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# Investigation of Dynamics of a Pipe Robot with Nonlinear Interactions of Its Elements

**Kazimieras RAGULSKIS*, Bronislovas SPRUOGIS**, Marijonas BOGDEVIČIUS***, Arvydas MATULIAUSKAS****, Vygantas MIŠTINAS*****, Liutauras RAGULSKIS********
**Kaunas University of Technology, K. Donelaičio Str. 73, Kaunas, Lithuania, E-mail: kazimieras3@hotmail.com*
***Vilnius Gediminas Technical University, Plytinės Str. 27, Vilnius, Lithuania, E-mail: bronislovas.spruogis@vgtu.lt*
****Vilnius Gediminas Technical University, Plytinės Str. 27, Vilnius, Lithuania, E-mail: marijonas.bogdevicius@vgtu.lt*
*****Vilnius Gediminas Technical University, Plytinės Str. 27, Vilnius, Lithuania, E-mail: arvydas.matuliauskas@vgtu.lt*
******Vilnius Gediminas Technical University, Plytinės Str. 27, Vilnius, Lithuania, E-mail: vygantas.mistinas@vgtu.lt*
*******Vytautas Magnus University, Vileikos Str. 8, LT-44404, Kaunas, Lithuania, E-mail: l.ragulskis@if.vdu.lt*

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## 1. Introduction

Systems in which vibrational displacements are limited because elastic forces increase practically up to unlimited values, are investigated. Here systems having one degree of freedom are investigated in which vibrations have specific qualities for the case of conservative systems as well as for the case of forced harmonic excitations. Typical expressions for nonlinear stiffness are proposed.

Systems of this type are important in the investigations of dynamics in elements of manipulators and robots. Theoretical investigations of different types of robots are performed by V. A. Glazunov [1]. The use of vibratory drives in robots is analysed in [2]. Resonant zones of dynamical systems are investigated in [3]. Stabilisation of nonlinear vibrating systems is investigated in [4]. Mechanical systems with impacts are analysed in [5]. Periodic orbits of vibrating systems are investigated in [6]. Nonlinear vibrations with impacts are analysed in [7].

Model of a pipe robot with limited values of displacements is presented and investigated. Motion of the pipe robot for different parameters of the system is obtained. Typical regimes of motion are investigated.

The obtained results are used in the process of design of pipe robots.

## 2. Model of the system with limited values of displacements

The investigated nonlinear vibrating system in general case is described by the following differential equation:

$$m\ddot{x} + H\dot{x} + \Pi'_x = Fsin\omega t, \tag{1}$$

where $x$ is the displacement; $m$ is the mass; $H$ is the coefficient of viscous damping; $\Pi'_x$ is the force of stiffness; $F$ is the amplitude of harmonic excitation; $\omega$ is the frequency of harmonic excitation, $t$ is the time variable, and the upper dot indicates differentiation with respect to it.

Nonlinear stiffness is determined as the derivative of potential energy $\Pi(x)$ with respect to $x$.

Further two typical expressions of nonlinearity are proposed in the paper as having the desired qualities of limited displacement type and as being based on elementary functions.

For nonlinear system of the following type:

$$\frac{1}{m}\Pi'_x = \frac{x}{1-x^2}, \tag{2}$$

it is obtained that:

$$\Pi = \frac{1}{m}\int \Pi'_x dx + const = \int \frac{x}{1-x^2} dx + const =$$
$$= -\int \frac{d\left(1-x^2\right)}{1-x^2} + const = -\ln\left(1-x^2\right) + const,$$
$$\text{when } x \neq 0. \tag{3}$$

For nonlinear system of the following type:

$$\frac{1}{m}\Pi'_x = tan\left(x\right), \tag{4}$$

it is obtained that:

$$\Pi = \int tan\left(x\right) dx + const = \int \frac{sinx}{cosx} dx + const =$$
$$= \int \frac{d\left(cosx\right)}{cosx} + const = -ln\left(cosx\right) + const. \tag{5}$$

Further results of investigations for some typical expressions of nonlinearities with limited values of displacements are presented.

## 3. Investigation of dynamics of the system for the first type of nonlinearity

The investigated nonlinear vibrating system is described by the following differential equation:

$$x'' + 2hx' + \frac{1}{1-x^2}x = fsin\nu\tau, \tag{6}$$

where: $x$ is the displacement; $h$ is the coefficient of viscous damping; $f$ is the amplitude of harmonic excitation; $\nu$ is the frequency of harmonic excitation; $\tau$ is the time variable and prime indicates differentiation with respect to it.

