

## Generating of digitised with the aid of Tetrobot module

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**crossref** <http://dx.doi.org/10.5755/j01.mech.18.6.3157>

### 1. Introduction

The paper presents a possible application for generation of surfaces/trajectories with the aid of a tetrahedral module (Fig. 1) from the structure of a robot type Tetrobot [1].

Any given surface can be digitized using graphical interpolation as described at [2]. The new data can then be used in finding the variation laws  $L_{N_1}(t)$ ,  $L_{N_2}(t)$  and  $L_{N_3}(t)$ , of the actuators from the tetrahedral module such that the end-effector ( $P_C$ ) will then recreate a scaled version of the surface.

### 2. Theoretical aspects regarding the generating of surfaces

To reproduce the scaled version of the surface with the aid of a robot, the surface needs to be modelled mathematically. The coordinates located on the newly modelled surface then become the coordinates of the characteristic point of the robot.

The method used to accomplish the approach of mathematical model is that of cubical interpolation. The most popular and easy to use methods are: Coons, Bezier, B-Splines [3].

The real surface is approximate by a spatial network of points. Each point from the surface is connected by the nearest points through interpolation segments [4].

As consequence of the modelling process many more parameters can be put in evidence (coordinates, tangents, nor-mal, and curvature) which concur to define the surface.

To follow or to set out certain curves/trajectories appeal can be made to the vectorial or parametric representation of these in space. In the case of analytical representation on parametric form requires reporting at a cartesian system and an allocation of numerical values for respective parameter.

To generate the trajectory, a tetrahedral module with three degrees of freedom from the structure of robots type Tetrobot [2] (Fig. 1) was chosen as a device.

The parametric equations of straight families which are passing through current point  $P_C(x(t), y(t), z(t))$  and fixed points (nodes)  $N_1$ ,  $N_2$  and  $N_3$  are determined with the aid of relations (1)

$$\begin{cases} \frac{X-x(t)}{X_{N_1}-x(t)} = \frac{Y-y(t)}{Y_{N_1}-y(t)} = \frac{Z-z(t)}{Z_{N_1}-z(t)} \\ \frac{X-x(t)}{X_{N_2}-x(t)} = \frac{Y-y(t)}{Y_{N_2}-y(t)} = \frac{Z-z(t)}{Z_{N_2}-z(t)} \\ \frac{X-x(t)}{X_{N_3}-x(t)} = \frac{Y-y(t)}{Y_{N_3}-y(t)} = \frac{Z-z(t)}{Z_{N_3}-z(t)} \end{cases} \quad (1)$$

The lengths between current point  $P_C(x(t), y(t), z(t))$  from the surface which will be modelled and the fixed points  $P_{N_1}(x_{N_1}, y_{N_1}, z_{N_1})$ ,  $P_{N_2}(x_{N_2}, y_{N_2}, z_{N_2})$  and  $P_{N_3}(x_{N_3}, y_{N_3}, z_{N_3})$  which define the Tetrobot base are calculated using the next relations

$$L_{N_i}(t) = \sqrt{(x(t) - x_{N_i})^2 + (y(t) - y_{N_i})^2 + (z(t) - z_{N_i})^2} \quad (2)$$

where  $i = 1-3$ .

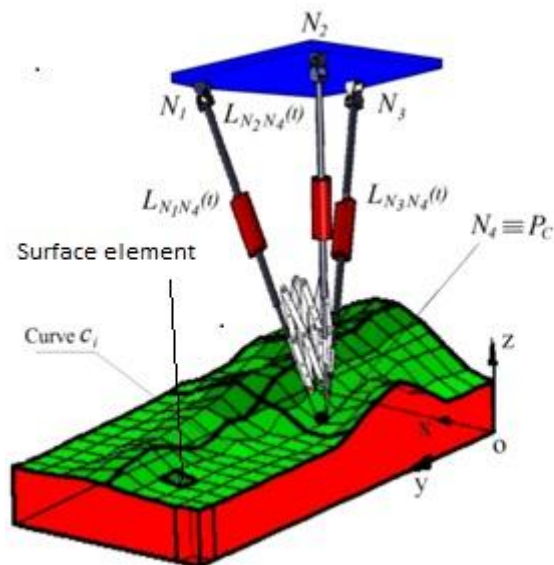


Fig. 1 Presentation of the model

During the reproduction process of a digitalised surface, the current point  $N_4$  is placed in the characteristic point  $P_C$  of the tetrahedral module. This lies at the intersection of all three axis elements that form the edges of the

module, and namely  $N_1N_4, N_2N_4, N_3N_4$ .

