

# Fuzzy connections in structural analysis

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## 1. Introduction

Civil engineering structures may be considered as systems whose inputs are external loads and corresponding outputs are the structural response or internal forces. In this regard, structural systems are described by their geometry and mechanical properties like stiffness. However, like other systems, the system description and system inputs may have uncertainties. The system uncertainty could be classified as uncertainties due to randomness or due to impreciseness. As an example wind or earthquake loads on the structures are not known in advance and hence in structural design, usually wind or earthquake loads are considered to be random and determined based on the statistical and probabilistic concepts. Impreciseness in structural systems usually arises from the complexity of the involved parameters. For example while theoretically it is possible to describe dead loads on the structures by using complicated mathematical expressions, but usually dead loads are simply described as uniform or concentrated loads. The terms uniform or concentrated load and their associated value refer to some concept that is used to describe the quantity and quality of the involved parameter (i.e. dead load). In many cases even it is not possible to have a definite value for the concept. While the first type of the uncertainty (randomness) is tackled by stochastic and statistical methods, the second type of the uncertainty (impreciseness) is not properly handled by these methods.

In 1965, Zadeh founded the basis of a new branch of mathematics which now is known as fuzzy theory [1]. During the past two decades, the fuzzy theory has been extensively and successfully used for analysis of the systems where imprecise (vague) parameters and cognitive uncertainties are involved. Some researchers have used this theory for analysis of civil engineering systems including modeling of reservoir operation [2], diagnosing cracks in RC structures [3], slope failure potential [4], bearing capacity of piles [5] and a number of applications in structural analysis and optimization [6-12]. In these researches, the fuzzy theory is used for description and analysis of the engineering systems in the presence of vagueness or impreciseness. For example, vagueness in structural analysis could be found in geometry, material properties, boundary conditions or loads. Some researches have discussed application of the fuzzy theory in finding the response of structures subjected to such uncertainties.

One of uncertainties which is encountered in structural analysis and design is the nature of the structural connections. Traditionally, structural connections are modeled ideally as fully hinged or fully rigid connections and the response of the structures are found using these ideal models for connections. In reality, however, the situation is

different, i.e. any hinged connection carries some moment and any rigid joint has some degree of flexibility. This fact is known for a long time and in many building codes (e.g. AISC) the concept of semi-rigid connections is recognized. Many researchers have investigated the behavior of the structural connections and literature in this regard is relatively reach [13-17]. But despite the recognition and investigations of the semi-rigid connections, there is no practical tool for analysis of the structure with the semi-rigid connections due to complexity of the problem and lack of precise describing for every single connection behavior. Many parameters like quality of welds, workers skill and type of the connecting elements affect the behavior of a joint. So, understanding the behavior of the joints in the real structures needs much time consuming and expensive tests which is practically impossible. In fact any single connection has its own behavior which needs a separate investigation for being described by traditional mathematical language. Instead, it seems that fuzzy linguistic variables [18, 19] are proper tools for describing the joint behavior. This research concerns on using fuzzy theory for analysis of the structures when the connection is to be considered as semi-rigid. After a brief introduction to the fuzzy theory, there is an overview of the common structural joints and their behavior. Then it has been illustrated how rigidity of a connection can be modeled as a fuzzy number or fuzzy linguistic variable. The problem of establishing the stiffness matrix and solving the resulting equations is studied next. For understanding the response of the structures with semi-rigid connections a FORTRAN program is developed and several structures was analyzed and conclusions were made based on the results.