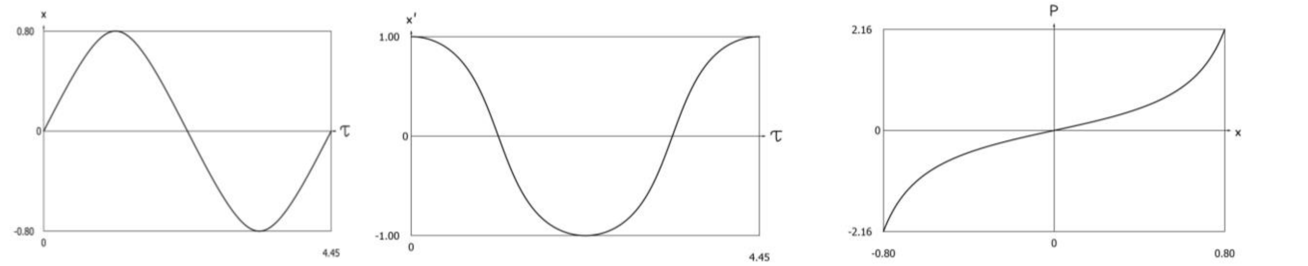
Conservative system is described by the following differential equation:

$$x'' + \frac{1}{1-x^2}x = 0. \tag{7}$$

3.1. Investigation of the conservative system

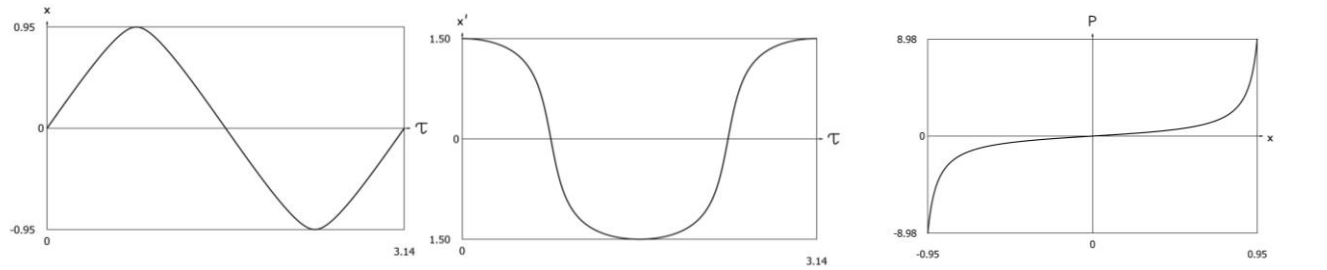
Results when $x(0)=0$, $x'(0)=1$ are presented in Fig. 1. Results when $x(0)=0$, $x'(0)=1.5$ are presented in Fig. 2.

From the presented results the influence of nonlinearity to the dynamical behaviour of the investigated conservative system is seen. With the increase of initial velocity, the nonlinear effect of stiffness of limited displacement type is greater and this is especially evident from the representation of force of stiffness as function of displacement.




a) Displacement as function of time b) Velocity as function of time c) Force of stiffness as function of displacement

Fig. 1 Investigation of the conservative system when $x(0)=0$, $x'(0)=1$




a) Displacement as function of time b) Velocity as function of time c) Force of stiffness as function of displacement

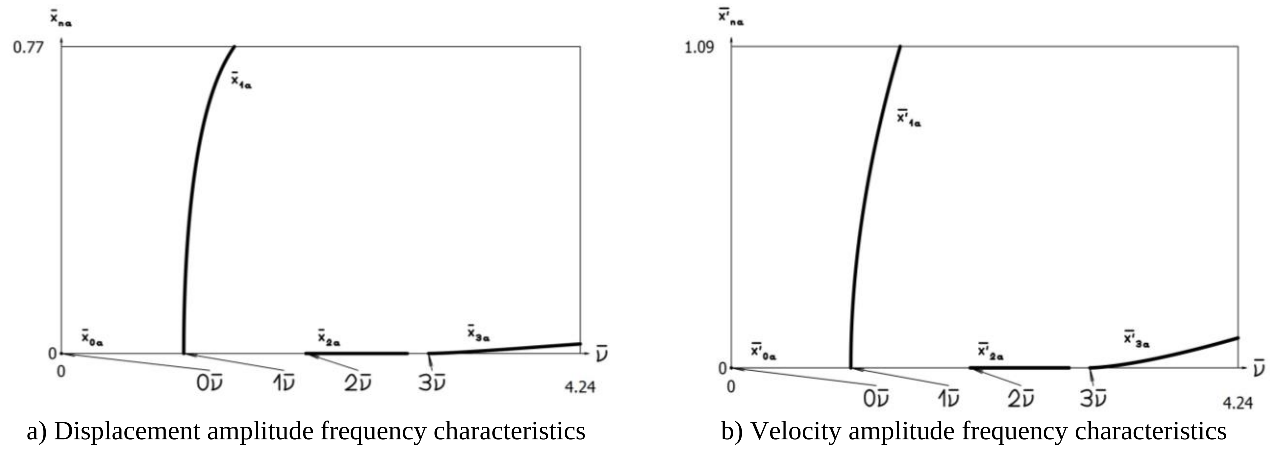
Fig. 2 Investigation of the conservative system when $x(0)=0$, $x'(0)=1.5$

3.2. Amplitude frequency characteristics of the conservative system

The displacement amplitude frequency characteristics as well as the velocity amplitude frequency characteristics are presented in Fig. 3.

