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On the rotation of orthotropic axes under uniaxial off-axis tension in sheet metals

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1. Introduction

The re-orientation of orthotropic axes due to the off-axis uniaxial tensile load has a major importance when the rate of this rotation is significant while the system of axes preserved. Kim observed that during the twisting of the cold drawn tubes [1], the orthotropic symmetry is maintained but the orthotropic axes rotate in the twisting direction. Boehler et al., Kim and Yin and Losilla et al. have shown that the cold rolled sheet metals under uniaxial off-axes tension loading remain approximately orthotropic but there is a large in-plane re-orientation of the orthotropic symmetric axes [2-4].

Bunge and Nielsen have used both experimental and orientation distribution function (ODF) measurement techniques to obtain the magnitude of the texture rotation of the orthotropic Aluminium cold rolled specimens subjected to off-axis stretching [5].

Tugcu and Neale have studied the orthotropic axes rotation using the orthogonal tensor *R* that specifies the polar decomposition of the deformation [6]. Attempts have also been made to solve the problem of orthotropic reorientation by Dafalias, Dafalias and Rashid and Cho and Dafalias [7-9].

The main theoretical tool to explain orientation of the orthotropic axes is the concept of plastic spin. This phenomenon is described and simulated by Dafalias using a simple theory of plasticity [10], which combines Hill's quadratic yield criterion for orthotropic sheet metals with the concept of plastic spin as an essential constitutive component for the orientational evolution of the anisotropic tensorial internal variables [11].

Itskov and Aksel have presented a constitutive model for orthotropic materials using plastic spin concept assuming that the anisotropies are preserved but the anisotropy axes can undergo a rigid rotation due to the large plastic deformations [12].

Tong et al. presented a simplified anisotropic plasticity theory which has been used to explain the anisotropic flow of the orthotropic polycrystalline sheet metals under off-axis uniaxial tension. Their theory was formulated in terms of the intrinsic variables of the principal stresses and a loading orientation angle for the uniaxial tension. They acquired a suitable analytical formula for the macroscopic plastic spin for orthotropic sheet metals with preserved but rotated orthotropic symmetry axes under off-axis uniaxial tension [13].

Also Tong has presented a phenomenological theory based on the plastic spin concept, Fourier series and concepts of microscopic polycrystalline plasticity for describing the anisotropic plastic flow of the orthotropic polycrystalline aluminium sheet metals under plane stress [14-15].

Hahm and Kim developed the Kim and Yin's work regarding to the work hardening of cold rolled sheet metals. They showed that Lankford's Ratio (*R*) under uniaxial off-axis tension remain approximately constant [16].

Kim D.-N. et al. proposed a constitutive model based on continuum energy considerations, the Lee decomposition and an anisotropic stored energy function of the logarithmic strains in which the rotation of the orthotropic axes is also considered [17].

In this paper, a re-orientation of the axes when orthotropic sheet metals are subjected to off-axis uniaxial tension is studied and formulated by using the following assumptions: The principal direction of plastic strain increment is the same as the sheet metal orthotropic axes, and Lankford's number remain unchanged. In many works, it was tacitly assumed that Hill's [11] suggestion of having the orthotropic directions aligned with the principal plastic stretch directions is valid, and this is indeed the case when loading maintains fixed principal stress directions from the onset of plastic deformation. The validity of the first assumption has been extensively discussed by Dafalias [10]. The second assumption has been considered using Hahm and Kim's experimental results on the low carbon steel sheet metals [16]. Based on these assumptions, a simple and applicable method is developed to obtain the rate of orthotropic axes rotation under uniaxial off-axis tension with plastic strain increment. Also a function of second pre-strain magnitude has been obtained which is similar to the one proposed by Kim and Yin [3]. The proper and explicit analytical form for rotation angle of orthotropic symmetry axes sheet metals under off-axis uniaxial tension is obtained and the associated material anisotropic constant are determined and compared with the experimental data of Kim and Yin [3]. Also, to determine the direction of the rotation of the orthotropic axes under mentioned loading and stress state a new simple method is presented.

2. Description of the problem

Are the orthotropic symmetries preserved when orthotropic sheet metals are subjected to in-plane off-axis uniaxial tension loading where the direction of loading is fixed with respect to the initial orthotropic axes? Moreover, if orthotropic symmetries are preserved, do the orthotropic axes remain the same, or they rotate? If yes, towards which direction? Finally, how fast do they rotate in relation to the plastic strain induced by the non-coaxial loading?

The above questions should be answered by experimental observations before developing a theory of anisotropic re-orientation. These answers can show whether the theoretical objective is worth pursuing or not.