## 2. Fuzzy sets vs crisp sets

A set is defined as a collection of objects with common properties or adjectives. Usually capital letters are used to name different sets. For example we may refer to set **A** of even numbers, set **B** of cities with low average temperature and set **C** of cities with average temperature higher than 25°C. All objects belonging to a set are called members of that set. For example 8 is a member of the set **A** (even numbers) and Paris with average temperature of 30°C is a member of set **C**, while Moscow with average temperature of 5°C is not a member of set **C**. With regard to membership of different objects, the sets are classified into two groups: Crisp sets and Fuzzy sets. If the membership of the all objects to a set is clear then the set is said to be a crisp set otherwise if membership of one or more objects to a set is not clear then the set is said to be a fuzzy set. For example the set **C** of cities with average temperature higher than 25°C is a crisp set because it is clear that

Paris belong to this set and Moscow does not belong to the set. On the other hand the set **B** of cities with low average temperature is not a crisp set because while Moscow with average temperature of 5°C definitely belongs to this set, but the membership of a city with average temperature of 12°C is not clear. So set **B** of cities with low average temperature is a fuzzy set. The classic theory of sets is essentially for crisp sets. In 1965, Prof. Zadeh understood that many sets (i.e. fuzzy sets) can not be studied by the classical set theory and hence he developed the theory and concepts of the fuzzy sets. He proposed that the membership of the objects to a fuzzy set can be assigned a value between 0 to 1. If an object certainly belongs to a set, a membership value of 1 is assigned to it and if an object certainly does not belong to a set a membership value of 0 is assigned to it. Other values between 0 and 1 are assigned to the objects whose membership to the set is not clear. Zadeh also defined common operations like intersection and union of two fuzzy sets [19]. A fuzzy set **B** is called a subset of fuzzy set **A** if membership of any object to set **B** is equal or less than its membership to set **A**. It is also possible to find the set of all objects whose membership to the fuzzy set **A** is equal or greater than a specified value  $\alpha$ . This set is called  $\alpha$ -cut of the set **A** [19].

In fuzzy theory, every fuzzy set is described by its elements and their membership function. For example the set **B** of low average temperatures may be represented as in Fig. 1. From this figure it is clear that a city with an average temperature of 3°C is a member of the set **B** with the membership value of 1 and a city with an average temperature of 14°C is a member of the set **B** with the membership value of 0.6 and a city with an average temperature of 20°C is not a member of the set **B** (membership value is 0). These can be shown  $\mu_B(3) = 1$ ,  $\mu_B(14) = 0.6$ ,  $\mu_B(20) = 0$  where  $\mu$  is called membership function of the set **B**.

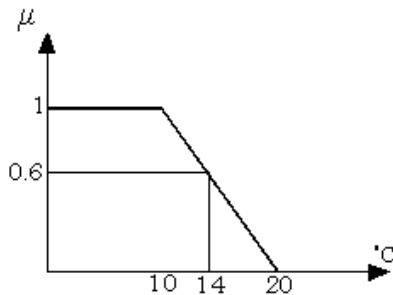


Fig. 1 Fuzzy set of low average temperature

A fuzzy number is a fuzzy set whose members are numbers and membership of numbers to the set follows some specific pattern. More information about fuzzy numbers and arithmetic operations on fuzzy numbers is found in [18]. The concept of fuzzy sets also leads to the definition of the linguistic terms and variables [19]. Linguistic terms are oral language words such as “low average temperature” or “more and less hot” and is described in a mathematical form by a fuzzy set like the one shown in Fig. 1 for “low average temperature”. A linguistic variable is a variable which can take linguistic terms as its value. For example, the average temperature may be considered as a linguistic variable which can take “low”, “mild” and “high” as its value (Fig. 2).