In the presented graphical relationships constant part and amplitudes of the first three harmonics as func

tions of frequency are seen. Hardening frequency responses are observed in the obtained graphical results. For small nonlinearity higher harmonics are negligible, while for higher nonlinearity the third harmonic increases substantially, though it is much smaller than the first harmonic.




a) Displacement amplitude frequency characteristics b) Velocity amplitude frequency characteristics

Fig. 3 Amplitude frequency characteristics of the conservative system: constant part and amplitudes of the first three harmonics

3.3. Investigation of dynamics of the system with harmonic excitation

The following values of the parameters of the investigated system were assumed:

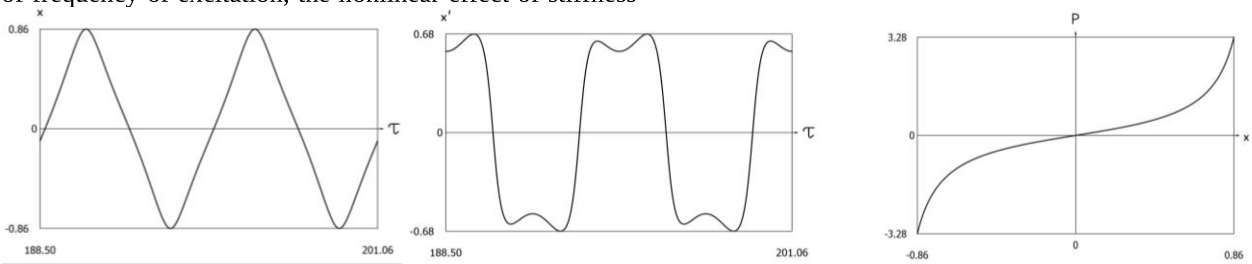
$$h = 0.1,\ f = 1. \tag{8}$$

Two periods of steady state motion are represented in the following figures.

Results when $\nu = 1$ are presented in Fig. 4. Results when $\nu = 1.5$ are presented in Fig. 5.

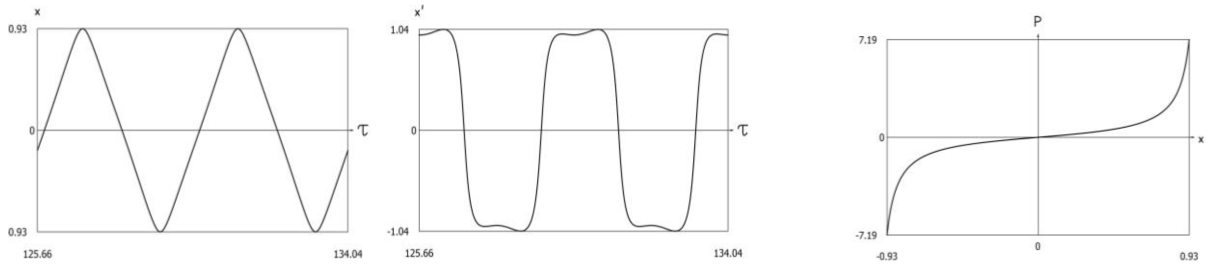
From the presented results the influence of nonlinearity to the dynamical behaviour of the investigated system with harmonic excitation is seen. With the increase of frequency of excitation, the nonlinear effect of stiffness of limited displacement type is greater and this is especially evident from the representation of force of stiffness as function of displacement.

Based on the presented results qualities of dynamic behavior of the investigated nonlinear vibrating system are observed.




a) Displacement as function of time b) Velocity as function of time c) Force of stiffness as function of displacement

Fig. 4 Dynamics of the system when $h = 0.1,\ f = 1,\ \nu = 1$




a) Displacement as function of time b) Velocity as function of time c) Force of stiffness as function of displacement

Fig. 5 Dynamics of the system when $h = 0.1,\ f = 1,\ \nu = 1.5$

**4. Investigation of dynamics of the system for the second type of nonlinearity**

The investigated nonlinear vibrating system is described by the following differential equation:

$$x'' + 2hx' + tan\frac{\pi}{2}x = fsin\nu\tau, \tag{9}$$

where: *x* is the displacement; *h* is the coefficient of viscous damping; *f* is the amplitude of harmonic excitation; *ν* is the frequency of harmonic excitation; *τ* is the time variable and prime indicates differentiation with respect to it.