Kim demonstrated that the answer for the first question is positive [1]. Kim and Yin's [3] experiments corroborate the results of Kim's experiment [1]. In the next step, they have provided answers to the second and third questions. If the answer to the third question is that the orthotropic axes do rotate but very slowly, again one may reason that for practical purposes, the orthotropic orientation may be assumed to remain fixed.

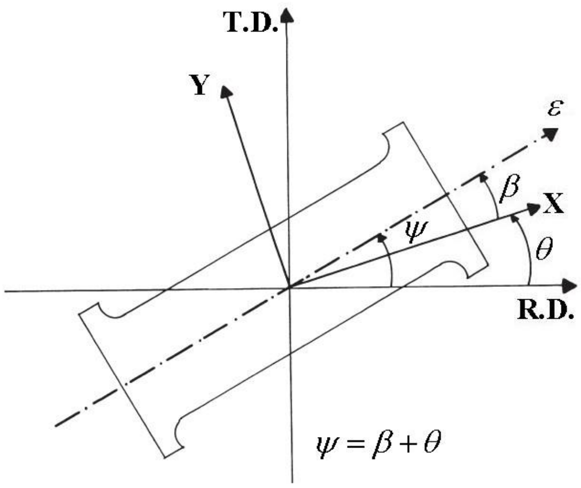

Fig. 1 Schematic presentation of the tensile specimen and the different directions and related angles, ψ, θ

Kim and Yin performed an experimental method to study the cold rolled sheet metals anisotropy with tensile tests at different angles to the rolling direction [3]. They have utilized the variation of the uniaxial yield stresses with the orientation of the tensile loading axis which can be used to set up orthotropic symmetry. For their tests, they have selected cold rolled sheets of low carbon steel which is widely used in the automotive industry. This alloy has moderate initial orthotropy. To increase the degree of orthotropy, full size sheets were stretched along the rolling direction up to 3% and 6% of tensile strains. Then, the stretched specimens were cut at an angle of ψ from the Rolling Direction (R.D.). Fig. 1 presents a schematic diagram of the specimens. The R.D. and T.D. (Transverse Direction) are the initial orthotropic axes. Three values of ψ were chosen: 30, 45 and 60 degrees and for each value of ψ, the specimens were subjected to a tensile second pre-strain ε along their axis with the magnitude of 1, 2, 5, and 10 percent. To investigate the possible re-orientation of the initial orthotropic directions R.D. and T.D. due to the mentioned pre-straining, small size specimens were cut from the pre-strained specimens at different angles and tested under tensile loading. For tracing the evolution and rotation of the orthotropic symmetries by following the "shift", with respect to the second pre-strain ε, the record of tensile yield stress distribution has been carried out for the small specimens at each ε and ψ. It was shown that the answers for two questions mentioned at the beginning of this section are positive. Also, it is possible to investigate the evolution of the orientation of the orthotropic axes X and Y with respect to the second pre-strain ε. This can be done by calculating the shift of the symmetrical yield stress distribution using the angle β from the direction of ε. (see Fig. 1).

3. Calculating the magnitude of rotation of orthotropic axes

In simulating the orientation of orthotropic axes of sheet metals due to the uniaxial off-axis tension (second pre-strain), Hahm and Kim's investigation [16] shows that the incremental plastic strain ratio (R) which is defined as:

$$R = -\frac{d\varepsilon_{22}^{p}}{d\varepsilon_{11}^{p} + d\varepsilon_{22}^{p}}, \tag{1}$$

remains unchanged. So that $d\varepsilon_{11}^{p}$ and $d\varepsilon_{22}^{p}$ are the plastic strain increments in the second pre-strain and transverse directions, respectively. Using the principal axes of the applied stress tensor in the selected Cartesian coordinate system which has been used in Kim and Yin's [3] experimental work (see Fig. 1), the expressions for the stress and strain increment tensors under uniaxial tension are:

$$\boldsymbol{\sigma} = \begin{pmatrix} \sigma & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}. \tag{2}$$

And the plastic strain increment tensor is:

$$\boldsymbol{d\varepsilon^{p}} = \begin{pmatrix} d\varepsilon_{11}^{p} & d\varepsilon_{12}^{p} & 0 \\ d\varepsilon_{12}^{p} & d\varepsilon_{22}^{p} & 0 \\ 0 & 0 & -\left(d\varepsilon_{11}^{p} + d\varepsilon_{22}^{p}\right) \end{pmatrix}. \tag{3}$$