The variation of the velocities from motor couples  $M_{N_1N_4}, M_{N_2N_4}$  and  $M_{N_3N_4}$  are achieved by derivation of the expressions from relation (2).

In Fig. 1 the equation of the curve  $c_i = z_i(v)$  is composed from a suite of  $n-1$  relations, presented in [2].

$$z_i(v) = A_i(v - v_i)^3 + B_i(v - v_i)^2 + C_i v + E_i, \quad (3)$$

where  $v_i < v < v_{i-1}$ .

Thus, the set of resulting curves will generate the virtual surface.

### 3. Aspects regarding the geometry of tetrahedral structures

The tetrahedral module is a part of Tetrobot structure.

Practically, Tetrobot is a structure type grillage beam, which it easily adapts to different types of applications by using actuators and a new joint system, called “concentrically multilink spherical joint”, shortly, couple or CMS joint [1].

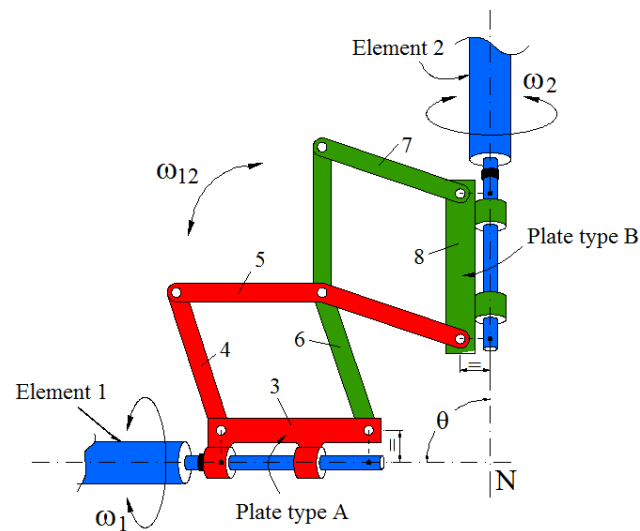


Fig. 2 CMS plane joint

Basically, this structure considering, elements 1 and 2 works up the movements of a classic ball joint.

This have 6 elements are connected through the rotation couples C, D, E, F, G, H, I, according with Fig. 3, with their axis perpendicular to the drawing plane. Elements 5 and 6 are bent with an angle  $\phi$  at the rotation couple F.

In this type of joint, elements 1 and 2 can have an angular rotation within an interval dependent on the geometrical parameters of the closed kinematic chain.

Practically, depending on the time which these components touch each other; it will determine the minimum and maximum limits for  $\theta$  angle.

According to the figure two configurations are formed which remain perfectly symmetric by FN axis, at the variation of  $\phi$  angle.

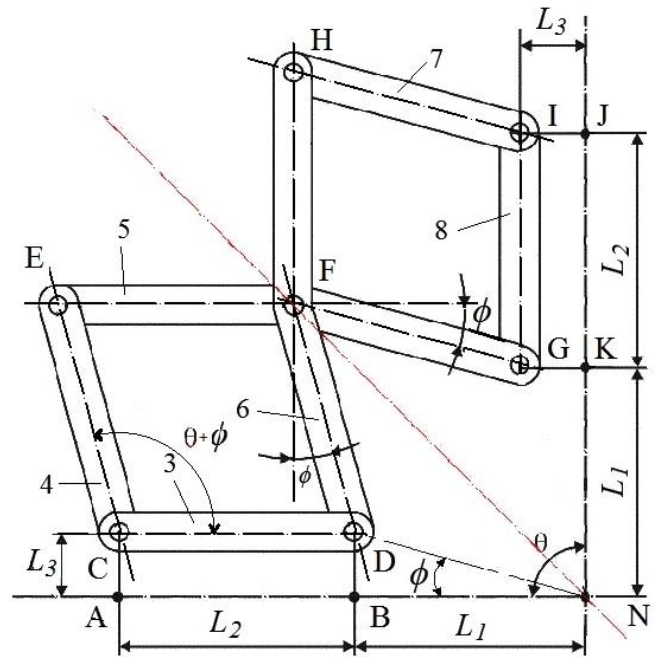


Fig. 3 The CMS plane joint

Considering that, any point from the axis of elements 1 and 2 execute rotation movements around the point N, the angle  $\phi$  it will be calculated, as shown in Fig. 3, with the following relation

$$\phi = \arctg \frac{L_3}{L_1} = const \quad (4)$$

Due to  $L_1 = const$ , meaning that the points B and K, at the variation of  $\theta$  angle are displaced on the same circle with the centre on point N, as a fact, any point from the axis of elements 1 and 2, as the components directly articulated on these axis, are rotated around the same point N (the centre of a fictive ball joint).

