Multi-objectives Optimization Design of A-type Frame in an Electric Mining Dump Truck Considering Multi-Source Uncertainties Based on the Interval Method

Chengji MI*, Zhonglin HUANG**, Wentai LI***, Di KANG****, Dong ZHANG*****,
Qishu YAO******, Xingzu MING*******.

*Department of Mechanical Engineering, Hunan University of Technology, Zhuzhou, 412007, China,
E-mail: michen@126.com
**Department of Mechanical Engineering, Hunan University of Technology, Zhuzhou, 412007, China,
E-mail: zhaoqishui@126.com
***Department of Mechanical Engineering, Hunan University of Technology, Zhuzhou, 412007, China,
E-mail: liwentai@126.com
****Hunan Academy of Forestry Sciences, Changsha, 410004, China, Email: 84289700@qq.com
*****Department of Mechanical Engineering, Hunan University of Technology, Zhuzhou, 412007, China,
E-mail: zhangdong0722@126.com
******Department of Mechanical Engineering, Hunan University of Technology, Zhuzhou, 412007, China,
E-mail: youqishui@126.com
*******Department of Mechanical Engineering, Hunan University of Technology, Zhuzhou, 412007, China,
E-mail: mxz69036@126.com
********Department of Mechanical Engineering, Hunan University of Technology, Zhuzhou, 412007, China,
E-mail: nizhengshan@126.com
crossref http://dx.doi.org/10.5755/j02.mech.24939

1. Introduction

The A-type frame that pertains to the truck’s steering system is a pivotal load-carring part, especially under downhill turning braking condition. This welded structure has to endure the cyclic loads caused by the random mine road surface, and thus its fatigue reliability needs to be guaranteed. The structural fatigue reliability is related to dimension, material and load, which are decided during design, manufacturing and use stage, respectively. However, there are uncertain factors in these procedures of production, such as dimensional tolerance, artificial processing technique and random excitation. Therefore, taking the multi-source uncertainties into account is indispensable to conduct on the optimization design of this component [1-2].

Numerous researches have been conducted on optimization design of engineering structure with uncertain parameters. Those approaches could be categorized into three major groups: probability [3-4], fuzzy [5] and interval optimization models [6]. The Gaussian mixture model was used to establish multimodal probability density functions of input random variables, and the uncertainties of input random variables were characterized by integrating of the sparse grid numerical approach and maximum entropy approach [7]. Based on the stress-strength interference model, the fatigue reliability of the design point was analyzed by the joint probability integral method, and then a reliability-based optimization design process was accomplished through the Kriging model considering both certain and uncertain variables [8-11]. A fuzzy multi-objective linear programming model with an interactive two-phase possibilistic linear programming was presented to approach for solving remanufacturing production system problems with multiple goals in a fuzzy environment [12]. In order to determine the observer gain, a fuzzy set-theory-based optimal approach was proposed by utilizing the fuzzy information of the uncertain bound [13]. However, compared with the two approaches mentioned above, less information is needed to characterize the range of variables and find the optimal solution through interval number, and its superiority exists in needless of constructing precise probability distribution function or fuzzy membership function. An approximate model between the design variables and stress was built by using TPS-HDMR, and the Kriging model was used to construct the relationship between the design variables and mass. The frame material density and Poisson’s ratio were considered as the uncertain variables, and the optimal solution was solved by the NSGA-II and IP-GA method [14]. An uncertain multi-objective optimization method for solving the optimization problem of sheet metal forming, and the hybrid optimization algorithm based on multi-objective genetic algorithm and sequential quadratic programming algorithm was utilized to find the Pareto set [15]. A fuzzy fatigue reliability analysis method based on self-configuring membership function was presented to accurately evaluate fatigue reliability of welded A-type frame, and the fuzzy fatigue reliability optimization [1-2]. However, this work mainly focused on the single optimization objective, and the uncertain variables were only considered as random parameters determined by the Latin Hypercube sampling method.