Like many other works, also it is assumed that Hill's suggestion of having the orthotropic directions aligned with the principal plastic strain directions is valid [11]. Under this assumption it can be written that:

$$cot\, 2\beta = \frac{d\varepsilon_{11}^{p} - d\varepsilon_{22}^{p}}{d\varepsilon_{12}^{p}}. \tag{4}$$

In which the subscript 1 denotes the second loading direction and subscript 2 shows the normal direction to the second loading direction (see Fig. 1). Kim and Yin suggest that in the absence of an appropriate theory [3], the rate of orientation change of the orthotropic axes is proportional to the shear strain rate with respect to the principal directions of stress as below:

$$d\beta = \left(1 + C\right) d\varepsilon_{12}^{p}. \tag{5}$$

where β is the angle between the X-axis direction and the tensile loading direction, as shown in Fig. 1, and C is a constant, the value of which is dependent upon the state of hardening as Kim and Yin pointed out [3]. It seems that this assumption is more specific and a more general form of the above equation can be used as:

$$d\beta = f\left(d\varepsilon_{11}^{p}\right)d\varepsilon_{12}^{p}, \tag{6}$$

where $f\left(\varepsilon_{11}^{p}\right)$ is an arbitrary function of second pre-strain magnitude. It should be highlighted that the orthotropic axes rotation under uniaxial off-axis tension in addition to the second pre-strain magnitude also depends on the loading angle (ψ). Last-mentioned dependence can be introduced in orthotropic axes re-orientation problem by implementing the boundary conditions for the governing differential equations.

As mentioned before, Hahm and Kim showed that for steel sheets which are widely used in automotive industries [16], the incremental plastic strain ratio R remains unchanged during the orthotropic axes re-orientation, approximately. Dividing the numerator and denominator of Eq. (4) by $d\varepsilon_{11}^{p}$ and using Eq. (6) leads to:

$$\int_{\psi}^{\beta} cot\, 2\beta d\beta = \frac{1}{2}\left(1+\frac{R}{1+R}\right)\int_{0}^{\varepsilon} f\left(\varepsilon_{11}^{p}\right)d\varepsilon_{11}^{p}. \tag{7}$$

In which the upper bound of integration, ε, is the second pre-strain magnitude in Kim and Yin's experiments [3]. Eq. (7) after analytical integration leads to:

$$\theta = \psi - \frac{1}{2} arc\,cot\left\{\left[1+cot^{2}\left(2\psi\right)\right]exp\left[-2\left(\frac{1+2R}{1+R}\right)\int_{0}^{\varepsilon} f\left(\varepsilon_{11}^{p}\right)d\varepsilon_{11}^{p}\right]-1\right\}^{\frac{1}{2}}, \tag{8}$$

where $\psi = \theta + \beta$. The simplest suggestion for function of $f\left(\varepsilon_{11}^{p}\right)$ is the one given by Kim and Yin [3], i.e. $f\left(\varepsilon_{11}^{p}\right) = 1 + C$ and by the substitution of this form of function into Eq. (8) and knowing that $R \approx 1.65$, it can be written as:

$$\theta = \psi - \frac{1}{2} arc\,cot\left\{\left[1+cot^{2}\left(2\psi\right)\right]exp\left[-3.245283018\ (1+C)\varepsilon\right]-1\right\}^{\frac{1}{2}}. \tag{9}$$

In the above expression C is a material constant and Eq. (9) must satisfy two following boundary conditions:

If $\varepsilon = 0$ then $\theta = 0$ where this condition was satisfied in Eq. (9).

For every ε when $\psi = 0$ or $\psi = \pi/2$ then $\theta = 0$ and this condition is satisfied in Eq. (9).

4. The direction of the re-orientation of orthotropic axes under uniaxial off-axis tensile loading

In addition to the rolling and transverse directions, there is another direction in the orthotropic sheet metals that if loading is applied on that direction, the orthotropic axes don't rotate. In this paper this direction that is shown in Fig. 1, called equivalent angle (ψ_{eq}). To define the re-orientation of the orthotropic axes under uniaxial off-axis tension, both the magnitude and direction of the orientation should be determined. Dafalias [10] has discussed the direction of the re-orientation of the orthotropic axes under uniaxial off-axis tension using the relation which Hill represented in the form of $tan^{2}\psi_{eq} = (g + 2h - 1)/(f + 2h - 1)$ where f, g and h are normalized coefficients of Hill's quadratic criteria [11]. Dafalias [10] showed that for the reported values of f, g and h by Kim and Yin [3], $\psi_{eq} = 44.68^{\circ}$.