Conservative system is described by the following differential equation:

$$x'' + tan\frac{\pi}{2}x = 0. \tag{10}$$

The following notation of the force of stiffness is introduced:

$$\frac{1}{m}\Pi'_x = tan\frac{\pi}{2}x. \tag{11}$$

4.1. Investigation of the conservative system

Results when $x(0)=0,\ x'(0)=1$ are presented in Fig. 6. Results when $x(0) = 0,\ x'(0) = 1.5$ are presented in Fig. 7.

From the presented results the influence of nonlinearity to the dynamical behaviour of the investigated conservative system is seen. With the increase of initial velocity, the nonlinear effect of stiffness of limited displacement type is greater and this is especially evident from the representation of force of stiffness as function of displacement.

4.2. Amplitude frequency characteristics of the conservative system

The displacement amplitude frequency characteristics as well as the velocity amplitude frequency characteristics are presented in Fig. 8.

In the presented graphical relationships constant part and amplitudes of the first three harmonics as functions of frequency are seen. Hardening frequency responses are observed in the obtained graphical results. For small nonlinearity higher harmonics are negligible, while for higher nonlinearity the third harmonic increases substantially, though it is much smaller than the first harmonic.

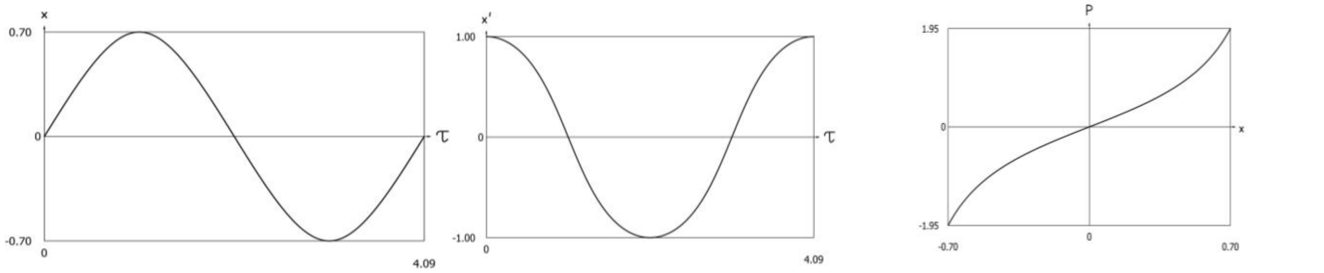
4.3. Investigation of dynamics of the system with harmonic excitation

The following values of the parameters of the investigated system were assumed:

$$h = 0.1,\ f = 1. \tag{12}$$

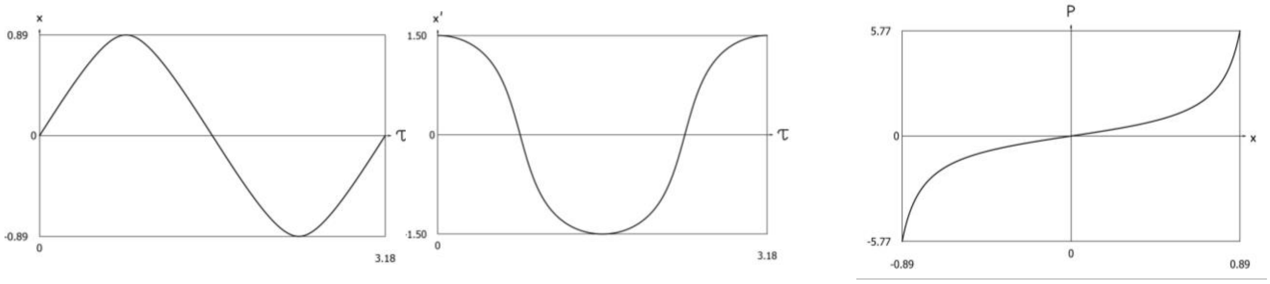
Two periods of steady state motion are represented in the following figures.

Results when $\nu$=1 are presented in Fig. 9. Results when $\nu$=1.5 are presented in Fig. 10.