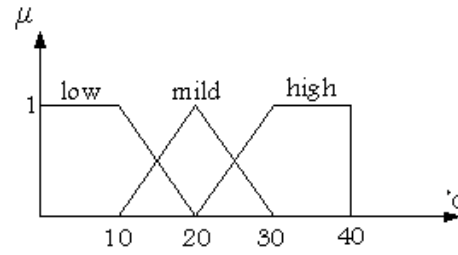


Fig. 2 Linguistic variable of low average temperature

### 3. Fuzzy structural connections

Many structural systems are constructed by assembling a number of prismatic elements (i.e. beams and columns) jointed together by structural connections. The behavior of beams and columns under various loads is well studied in the literature and relatively precise description of these elements can be reached by FEM or slope deflection method. However overall behavior of a building frame not only depends on behavior of the beams and columns but also on the behavior of structural connections (joints). In conventional analysis and design of the frames, it is a common practice to model these connections as dimensionless ideal limiting cases of fully hinged or fully rigid joints. However the reality is different: any real hinged connection has some degree of rotational stiffness and every so-called rigid connection has some degree of flexibility. Hence realistic evaluation of member forces and displacements should be done considering uncertainty involved in rigidity of the structural joints. For modeling this kind of uncertainty, which is due to complex behavior of joints, it seems that fuzzy theory provides a powerful analytical tool. In this research, various types of common connections are modeled as fuzzy numbers through their fixity factor [13]. Many researchers have modeled structural connections as the rotational springs (Fig. 3).

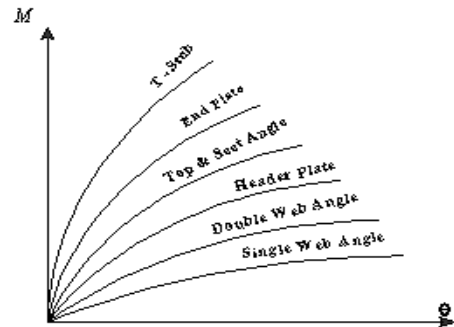


Fig. 3 Moment-rotation curve of connections

Fixity factor  $\gamma$  of a joint connected to a member and modeled as a rotational spring is defined as the ratio of rotation at the two sides of the joint connecting two members (Fig. 4). On the other hand the stiffness of the spring itself is another way of describing the joint behavior. For this purpose experimental works have been done on different types of the connections and  $M-\theta$  curves have been plotted for the connections where  $M$  is the moment applied to the connection and  $\theta$  is the rotation induced by the moment. Typical common structural connections is studied in [14, 20] and indicates the complex behavior of these connections.



Fig. 4 Beam-column element with flexible connections

Many parameters like quality of welds, workers skill and type of the connecting elements affect the behavior of a joint. So, understanding the behavior of the joints in real structures needs much time consuming and expensive tests which is practically impossible. Hence for a realistic analysis of structures, a systematic approach is needed to look after the uncertainty in the joints behavior. In this paper, the Fixity factor  $\gamma$  of a connection connected to a

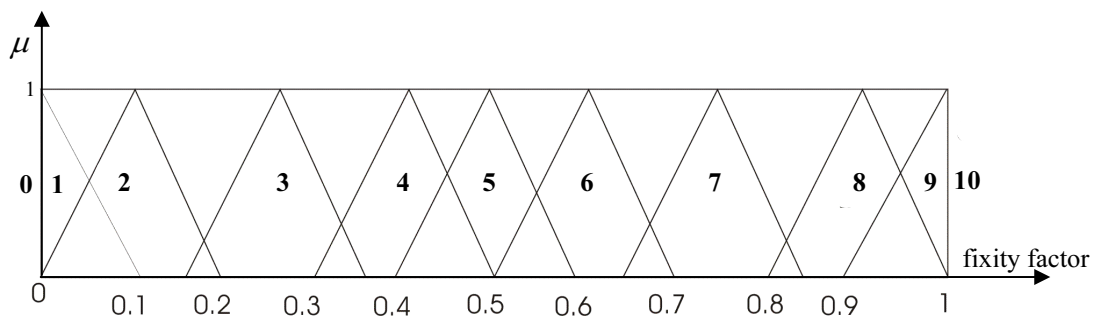


Fig. 5 Fuzzy Number corresponds to linguistic variable of restraint

#### 4. Solving the system equation

As said, the connection stiffness can be modeled as a fuzzy number to present the ambiguity involved. As a result the stiffness matrix of a beam element with such connections becomes fuzzy. Deriving fuzzy stiffness matrix for a beam element is lengthy and the final result is reported in [21]. Having the fuzzy stiffness matrix for individual elements, it is possible to form the overall stiffness matrix  $\tilde{K}$  for a frame by assembling stiffness matrix of the elements combining finite element concepts and fuzzy arithmetic. The response of the structural system to any external loading then is found by solving the system equilibrium equation