In this research work, we propose a simple relationship which can be used to determine the direction of the re-orientation of the orthotropic axes under off-axis uniaxial tension. To determine the relation for ψ_{eq}, we suppose that the yield stress for the initial sheet in ψ_{eq} is σ, so the following equilibrium equations are satisfied:

$$\left.\begin{aligned}\sigma\, sin^{2}\psi_{eq} &= \sigma_{T.D.}^{Y}\\ \sigma\, cos^{2}\psi_{eq} &= \sigma_{R.D.}^{Y}\end{aligned}\right\}, \tag{10}$$

where $\sigma_{R.D.}^{Y}$ and $\sigma_{T.D.}^{Y}$ are the yielding stress for initial sheet in the rolling and transverse directions respectively. Dividing both sides of the equilibrium equations gives:

$$\psi_{eq} = arctan\sqrt{\frac{\sigma_{T.D.}^{Y}}{\sigma_{R.D.}^{Y}}}. \tag{11}$$

5. Results

The re-orientation of the orthotropic axis of the sheet metals under uniaxial off-axis tension is obtained using Eq. (9) with different material constants. The results are presented in Figs. 2-4. These results are compared with the experimental data given by Kim and Yin for a metal sheet by 3% initial pre-strain [3]. Note when the loading angle is greater than the equivalent angle (ψ_{eq}) and consequently the orthotropic axes rotate clockwise then the positive direction of θ should be calculated from the initial *T.D.* towards the Y-axis which rotates clockwise. Therefore, the value of θ is positive in every condition.

The Eq. (9) is explicit formulation of the state and orientation of the anisotropic orthotropy under off-axis uniaxial tensile loads. The rotation of the orthotropic axes can be obtained using this equation based on two variables of loading angle ψ and secondary pre-strain ε. The secondary pre-strain consists of elastic and plastic strains but the former is neglected due to its low value. In addition to these variables, one other constant C is required which can be obtained using experimental data with given initial anisotropy for any specific material. This material constant is exactly the same as the one used by Kim and Yin [3].

To compare the calculated values with experimental data, the constants of Eq. (9) are extracted for the orthotropic sheet metal using Kim and Yin's experimental data [3]. Figs. 2-4 show the rotation of the orthotropic axis

