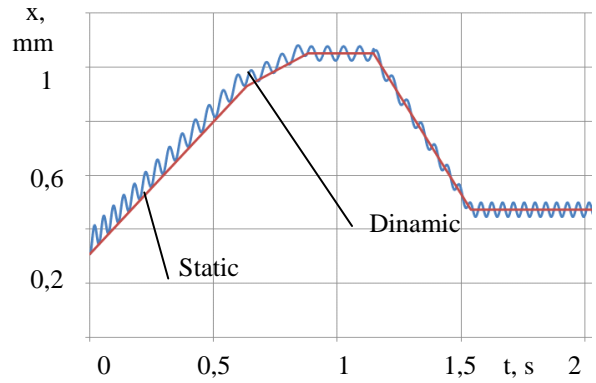


Fig. 9 Displacement curve, project 8

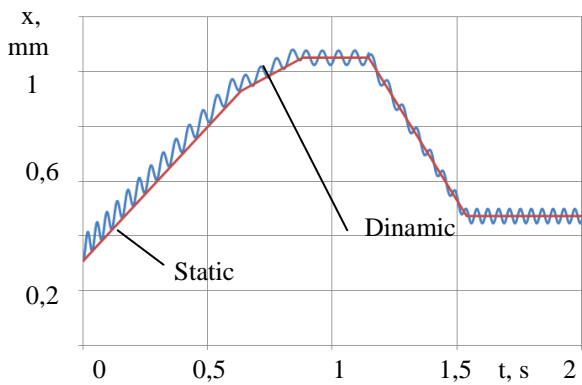


Fig. 10 Displacement curve, project 9

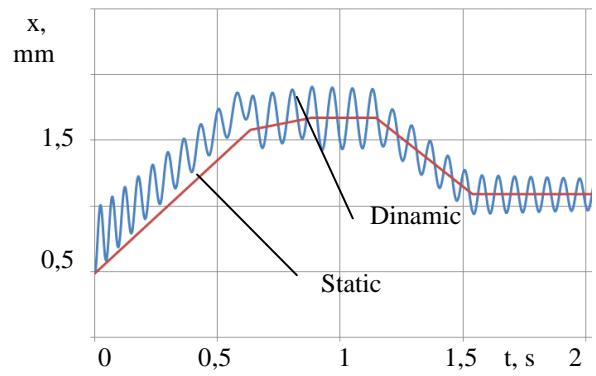


Fig. 11 Displacement curve, project 10

## 5. Conclusions

In the article a typical structural layout diagram and the mathematical model of operation of filling system with weighing type batcher is presented. Sensitive element of weight batcher is regarded as an elastic system with one degree of freedom, operating together with the variable mass body attached to it. The mathematical model describes operation of weight batcher throughout its full working cycle, covering all five stages: stage one- filling at increased productivity, stage two- filling at low productivity, stage three – suspension of filling (pause before discharge), stage four – discharge of product and stage five- suspension of discharge (pause after discharge). The model incorporates all key elements affecting performance of the batcher and is fit for theoretical analysis of machine operation, influenced by various design and process factors. Analysis results representing for ten projects is presented.

Structural scheme of the system with weighing filling machine for dry products is analyzed. Sensitive element of weighing filling machine is regarded as elastic system with one degree of freedom, on which the variable mass body operates. Mathematical model for describing the operation of the weighing filling machine for dry products is shown. Working cycle of weight batcher consists of five stages. Step one is filling at increased productivity. Step two is filling at low productivity (accurate filling). The third – is suspension of filling (pause before discharge). Step four is discharge of product. Step five – the discharge is suspended (pause after discharge).

The model describes operation of the batcher at various working conditions and can be used for theoretical considerations at various development stages of dosing process and the batcher itself in order to eliminate its critical elements and find the right setup of the system. Several projects are compared. The modelling results, representing curves of body displacement, are presented. The curves show that both the design and process parameters have an influence and change the nature and size of movement.

The obtained results show the need for further research. Detailed analysis is to be carried out in order to assess the effect each of the parameters makes and how this would influence the process of dosing, productivity and accuracy, etc.

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BIRIŲJŲ PRODUKTŲ SVORINIO DOZATORIAUS DARBO MATEMATINIS MODELIS

Re z i u m ė

Straipsnyje pateikta biriųjų produktų pakavimo sistemos su svoriniu dozatoriumi tipinė struktūrinė schema ir dozavimo proceso matematinis modelis, kaip vieno laisvės laipsnio tamprinė sistema, kurią veikia kintamos masės kūnas. Pateiktas svorinio dozatoriaus darbo per visą ciklą matematinis modelis. Svorinio dozatoriaus darbo ciklą sudaro penki etapai. Pirmasis etapas – tai produkto tiekimas didesniu našumu, antrasis – produkto tiekimas mažo našumu, trečiasis – produkto tiekimo sustabdymas (paūzė

prieš išpylimą), ketvirtasis – produkto išpylimas, penktasis – produkto išpylimo sustabdymas (pauzė po išpylimo). Šis svorinio dozatoriaus darbo matematinis modelis leidžia teoriškai nagrinėti dozatoriaus darbo procesą ir dozatoriaus schemos bei konstrukcijos ypatumus. Straipsnyje pateikta dešimties svorinių dozatorių darbo lyginamoji analizė. Pateiktos šių projektų kaušo judėjimo trajektorijos visuose penkiuose etapuose, sudarančiuose svorinio dozatoriaus darbo ciklą. Atliktas tyrimas rodo, jog kaušo judėjimo trajektorija ir jos dydis priklauso ir nuo svorinio dozatoriaus techninių parametru, ir nuo dozavimo proceso technologinių režimų. Gauti rezultatai rodo, jog reikalingi tęstiniai tyrimai. Būtina detaliai ištirti, kaip kiekvienas techninis ir technologinis parametras veikia dozavimo procesą.

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#### MATHEMATICAL SIMULATION OF OPERATION OF THE WEIGHING TYPE FILLING MACHINE FOR DRY PRODUCTS

#### S u m m a r y

In the article a typical structural layout diagram and the mathematical model of operation of filling system

with weighing type batcher is presented. Sensitive element of weight batcher is regarded as an elastic system with one degree of freedom, operating together with the variable mass body attached to it. The mathematical model describes operation of weight batcher throughout its full working cycle, covering all five stages: stage one - filling at increased productivity, stage two - filling at low productivity, stage three – suspension of filling (pause before discharge), stage four- discharge of product and stage five - suspension of discharge (pause after discharge). The model incorporates all key elements affecting performance of batcher and is fit for theoretical analysis of machine operation, influenced by various design and process factors. Analysis results representing comparison of shovel displacement curves throughout the working cycle for ten different projects of batchers are presented. The curves show that both the design and process parameters have influence and change the nature and size of movement. The results indicate the need for further research. Detailed analysis is to be carried out in order to assess the affect each of the parameters makes and how this would influence the entire dosing process.

**Keywords:** packing, weight dosing, mathematical model.

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