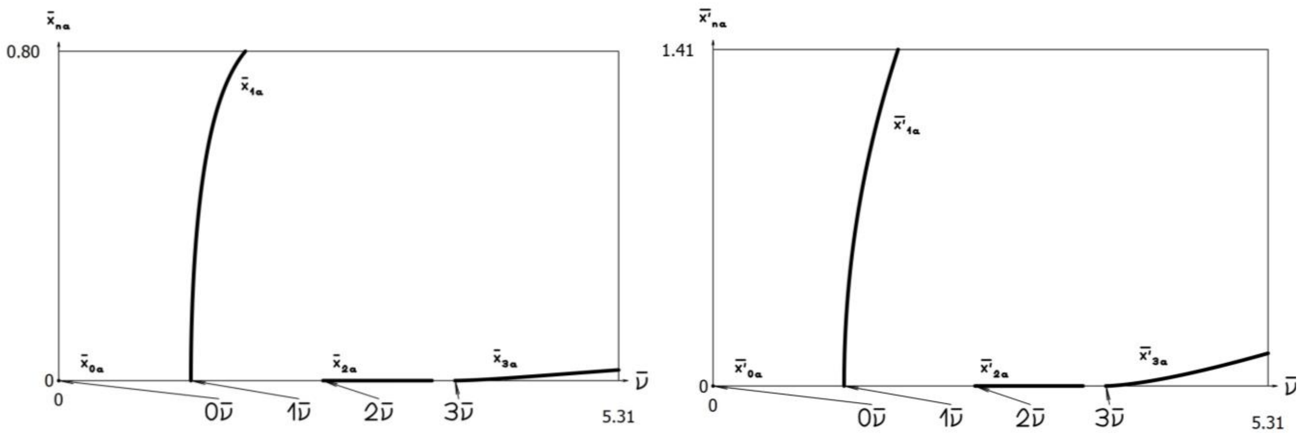

a) Displacement as function of time b) Velocity as function of time c) Force of stiffness as function of displacement

Fig. 6 Investigation of the conservative system when $x(0)=0$, $x'(0)=1$




a) Displacement as function of time b) Velocity as function of time c) Force of stiffness as function of displacement

Fig. 7 Investigation of the conservative system when $x(0)=0$, $x'(0)=1.5$

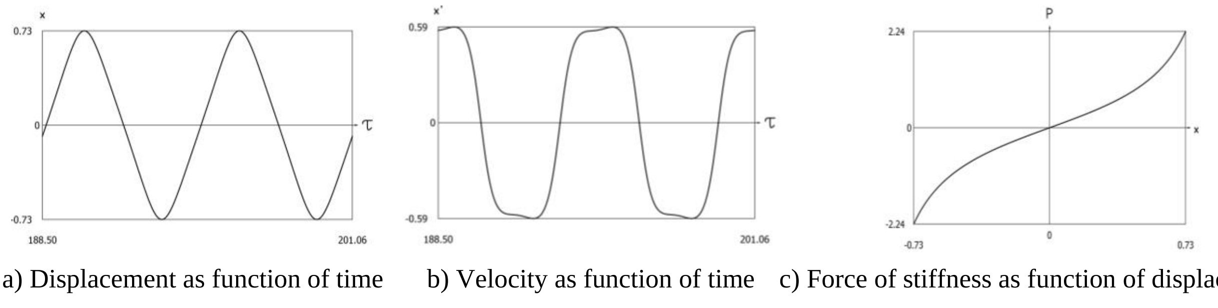



a) Displacement amplitude frequency characteristics b) Velocity amplitude frequency characteristics

Fig. 8 Amplitude frequency characteristics of the conservative system: constant part and amplitudes of the first three harmonics

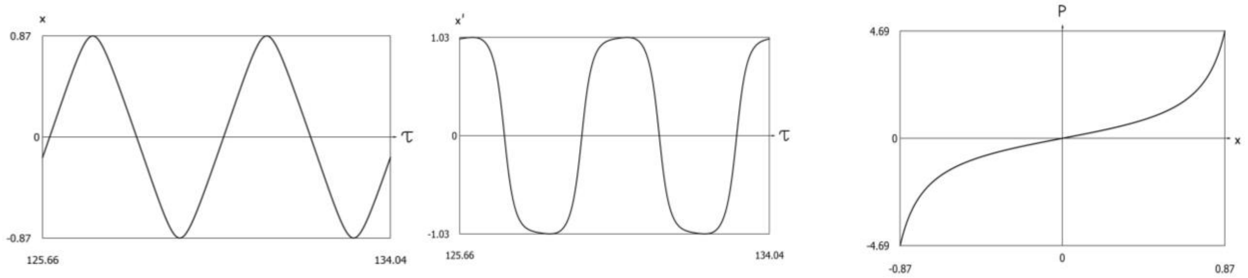
From the presented results the influence of nonlinearity to the dynamical behaviour of the investigated system with harmonic excitation is seen. With the increase of frequency of excitation, the nonlinear effect of stiffness of limited displacement type is greater and this is especially evident from the representation of force of stiffness as function of displacement.