In this paper, the interval multi-objectives optimization function for improving fuzzy fatigue reliability and reducing weight of A-type frame in an electric mining dump truck considering multi-source uncertainties from design, manufacturing and use stage was firstly defined. Then, the relationship between optimization objectives and design variables was analyzed based on response surface method. Finally, the optimal solution was found through the non-dominated sorting genetic algorithm.
2. Interval optimization method with uncertain variables

If the ordered real number was utilized to define the interval of uncertain variables, the uncertain interval optimization function could be described as [16]:

\[
\begin{align*}
\min & \quad f(X,U) \\
\text{s.t.} & \quad g_i(X,U) \leq b_i^j = [b_i^l, b_i^u] \\
& \quad i = 1, 2, \ldots, l; X \in \Omega^i, \\
& \quad U \in U^i = [U_i^l, U_i^u], U_i = U_i^l = [U_i^l, U_i^u] \\
& \quad i = 1, 2, \ldots, q
\end{align*}
\]  

(1)

where \( f \) and \( g \) is target function and constraint function, respectively, which is a continuous function about \( X \) and \( U \). \( X \) is n-dimensional design variable, and belongs to \( \Omega^i \). \( U \) is q-dimensional uncertain vector, and its uncertainty is characterized by q-dimensional interval vector \( U^i \). \( b_i \) is i-th allowable interval of uncertain variables, and is the continuous function about \( U \) within its range.

\( f(X, U) \) and \( g(X, U) \) is the continuous function about \( U \), and the fluctuation range of \( U \) belongs to an interval vector. For any certain \( x \), their possible values all form an interval instead of uncertain values [16]:

\[
f^l(X) = [f^l(X), f^u(X)].
\]  

(2)

In general, the rational interval \( f(X) \) depends on the middle value \( f(X) \) and the radius value \( f^u(X) \). The mean value of target function could be improved by optimizing the middle value \( f(X) \), and the sensitivity of the objective functions on uncertainties could be reduced by optimizing the radius value \( f^u(X) \). Then, Eq. (2) can be changed into [16]:

\[
f^l(X) = [f^l(X), f^u(X)] = \left\{ f^l(X), f^u(X) \right\},
\]  

(3)

\[
f^* = \frac{f^l(X) + f^u(X)}{2},
\]  

(4)

\[
f^{**} = \frac{f^u(X) - f^l(X)}{2}.
\]  

(5)

Eq. (1) could not be solved by the traditional certain approach, however, the design variables can be evaluated by the middle value \( f(X) \) and the radius value \( f^u(X) \) mentioned above. If the design vector \( x^{(1)} \) was better than the design vector \( x^{(2)} \), the interval of target function with the design vector \( x^{(1)} \) could be better than the design vector \( x^{(2)} \). Thus, in order to find an optimal design variable for minimizing the middle value \( f(X) \) and the radius value \( f^u(X) \) during the interval of uncertain target function, the uncertain target function could be converted as the certain multi-objectives optimization function [16]:

\[
\begin{align*}
\min & \quad (f^*(X), f^{**}(X)) \\
& \quad f^*(X) = \frac{\min(f(X,U)) + \max(f(X,U))}{2} \\
& \quad f^{**}(X) = \frac{\max(f(X,U)) - \min(f(X,U))}{2}
\end{align*}
\]  

(6)

Two optimizations needed to find the interval of uncertain optimization target function. In fact, the middle value \( f^*(X) \) and the radius value \( f^{**}(X) \) were considered as the interval attribute for the same variable, so that they could be integrated during the optimization process. Then, the interval optimization model was transformed to be single optimization function as following [16]:

\[
\min f(X) = \lambda f^*(X) + (1-\lambda) f^{**}(X),
\]  

(7)

where \( \lambda \) stood for being partial to the mean value or robustness of target function. \( \lambda > 0.5 \) denoted that the objective function tended to be mean value, and in contrast \( \lambda < 0.5 \) meant it followed the robustness.