Knowing the parameters  $L_1, L_2$  and  $L_3$ , as fixed measures and taking into the parallelism of the components of CDFE and GFHI parallelograms, the lengths of the components of plane articulation are

$$\left. \begin{aligned} |AB| &= |CD| = |EF| = |FH| = |GI| = |JK| = L_2 \\ |CE| &= |DF| = |FG| = |HI| = \sqrt{L_1^2 + L_3^2} \\ |AC| &= |BD| = |KG| = |JI| = L_3 \end{aligned} \right\} \quad (5)$$

By linking together of a three types of joints, CMS plane, a CMS joint with three elements is obtained Fig. 4, in which, each element taken individually, will execute the movements as a classic ball joint.

Considering four CMS joints shaped like those in Fig. 4 linked together with the aid of linear actuators, a tetrahedral module will be achieved according with the figure illustrated bellow. The centres of the tetrahedral module noted with  $N_i = (i=1-4)$ , will be placed at the intersection of all three elements axis that form the respective node, Fig. 5.

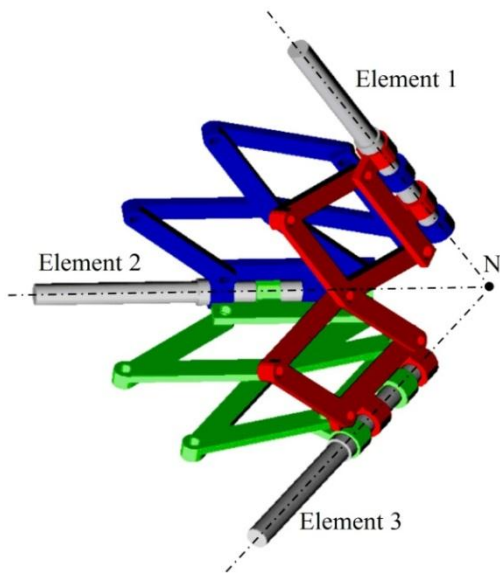


Fig. 4 CMS joint with three elements

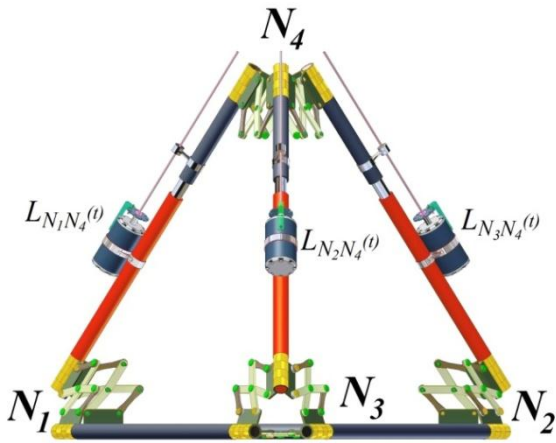


Fig. 5 Tetrahedral module

Basically, the tetrahedral structure is statically determined, namely when there is no actuator power-operated, the degree of freedom is zero. Fact confirmed by the calculus of degree of freedom  $F$ , with the aid of relation [1,5]

$$F = \lambda(l - j - 1) + \sum_{i=1}^j f_i \tag{6}$$

where  $\lambda$  – mechanism grade, which is representing the number of permitted movements of the elements that compound the respective mechanism,  $l$  – the number of elements of the structure,  $j$  – the number of articulations that are linking together two elements,  $f_i$  – the number of degree of freedom of the couple.

Considering the module as a spatial structure formed by ball joints, then  $\lambda = 6$  and  $f_i = 3$  (the number of limited movements for a ball joint), thus the relation (6) becomes

$$F = 5(l - 3)j - 6 = 0 \tag{7}$$

It was considered that, the tetrahedral compound from  $l = 6$  elements and  $j = 8$  ball joints, in according with the figure.

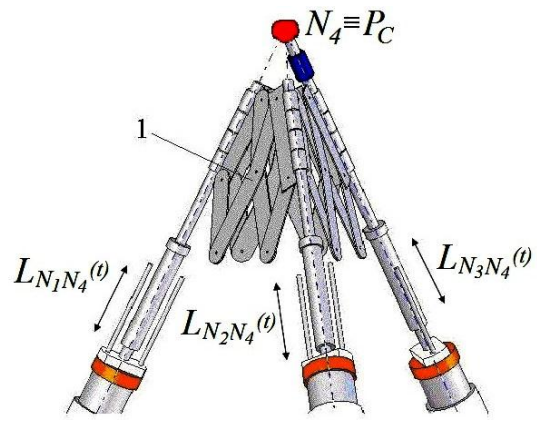


Fig. 6 The end-effector of tetrahedral module

The workspace of the Tetrobot represents the total volume swept by the end-effector, when actuators of the components of the module can execute all possible motions.