Based on the presented results qualities of dynamic behavior of the investigated nonlinear vibrating system are observed. From the presented results of investigations, it is concluded that qualitatively the behavior of systems with both types of nonlinearities of limited displacements type is mutually similar, but quantitative differences can be observed.




a) Displacement as function of time b) Velocity as function of time c) Force of stiffness as function of displacement

Fig. 9 Dynamics of the system when $h = 0.1$, $f = 1$, $\nu = 1$




a) Displacement as function of time b) Velocity as function of time c) Force of stiffness as function of displacement

Fig. 10 Dynamics of the system when $h = 0.1,\ f = 1,\ \nu = 1.5$

## 5. Model of the pipe robot with limited values of displacements

Schematic diagram of a pipe robot with nonlinear interactions of limited displacement type is presented in Fig. 11.

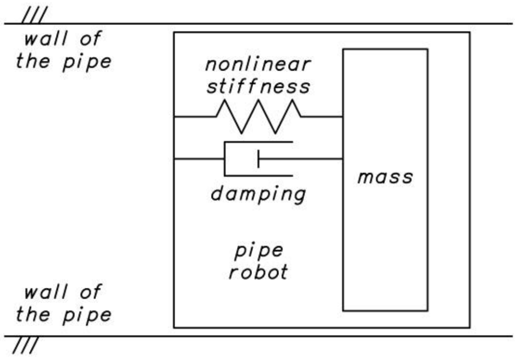
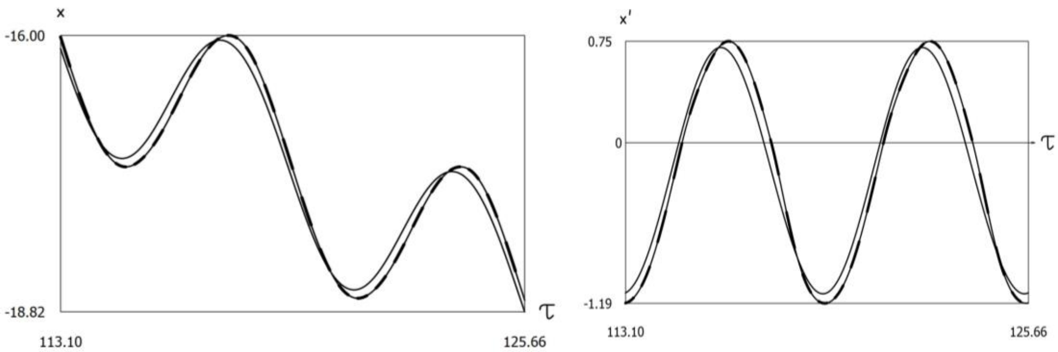



Fig. 11 Schematic diagram of a pipe robot with nonlinear interactions of limited displacement type

The investigated nonlinear vibrating system describing the motion of a pipe robot is represented by the following differential equations:

$$\begin{cases} x_1'' + h\left(x_1' - x_2'\right) + \dfrac{1}{1-\left(x_1 - x_2\right)^2}\left(x_1 - x_2\right) = \\ = f_0 sin\nu\tau, \\ \mu x_2'' + h\left(x_2' - x_1'\right) + \dfrac{1}{1-\left(x_1 - x_2\right)^2}\left(x_2 - x_1\right) + \\ + \begin{Bmatrix} h_1 x_2' \text{ when } x_2' > 0 \\ h_2 x_2' \text{ when } x_2' < 0 \end{Bmatrix} = 0, \end{cases} \tag{13}$$

where: $x_1$ is the displacement of the vibrating mass inside the pipe robot; $x_2$ is the displacement of the pipe robot; $\mu$ is the mass of the case of the pipe robot; $h$ is the coefficient of viscous damping between the vibrating mass inside the pipe robot and the case of the pipe robot; $h_1$ is the coefficient of viscous damping of the case of the pipe robot with respect to the pipe itself for positive velocity of motion of the pipe robot; $h_2$ is the coefficient of viscous damping of the case of the pipe robot with respect to the pipe itself for negative velocity of motion of the pipe robot; $f_0$ is the amplitude of harmonic excitation; $\nu$ is the frequency of harmonic excitation; $\tau$ is the time variable, and the prime indicates differentiation with respect to it.




a) Displacement as function of time b) Velocity as function of time

Fig. 12 Dynamics of the pipe robot when $\mu = 0.1,\ h = 0.1,\ h_1 = 0.2,\ \nu = 1,\ f_0 = 1,\ h_2 = 0.1$