Based on the interval optimization method mentioned above, the certain design variables, uncertain design variables and constraints could be firstly determined. Then, the sample points of design variables and uncertain variables could be obtained by the Latin Hypercube method, and the corresponding response values could be calculated by the finite element analysis. The approximate model between design variables and optimization objectives could be constructed through the second-order response surface function. Finally, the optimization problem was solved by the NSGA. All steps were shown in Fig. 1.

![Fig. 1 Optimization flowchart](image)

3. Multi-objectives optimization design

3.1. Definition of optimization objectives and variables

According to the author’s previous work [1-2], the fatigue fuzzy reliability of A-type frame was improved by increasing the thickness of several welded sheet plates. To a certain extent, this optimization method would gain more materials for enduring cyclic loads and reducing fatigue stress, but it is adverse to the lightweight of this structure. Therefore, based on the multi-objectives optimization method mentioned above and stress-strength interference model, in this paper the maximum equivalent fatigue stress obtained from finite element analysis under cyclic loading was considered as optimization target, as well as the weight of A-type frame at the same time [1-2]. The allowable range of fatigue strength of high strength steel welded joint was found to be [142.32, 256.85] MPa, which was determined...
by the blind number method for considering section size, loads and mechanical behavior of material [2]. Then, the lower bound would be taken as the constraint boundary of optimization target in the following optimization process. The two-optimization objectives mainly depended on the thickness of welded sheet plates, and thus the thickness of tail, roof, lateral and base sheet plates was considered as the design variables, just like what the previous work did [1-2]. The maximum equivalent fatigue stress was also related to the random cyclic load and material’s parameters. In order to take the multi-source uncertainties caused in the use and manufacturing stage into account, the load at front traction joint having the most important influence on the stress level was regarded as the uncertain variable, as well as the Young’s modulus and density of welded joints, which would mainly affect the mechanical responses and weight, respectively [1-2]. The finite element model was built by software HYPERMESH, and had 84948 elements and 134564 nodes, which included 82046 C3D8 elements and 2902 C3D6 elements, as shown in Fig. 2 [1-2]. The model’s accuracy of this A-type frame was testified by the experimental work and discussed in my previous work as well [1-2]. The mark A1, A2, A3 and A4 were the connection sites at different location: front traction joint, right steering joint, left steering and front lateral stabilizer joint, and the corresponding loads at those joints were marked by F1, F2, F3 and F4 [1-2].

3.2. Determination of approximate model

In order to construct the approximate model between the design variables and optimization objectives, the design variables and uncertain variables were firstly obtained through the Latin Hypercube sampling method. There were thirty sample points for the design variables, and ten points for the uncertain variables. Then, according to the sample points, the maximum equivalent fatigue stress under cyclic loads acquired by the multi-bodies dynamic analysis was determined by the finite element analysis [1-2], while the weight could be calculated by the density and volume of welded sheet plates. The six sample values were listed in Table 1.

![Finite element model of A-type frame](image)

**Fig. 2 Finite element model of A-type frame**

**Table 1**

<table>
<thead>
<tr>
<th>Sample data of variables and responses</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thickness of tail plate $x_1$, mm</td>
</tr>
<tr>
<td>Thickness of roof plate $x_2$, mm</td>
</tr>
<tr>
<td>Thickness of lateral plate $x_3$, mm</td>
</tr>
<tr>
<td>Thickness of base plate $x_4$, mm</td>
</tr>
<tr>
<td>Density of material $w_1$, $\times 10^3$ kg/m$^3$</td>
</tr>
<tr>
<td>Young’s modulus $w_2$, MPa</td>
</tr>
<tr>
<td>Force at front traction joint $w_3$, N</td>
</tr>
<tr>
<td>Stress response value $s$, MPa</td>
</tr>
<tr>
<td>Weight response value $w$, t</td>
</tr>
</tbody>
</table>

According to Eqs. (6) and (7), the uncertain objective function of the maximum equivalent fatigue stress could be transformed into the certain one as following:

$$
\min S(X) = \lambda S'(X) + (1 - \lambda) S''(X),
$$

where the middle value $S'(X)$ and the radius value $S''(X)$ could be determined as following:

$$
S'(X) = \frac{\min(S(X, U)) + \max(S(X, U))}{2},
$$

$$
S''(X) = \frac{\max(S(X, U)) - \min(S(X, U))}{2}.
$$

Then, based on the second-order response surface approach, the approximate model between the certain optimization target and design variables could be ascertained.