The workspace can be determined by taking into account the geometric model and the constraints imposed by the limitations of the CMS joints.

Giving values for all three generalised coordinates in a wide range of limits all points of a surface that the end-effector can touch them will be achieved.

The workspace for a tetrahedral module with the end-effector situated according with the Fig. 5 exactly in the characteristic point, that is,  $N_4$  node, when the base is considered fixed, and is illustrated in the figure bellow.

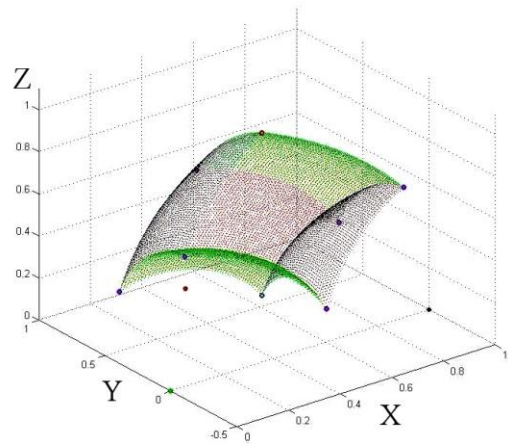


Fig. 7 The workspace – the tetrahedral module with a fixed base

#### 4. Accomplishment of generating simulation of the developed mathematical model

To simulate the surface the points which will be touch by the characteristic point of the tetrahedral module with fixed base will describe the presented surface (Fig. 1 [2]), where the coordinates of the points which must be touched and which will be read from a text file.

For 3D simulation, was used the OpenGL libraries for Delphi, and namely GLScene that are freeware.

The active window of the achieved program is presented in Fig. 8.

The program was created in Borland Delphi 7 programming language of the Borland company, because of the facilities offered by the programming environment

and its flexibility. Thus, the characteristic point was programmed to track longitudinal trajectories as like that presented in [2].

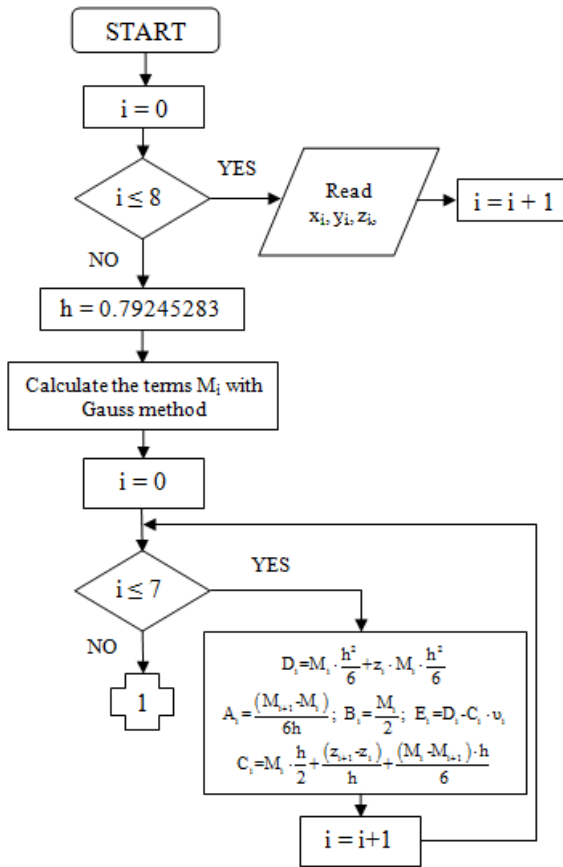


Fig. 8 Logic diagram

At the end of curve, the characteristic point will execute a transverse feed with a pitch to score the next curve, finally resulting the surface physical generated like type of that from Fig. 1.