$$KX = P \quad (1)$$

where the vector  $X$  is the displacement of the system and  $P$  is the force vector resulting from the external loads. The vector of forces  $P$  also can be found by assembling the nodal forces calculated by slope deflection method for each element. For finding the nodal displacements and element forces, the system of Eq. (1) should be solved. A general solution for Eq. (1) when  $K$  is fuzzy is not available, although some methods have been proposed [22, 23], but these methods are not practically applicable. Rao et al. proposed an algorithm for solving equation 1 which is based on optimization concepts [8]. The method is used for simple structures like a beam successfully, but for large structures the solution does not provide acceptable results. Ayyube and Chao used Fuzzy arithmetic and permutation

member is assumed to be a fuzzy number. Considering fixity factor of a connection as a fuzzy number, the common connections in steel structures can be defined by linguistic terms such as rigid, very rigid or more and less rigid, etc. [21]. Eleven linguistic terms are used and each one is assigned a number from 0 to 10. These include 0-Ideal Hinged (Absolutely Hinged), 1-Very Hinged (e.g. single web angle), 2-Almost Hinged (e.g. single web plate), 3-Fairly Hinged (e.g. double web angle), 4-More and Less Hinged (e.g. header plate), 5-Half Rigid-Half Hinged (e.g. top & seat angle), 6-More and Less Rigid (e.g. top plate & seat angle), 7-Fairly Rigid (e.g. top & seat plate), 8-Almost Rigid (e.g. end plate), 9-Very Rigid (e.g. t-stub & web angle), 10-Ideal Rigid (Absolutely Rigid). These terms were considered to be triangular fuzzy numbers with 20% absolute spread [18] as shown in Fig. 5.

for analyzing structures having fuzzy modulus of elasticity [8]. Their first method which is based on the fuzzy arithmetic yields approximate solutions which are extremely wide. The permutation method is not practical because it needs  $2^{n(n+1)}$  stages (i.e. solving  $2^{n(n+1)}$  linear systems of  $n$  equations) if all the  $n(n+1)$  coefficients of the complete matrix are fuzzy parameters. Taghuchi algorithm is utilized in [11] for solving fuzzy systems. The method put some limits on the fuzziness of the system parameters which is not desirable. Abdel-Tawab and Noor used combinatorial method for solving system equations in a dynamic thermo elasto-viscoplastic analysis [6].

When the  $K$  matrix is fuzzy, considering various methods, it is found that Eq. (1) can not be solved practically or yields the fuzzy numbers which are extremely wide. However, it was found that for structural displacements with the fuzzy connections, the combinatorial method [24] offers a practical solution while avoiding unmanageable computational efforts and provides exact solution. With  $n$  Fuzzy parameters (i.e.  $n$  fuzzy connections), the combinatorial method requires  $2^n$  stages (i.e. Solving  $2^n$  linear equations) for finding hull of the fuzzy displacements. Having structural displacements expressed in terms of fuzzy numbers, it is possible to calculate element forces by either of the following techniques: a) at each stage the element forces are calculated using fuzzy displacements obtained at that stage. The hull of fuzzy element forces is then obtained from element forces from all stages, b) the hull of fuzzy element forces also can be calculated from hull of fuzzy structural displacements. The two techniques yield different answers. However,

technique 1 is more consistent with combinatorial techniques. In numerical examples first technique and second techniques are referred to as COMB-1 method and COMB-2 method, respectively.

5. Numerical examples

For evaluating the effect of the fuzzy connections in structures, a program named FCICE is developed for solving large structures with many fuzzy connections. The program allows the user to choose structural connections either from a list of common type of connections or as a linguistic terms as described previously. The program also solves the fuzzy system of equations by either optimization method or combinatorial method selectable by user. The results obtained from two numerical examples are reported here.