The 3-dimentional response surface relationship between the certain stress and the thickness of lateral and base sheet plates was shown in Fig. 3, and the main effect relationship between the certain stress and all design variables was displayed in Fig. 4. Fig. 3 showed that there was nonlinear relationship between the optimization target and the two de-
sign variables. The other one illustrated that there was positive correlation between the certain stress and the thickness of lateral and base sheet plates, which implied that decreasing their thickness could be contributed to reduce the certain fatigue stress of welded A-type frame. In addition, there was negative correlation between the certain stress and the thickness of tail and roof sheet plates. There was parallel relationship for the interaction effect on certain stress between thickness of tail sheet plate and thickness of roof sheet plate, shown in Fig. 5, which signified that they were independent of having an impact on the optimization target.

According to the sample data and the fitting method, the 3-dimensional response surface relationship between the certain weight and the thickness of lateral and base sheet plates was shown in Fig. 6. It also had the non-linear relationship between the certain weight and the two design variables. The all design variables had a positive correlation with the optimization target, seen in Fig. 7. The thickness of tail sheet plate and the thickness of roof sheet plate were independent of having an impact on the certain weight, because they were parallel as shown in Fig. 8.

Similarly, the uncertain objective function of the weight of welded A-type frame could be transformed into the certain one as following:

\[
\min W(X) = \lambda W^r(X) + (1 - \lambda) W^w(X),
\]

(11)

where the medium value \( W^r(X) \) and the radius value \( W^w(X) \) could be also determined as following:

\[
W^r(X) = \frac{\min(W(X, U)) + \max(W(X, U))}{2},
\]

(12)

\[
W^w(X) = \frac{\max(W(X, U)) - \min(W(X, U))}{2}.
\]

(13)

According to the sample data and the fitting method, the 3-dimensional response surface relationship between the certain weight and the thickness of lateral and base sheet plates was shown in Fig. 6. It also had the non-linear relationship between the certain weight and the two design variables. The all design variables had a positive correlation with the optimization target, seen in Fig. 7. The thickness of tail sheet plate and the thickness of roof sheet plate were independent of having an impact on the certain weight, because they were parallel as shown in Fig. 8.

In order to testify the accuracy of those approximate models, the other ten sample points were chosen to compare the predicted results obtained from the response surface model with the simulated ones directly calculated in
the finite element model of welded A-type frame. The comparison of both mean value and radius value for two optimization objectives were shown in Fig. 9. It was clearly seen that the estimated data located at around the 45° equal line all agreed well with the simulated results, which indicated that the constructed response surface models were precise, and could be utilized to conduct on the multi-objectives optimization design in the following part.

![Graphs showing comparison of stress and weight values](image)

**Fig. 9 Verification of four approximate models**

4. Optimized results

4.1. Multi-objectives optimization based on NSGA

In this paper, based on the software Isight, the NSGA method was utilized to search for the optimal solution of the multi-objectives optimization problem. The number of populations was 50, and the number of iterations was equal to 100 [1-2]. The weight coefficient of certain stress was defined by 0.6, and the other one was 0.4. The design variables could be varied within ±25%. The constraint condition of certain fatigue stress was expressed as following:

\[ S(x) \leq \lambda B, \]  

where: \( B \) was the lower bound of interval fatigue strength, and \( \lambda \) was taken as 0.5 [16].

The constraint condition of certain weight could be also described as following:

\[ W(x) \leq \lambda T, \]  

where: \( T \) was allowable weight of welded A-type frame, and was taken as 2.0 in this paper [1-2].