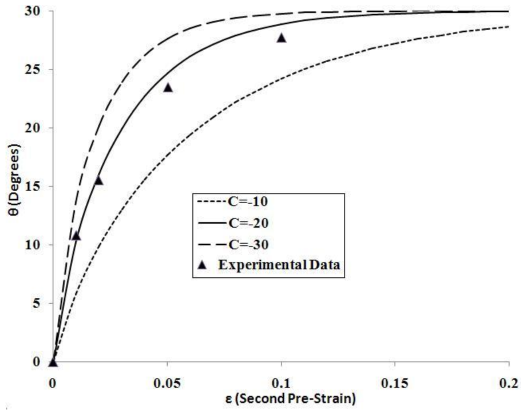

Fig. 2 The rotation of the orthotropic axis for loading angles of 30°

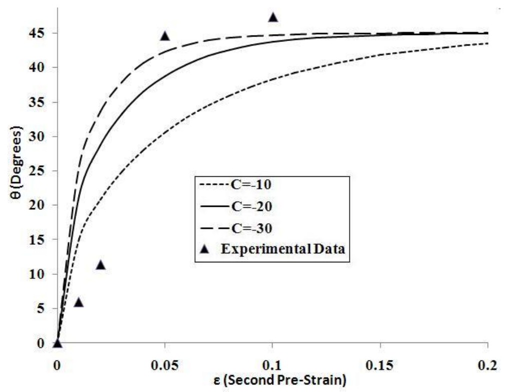

Fig. 3 The rotation of the orthotropic axis for loading angles of 45°

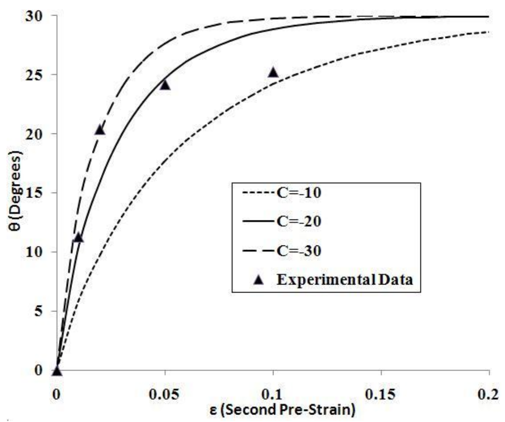

Fig. 4 The rotation of the orthotropic axis for loading angles of 60°

for three loading angles of 30°, 45° and 60°respectively. These results have been obtained using Eq. (9) for a low carbon steel sheet with 3% initial pre-strain. The material constant is given in each figure. The agreement with experimental data is good for the loading angle of 30°, because of loading angle is smaller than the equivalent angle ($\psi < \psi_{eq}$); and, the difference between the loading angle and equivalent angle is significant. For the same reasons, for the loading angle of 60°, the results compare well with the experimental data in spite of the fact that the equivalent angle is larger than the loading angle ($\psi > \psi_{eq}$). But there is a relatively large difference between the experimental data and the calculated values for the loading angle of 45°. The main reason for this difference is that the loading angle is close to the equivalent angle, which we can consider that these two angles are coinciding.

6. Discussion

In the course of a work on the subject of this paper, Dafalias elaborately discussed the re-orientation of orthotropic axes under uniaxial off-axis tensile loading [10]. He entirely connected the problem of the re-orientation of orthotropic axes to plastic spin and also the concept of rotation of substructure texture. He especially discussed the concept of equivalent angle and direction of re-orientation of orthotropic axes. However, the model proposed by Dafalias is difficult to use due to its complexity [10]. On the contrary, the formulation presented in this paper is very simple, succinct and also applicable. The model presented in this work is based on Kim and Yin's experimental data and so Dafalias's work [3, 10].

To make the proposed model more flexible, other forms of function $f\left(\varepsilon_{11}^{p}\right)$ can be used which may require more material constants rather than only one "*C*" and this can be regarded as a future work for this research program.

7. Conclusion

The value and direction of the re-orientation of orthotropic axis under off-axis uni-axial loading is obtained using a simple formulation. This method is based on the following assumptions: The principal direction of plastic strain increment is the same as the sheet metal orthotropic axes, and Lankford's number remain unchanged. Also, associated material anisotropic constant has been obtained and compared with those given by Kim and Yin which has been obtained based on experimental tests [3].

The results show that by using this formulation, the orientation and intensity of the rotation of the orthotropic axis can be calculated for uniaxial off-axis loading with good agreement with experimental data. To use this model, only one material constant should be determined based on affordable number of tests and this adds to the advantages of the model.

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ORTOTROPINIŲ AŠIŲ SUKIMASIS SU ĮTEMPIŲ KRYPTIMI NESUTAMPANČIUOSE METALO LAKŠTUOSE

R e z i u m ė

Eksperimentiniai tyrimai rodo, kad kai ortotropiniai metalo lakštai yra veikiami įtempių, nesutampančių su jų kryptimi, ašys persiorientuoja, o ortotropinė simetrija išsaugoma. Šiame darbe šis reiškinys tiriamas ir formuluojamas remiantis tokia prielaida: pagrindinė plastinių deformacijų didėjimo kryptis yra ta pati kaip ir metalo lakšto ortotropinių ašių kryptis ir Lankfordo skaičius lieka nepakitęs. Pateikiamas paprastas ir pritaikomas metodas, kuris susieja ortotropinės ašies sukimosi greitį veikiant neašiniam įtempiui, sukeliančiam plastinės deformacijos prieaugį. Taip pat pasiūlytas naujas supaprastintas metodas ortotropinės ašies sukimosi krypčiai nustatyti, kai įtempiai nesutampa su ašies kryptimi. Parodyta, kad pasiūlytieji metodai gali būti sėkmingai taikomi ortotropinių ašių sukimosi greičiui ir krypčiai nustatyti, o gaunami rezultatai gerai sutampa su eksperimentiniais.

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ON THE ROTATION OF ORTHOTROPIC AXES UNDER UNIAXIAL OFF-AXIS TENSION IN SHEET METALS

S u m m a r y

Experimental investigations show that when orthotropic sheet metals are subjected to off-axis uniaxial tension, a re-orientation of the axes occurs, while orthotropic symmetries are preserved. In this paper this phenomenon is investigated and formulated based on the following assumptions: the principal direction of the plastic strain increment is the same as the direction of the orthotropic axes of the sheet metal and Lankford's number remains unchanged. A simple and applicable method is presented which relate the rate of change of orthotropic axes rotation under uniaxial off-axis tension with plastic strain increment. Also, a new and simple method is developed to obtain the direction of the orthotropic axes rotation under uniaxial off-axis tension. It is shown that the proposed methods can successfully be used to obtain the magnitude and direction of the rotation of the orthotropic axes and the results show good agreement with the experimental.

Keywords: Orthotropic axes, Plastic Strain, Re-Orientation, Sheet metal.

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