5.1. Example 1

This example was designed for assessment of the program itself as well as different techniques used in the Program. In this example a one fixed end, one pinned end beam as shown in Fig. 6 is investigated. The support conditions at both end as well as beam properties are listed in Table 1. The last two columns in the table are connection type at the beam nodal points which here are considered type 0 (ideal hinge) and type 5 (half rigid-half hinged) connections. The following results were obtained by analyzing the beam by FCICE program:

1. As shown in Fig. 7, the optimization method yields a more wider intervals for the fuzzy displacements compared to combinatorial methods.

2. The horizontal displacement at node 1 is determined to be the non-fuzzy number 1.9048 mm, which is equal to the beam axial displacement using the simple formula  $\delta = Pl / EA$ . It can be concluded that the single fuzzy connection in this case has no effect on the horizontal displacement of the node 1. However it can be seen in Fig. 8 that the optimization method produces unwanted errors due to complicated numerical calculations.

3. Although the connection at node 1 is an ideal hinge, but the COMB-2 method and optimization method yields zero fuzzy numbers for the moment at this node (Fig. 9). Again the answer of optimization method has wider intervals. The COMB-1 method resulted in a non-fuzzy zero for the same moment, which is more acceptable (Fig. 9).

It can be concluded that the combinatorial method yield better results compared to optimization method. Also element forces obtained by the COMB-1 method have narrower intervals compared to COMB-2 method. The results of the program are in agreement with expectations that intervals width should satisfy the following inequalities:

- Optimization method  $\geq$  COMB-2 method;
- COMB-2 method  $\geq$  COMB-1 method.

Also COMB-1 method has shown results that are more consistent with the ideal-hinged end of the beam when compared to Optimization method and COMB-2 method.

Table 1

Characteristics of example one

Element number	Cross section, mm <sup>2</sup>	Moment of inertia, mm <sup>4</sup>	Fixity factor, Nod i	Fixity factor, Nod j
1	2.00E+03	8.70E+06	0	5

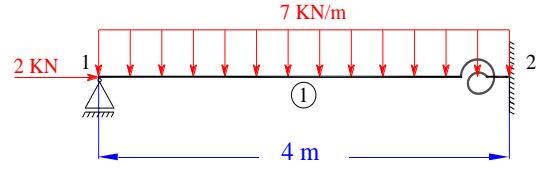


Fig. 6 Example 1 (one fixed end, one pinned end beam)

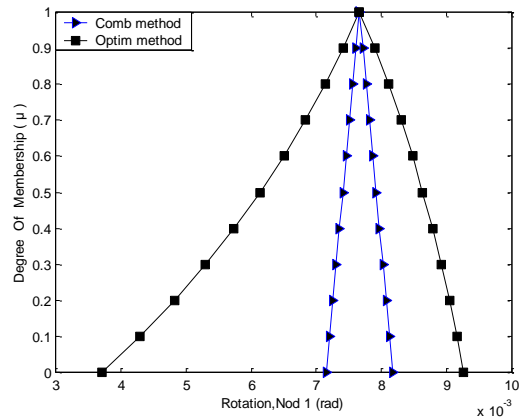


Fig. 7 Rotation of node 1

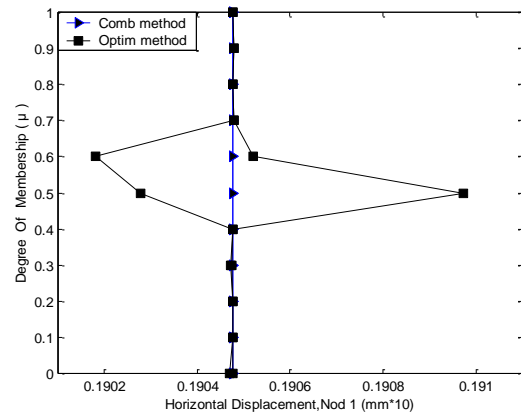


Fig. 8 Horizontal displacement of node 1

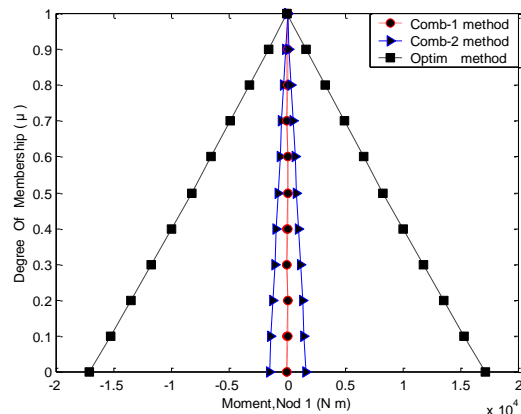


Fig. 9 Left moment (node 1)