4.2. Results

After the iteration was run about fifty minutes, the optimized results was obtained and listed in Table 2. It was clearly shown that the target value of certain stress was reduced by 10.70% through increasing the thickness of tail and roof sheet plates and decreasing the thickness of lateral and base sheet plates. In the meanwhile, because of the unequal weight coefficient, the target value of certain weight was increased by 7.41%, which was mainly caused by enlarging the thickness of tail and roof plates. However, the optimized result of certain weight was still within the range of allowable value. It was acceptable for ensuring the fuzzy fatigue reliability of welded A-type frame at expense of some weight. The initial and optimized maximum equivalent fatigue stress under cyclic loadings was shown in Figs. 10 and 11, respectively, which revealed that the stress had cut down from 135.80 MPa to 110.50 MPa.
### Table 2

<table>
<thead>
<tr>
<th></th>
<th>Initial</th>
<th>Optimized</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thickness of tail plate $x_1$, mm</td>
<td>36</td>
<td>41</td>
</tr>
<tr>
<td>Thickness of roof plate $x_2$, mm</td>
<td>32</td>
<td>40</td>
</tr>
<tr>
<td>Thickness of lateral plate $x_3$, mm</td>
<td>20</td>
<td>15</td>
</tr>
<tr>
<td>Thickness of base plate $x_4$, mm</td>
<td>20</td>
<td>15</td>
</tr>
<tr>
<td>Target value of certain stress $s$, MPa</td>
<td>70.93</td>
<td>63.33</td>
</tr>
<tr>
<td>Target value of certain weight $w_t$, t</td>
<td>0.9217</td>
<td>0.9896</td>
</tr>
</tbody>
</table>

In my previous work [2], a self-configuring membership function was presented, and was utilized to estimate the fuzzy fatigue reliability of optimized welded A-type frame. Before running the MATLAB code to calculate the reliability, the 100,000 random sampling points of thickness of tail and roof sheet plates were firstly implemented for the statistic analysis, whose results were shown in Fig. 12 and Fig. 13, respectively. It was obviously shown that the distribution of sampling data appeared as the normal one. Then, based on the fuzzy theory and proposed model, the fuzzy fatigue reliability of optimized welded A-type frame could increase from 69.47% to 98.45%, which meant that the optimization target achieved just by spending a little weight [2].

![Fig. 10 Stress contour of the initial welded A-type frame](image1)

![Fig. 11 Stress contour of the optimized welded A-type frame](image2)

![Fig. 12 Distribution of random sampling for thickness of rail sheet plate](image3)

![Fig. 13 Distribution of random sampling for thickness of roof sheet plate](image4)

### 5. Conclusion

In this paper, an interval multi-objects optimization method was presented to improve fuzzy fatigue reliability and make weight reduction of A-type frame in an electric mining dump truck considering multi-source uncertainties from design, manufacturing and use stage. Based on the Latin hypercube sampling method and finite element analysis, the uncertain and certain variables and their response values were all obtained. Then, the approximate models between the optimization target and design variables was constructed through the second-order response surface approach, and their accuracies were all testified by comparing the predicted results with simulated ones. Based on the NSGA, the multi-objects optimization design was conducted under the determined constraints of variables and optimization target. The maximum equivalent fatigue stress of optimized A-type frame was decreased by 18.63%, so that its fuzzy fatigue reliability reached up to 98.45% at expense of a very little weight caused by increasing the thickness of tail and roof sheet plates, which would be helpful for modifying the design of this engineering structure.

### Acknowledgment

The authors gratefully thank for the support of Excellent Youth Project of Hunan Education Department (Grant No.:18B301) and Natural Science Foundation of Hunan Province (Grant No.:2020JJ6075 and 2020JJ6076) and Science and Technology Innovation Project of Hunan Province (Grant No.:2018NK1030) and National Natural Science Foundation of China (Grant No.:51975192).

### References


This article is an Open Access article distributed under the terms and conditions of the Creative Commons Attribution 4.0 (CC BY 4.0) License (http://creativecommons.org/licenses/by/4.0/).