The following values of the parameters of the investigated system were assumed:

$$\mu = 0.1,\ h = 0.1,\ h_1 = 0.2,\ \nu = 1. \tag{14}$$

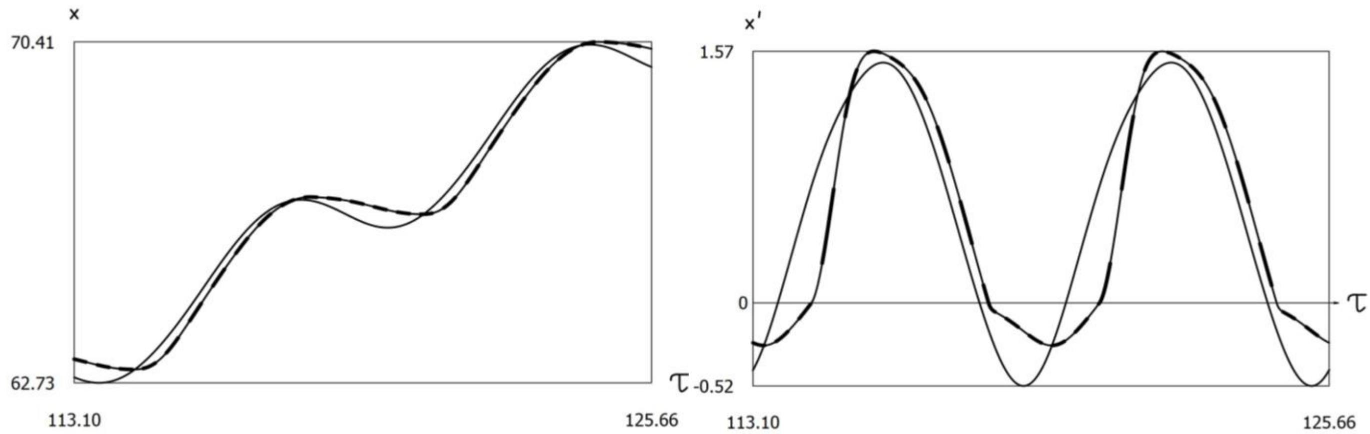
Two periods of steady state motion are represented in the following figures. Motion of the vibrating mass inside the pipe robot is represented by continuous lines and motion of the case of the pipe robot is represented by dashed lines.

5.1. Investigation of dynamics of the system for low amplitude of excitation

The following value of amplitude of excitation is assumed:

$$f_0 = 1. \tag{15}$$

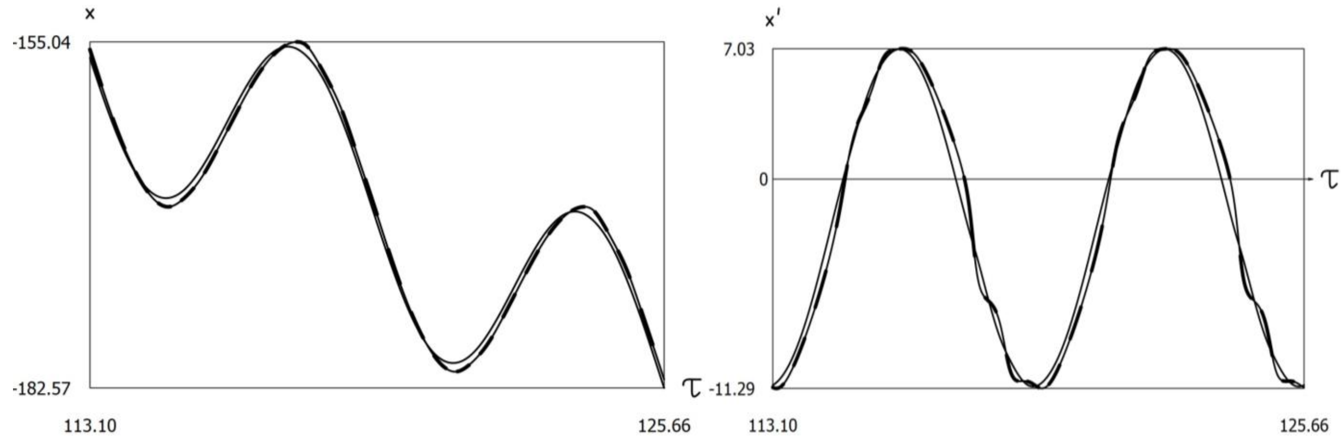
Results when $h_2$=0.1 are presented in Fig. 12. Results when $h_2$=2 are presented in Fig. 13.