5.2. Example 2

This example is designated to show some aspects of considering fuzziness in the structural connections. The two storey building shown in Fig. 10 is considered to have “very rigid” supports while the beam-column connections are assumed to be “almost rigid”. Other properties of the frame are shown in Table 2. Nodal displacements and element forces in this frame are calculated with two different techniques: In the first technique (labeled as METHOD-1 in the figures), the gravitational and lateral loads are applied simultaneously and the total displacements and forces were obtained using FCICE program. In the second technique (labeled as METHOD-2 in the figures), the program calculates the displacements and forces due to gravitational loads and lateral loads separately and then the resulting fuzzy numbers are added to find total displacements and element forces. The following results for nodal displacements and element forces were obtained by analyzing the frame:

1. Figs. 11-13 shows the horizontal displacements of node 5, support moment for element 1 and base shear for element 1. As it can be seen, the interval width is larger when METHOD-2 is used. The fact that results of the METHOD-1 and METHOD-2 are different, is an interesting fact by itself since it demonstrates that although a linear analysis is carried out, but the principle of the superposition is not held due to fuzzy connections. This needs to be investigated more carefully.

2. Table 3 compares the moments, shears and axial forces obtained by conventional analysis assuming non-fuzzy connections with those obtained by FCICE program assuming fuzzy connections. In the later case the fuzzy number is represented by its center of gravity defuzzifier. This table reveals that considering connections as fuzzy numbers has significant effects on the element forces. For example, end moments of the elements connected to node 3 are reduced by 11%.

3. For evaluating fuzziness inducted in the element forces due to fuzzy connections; the concept of fuzzy entropy [20] may be used. In Fig. 14, the fuzziness of various element forces is measured by fuzzy entropy where the vertical axis is the fuzzy entropy and the horizontal axis represents element number. A greater fuzzy entropy in an element shows greater uncertainty in the element forces. This implies a greater safety factor (or load factors) for designing such elements when cognitive uncertainty is involved.

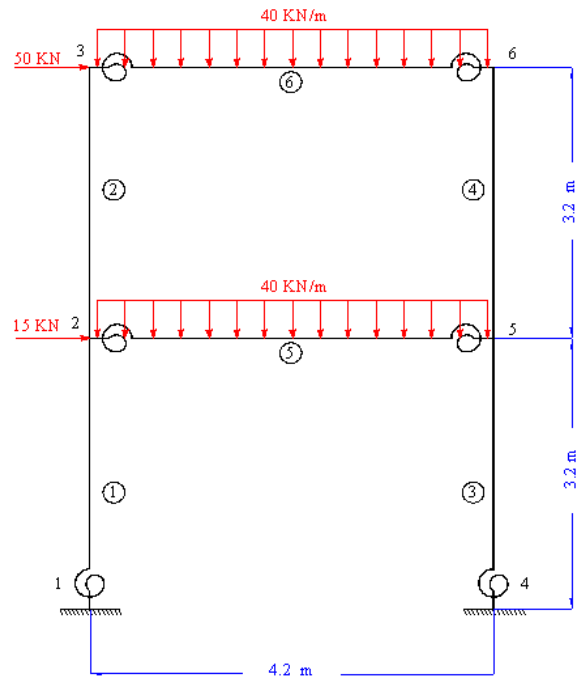


Fig. 10 Example 2 (bending frame)

Table 3

Fuzzy vs crisp analysis

Element number	Axial Force, N	Shear force Nod i, N	Moment Nod i, Nm	Shear force Nod j, N	Moment Nod j, Nm
1	134794	24916	-50198	24916	-29534
	135210	25445	-51722	25445	-29705
2	79235	-13926	23257.5	13926	21304.2
	75922	-12875	22308	12875	18891
3	210121	40084	-66335	40084	-61933
	200790	39555	-66586	39555	-60012
4	91765	28926	-38646	28926	-53916
	92078	27875	-36391	27875	-52814
5	11158	58558	6276.8	109442	100578
	11679	59282	7411.8	108720	96420
6	28935	76235	-21304	91765	53916.1
	27875	75922	-18891	92078	52814

Table 2

Characteristics of example two

Element number	Cross section, mm <sup>2</sup>	Moment of inertia, mm <sup>4</sup>	Fixity factor, Nod i	Fixity factor, Nod j
1	3.30E+03	1.08E+07	9	10
2	3.30E+03	1.08E+07	10	10
3	3.30E+03	1.08E+07	9	10
4	3.30E+03	1.08E+07	10	10
5	2.40E+03	2.03E+07	8	8
6	2.40E+03	2.03E+07	8	8

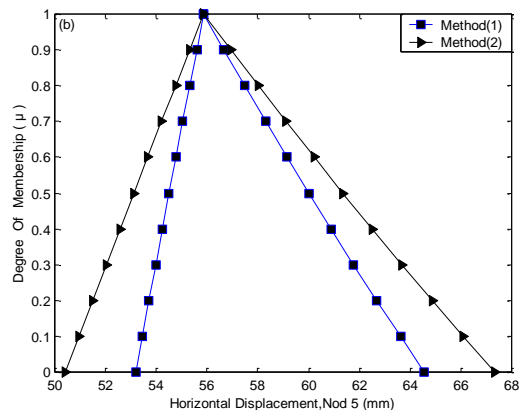


Fig. 11 Lateral displacement of node 5

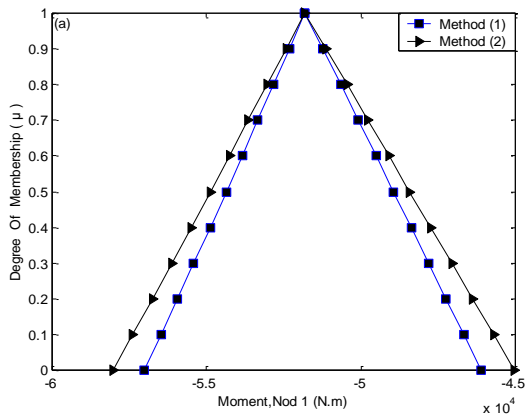


Fig. 12 Base moment (element 1)

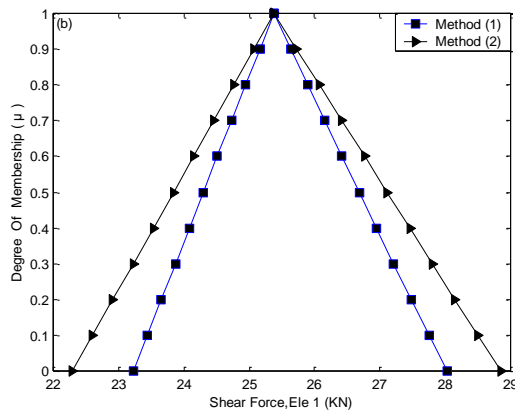


Fig. 13 Shear force (element 1)