a) Displacement as function of time

b) Velocity as function of time

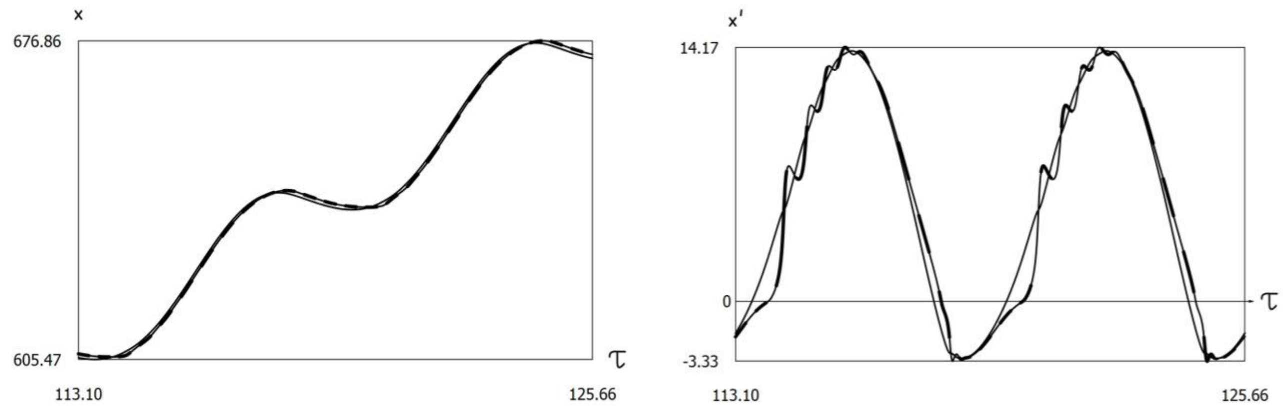
Fig. 13 Dynamics of the pipe robot when $\mu = 0.1,\ h = 0.1,\ h_1 = 0.2,\ \nu = 1,\ f_0 = 1,\ h_2 = 2$




a) Displacement as function of time

b) Velocity as function of time

Fig. 14 Dynamics of the pipe robot when $\mu = 0.1,\ h = 0.1,\ h_1 = 0.2,\ \nu = 1,\ f_0 = 10,\ h_2 = 0.1$




a) Displacement as function of time

b) Velocity as function of time

Fig. 15 Dynamics of the pipe robot when $\mu = 0.1,\ h = 0.1,\ h_1 = 0.2,\ \nu = 1,\ f_0 = 10,\ h_2 = 2$

In Fig. 12 motion of the pipe robot in the negative direction of the *x* axis is observed, while in Fig. 13 motion of the pipe robot in the positive direction of the *x* axis is observed.

5.2. Investigation of dynamics of the system for high amplitude of excitation

The following value of amplitude of excitation is assumed:

$$f_0 = 10. \tag{16}$$

Results when $h_2$=0.1 are presented in Fig. 14. Results when $h_2$=2 are presented in Fig. 15.

In Fig. 14 motion of the pipe robot in the negative direction of the *x* axis is observed, while in Fig. 15 motion of the pipe robot in the positive direction of the *x* axis is observed.

For high amplitude of excitation, the distance travelled by the pipe robot is much greater than for low amplitude of excitation.

The obtained results are used in the process of design of pipe robots of advanced type.

## 6. Conclusions

Systems in which vibrational displacements are limited because elastic forces increase practically up to unlimited values, are investigated. Here systems having one degree of freedom are investigated in which vibrations have specific qualities for the case of conservative systems as well as for the case of forced harmonic excitations.

Typical expressions for nonlinear stiffness are proposed. Displacement as function of time, velocity as function of time and nonlinear stiffness as function of displacement are represented for various initial velocities of the conservative system. Also, the same quantities for various frequencies of excitation of non-conservative systems in steady state regimes of motion are investigated.

Model of a pipe robot with limited values of displacements is presented and investigated. The model has two degrees of freedom: the displacement of the vibrating mass inside the pipe robot and the displacement of the pipe robot itself. Motion of the pipe robot in the negative direction as well as motion of the pipe robot in the positive direction for different parameters of the system is observed. For high amplitude of excitation, the distance travelled by the pipe robot is much greater than for low amplitude of excitation.

The obtained results are used in the process of design of pipe robots of advanced type.

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K. Ragulskis, B. Spruogis, M. Bogdevičius,
A. Matuliauskas, V. Mištinas, L. Ragulskis

INVESTIGATION OF DYNAMICS OF A PIPE ROBOT WITH NONLINEAR INTERACTIONS OF ITS ELEMENTS

S u m m a r y

Systems in which vibrational displacements are limited because elastic forces increase practically up to unlimited values, are investigated. Here systems having one degree of freedom are investigated in which vibrations have specific qualities for the case of conservative systems as well as for the case of forced harmonic excitations. Typical expressions for nonlinear stiffness are proposed. Model of a pipe robot with limited values of displacements is presented and investigated. The model has two degrees of freedom: the displacement of the vibrating mass inside the pipe robot and the displacement of the pipe robot itself. Motion of the pipe robot in the negative direction as well as motion of the pipe robot in the positive direction for different parameters of the system is observed. For high amplitude of excitation, the distance travelled by the pipe robot is much greater than for low amplitude of excitation. The obtained results are used in the process of design of pipe robots.

**Keywords:** vibrations, conservative system, non-conservative system, forced harmonic excitation, displacement limiters, nonconservative qualities, pipe robots.

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