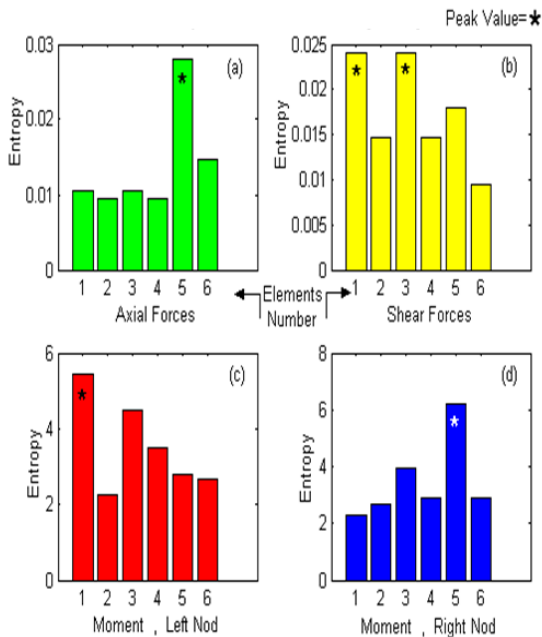


Fig. 14 Fuzzy entropy for element forces

## 6. Conclusion

In this research it was proposed that the vagueness in the rigidity of the structural connections is to be accounted for in structural analysis involving determining nodal displacements and element forces. The uncertainty of the connections was represented by the fuzzy numbers which seems to be an effective tool for modeling such uncertainties. Common structural connections were linked to

the linguistic terms such as fairly rigid or more and less rigid. A triangular fuzzy number then is assigned to each linguistic term which results in a fuzzy stiffness matrix for a structure possessing such connections. A program was written for analyzing the structures with the fuzzy stiffness matrix. The program uses the combinatorial as well as optimization methods for solving the resulting fuzzy system of equations. The credibility of the program was assessed by analyzing some simple numerical examples and then more complicated structures were analyzed by the program. Results of the numerical examples are summarized below:

1. Fuzzy theory provides effective tools in modeling uncertainty involved in the rigidity of the structural connections. This allows such uncertainties can be modeled and incorporated in the structural analysis.

2. Optimization method for solving fuzzy system of equations results in fuzzy numbers which have wider intervals comparing to the better combinatorial method. The optimization method also becomes incredible when the number of fuzzy connections in a structure is large.

3. The principle of superposition is not held when fuzziness of the connections are taken into account in structural analysis. This implies more investigations for possible practical applications of the fuzzy theory in the structural analysis and design.

4. Fuzzy entropy can be considered as a safety factor for designing of the structural elements when cognitive uncertainties involved.

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## “NEAPIBRĖŽTI” RYŠIAI KONSTRUKCIJŲ ANALIZĖJE

### R e z i u m ė

Konstruciniai ryšiai modeliuojami kaip nejudami ar įtvirtinti šarnyriškai. Tačiau šarnyriniai ryšiai taip pat būna veikiami lenkimo momentų, o nejudami pasižymi liaunumu. Tai yra įvertinama pusiau nejudamais sujungimais konstrukcijos brėžiniuose, tačiau trūksta praktinio įrankio konstrukcijų su pusiau nejudamomis jungtimis analizei, nes bet kuri jungtis pasižymi savais ypatumais, kuriuos sunku matematiškai griežtai apibrėžti. Šie tyrimai siejami su “neapibrėžtų” ryšių teorijos pritaikymu konstrukcijų su pusiau nejudamais sujungimais analizei. Parodyta, kad sujungimo standumas gali būti modeliuojamas kaip “neapibrėžtas” skaičius arba lingvistinis kintamasis. Analizuojama “neapibrėžto” standumo matrica lygčių sprendimui. Keletas konstrukcijų išanalizuota kompiuteriu ir pasiūlyti apibendrinimai.

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## FUZZY CONNECTIONS IN STRUCTURAL ANALYSIS

### S u m m a r y

Structural connections are modeled as hinged or rigid. However, hinged connection carries some moment and rigid joint has some flexibility. This is recognized by concept of semi-rigid connections in building codes. But, there is no practical tool for analysis of the structures with the semi-rigid connections as any connection has its own behavior which is not mathematically well defined. This research concerns on using fuzzy theory for analysis of the structures with semi-rigid connections. It is illustrated that rigidity of a connection can be modeled as a fuzzy number or linguistic variable. The fuzzy stiffness matrix and solving the equations is studied. Several structures were analyzed by a computer program and conclusions drawn.

**Keywords:** semi-rigid connection, fuzzy, vagueness.

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