The program included to the visualisation of the simulation of a type of physical generation with the aid of a active window is illustrated in Fig. 9, where the basic commands are presented and the functions are accomplished:

- *visualization of the model* (front, back, left, right, top and isometric);
- the program allows to load other text files with coordinates in the same format as the default, to visualise as soon as possible other sections from the generating of desired surface simulation. In case of loading of other types of files different like the default type of the program, a warning message will appear;
- *simulation* – generate the digitalised surface according with the input file previously selected;
- *visualisation of the workspace* – the maximum workspace, calculated in according with the strokes on each actuator form the structure, and also of the minimum angle allowed by the CMS joint from the peak of tetrahedral module, noted with 1 in Fig. 5;
- *reset* – stop the simulation and return the model in the initial position;
- *visualisation of the graphics* – the variation of

displacements from motor couples, during the simulation, depending by the read nodes from text file that which it was make the initialisation).

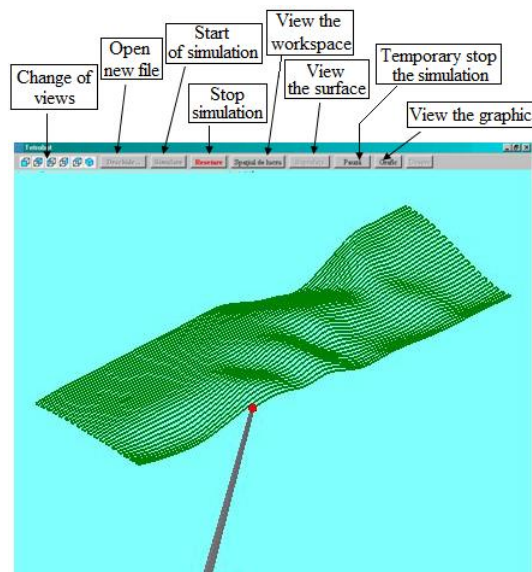


Fig. 9 The active window of the PC

The variation strokes on each actuator are presented in Fig. 10 differentiated by corresponding colour for all three motor couples.

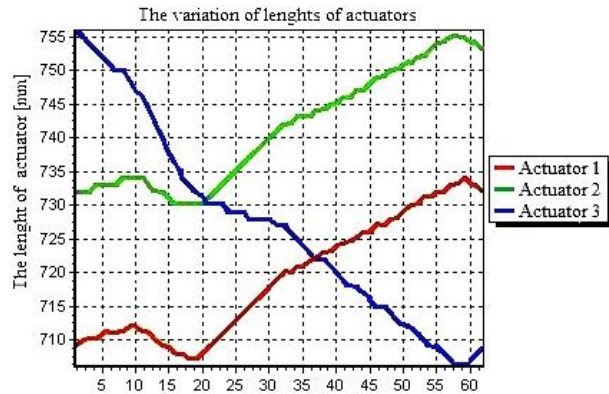


Fig. 10 The strokes of actuators

The simulation accuracy is directly proportional with the number of the points, from text file from which the presented application will be generate interpolated curves of the surface.

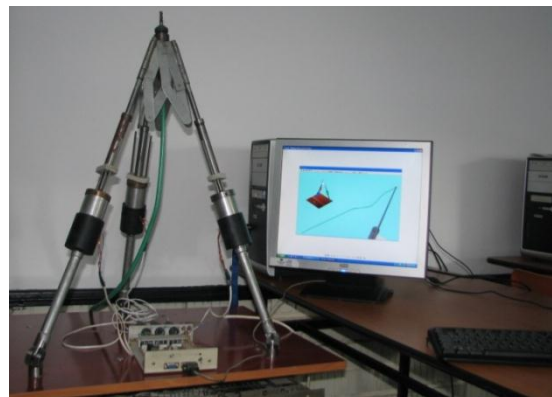


Fig. 11 The prototype robot

Results of simulation are verified by a physical structure shown in Fig. 11.

## 5. Conclusions

In the area of physical generation of surfaces, one of the initial steps is to model them virtually. For solving the aforementioned step, a mathematical approximation of a surface using spline-curves has been created, fact which allowed virtual representation of a surface by using the Borland Delphi 7 programming language.

The data obtained behind the discretisation of the surface are loaded in text file, which subsequently is appealed to command all three actuators from the structure of a tetrahedral module.

In the same time with the appeal of the files with data to generate the surface, the application allows to view the workspace, the strokes of the actuators and of the tetrahedral structure from different angles.

And last but not least, the program through the created interface allow loading other text files, with the points (nodes) in three coordinates, which are belong to other digitised surfaces that, can be view in a virtual way.

## References

1. **Sandor, G.; Erdman, A.** 1984, *Advanced Mechanism Design: Analysis and Synthesis*, Prentice Hall, Englewood Cliffs, New Jersey.
2. **Teutan, E.; Bara, M.; Ardelean, I.** 2010. Theoretical considerations regarding the virtual modelling of surfaces used in topography, *Mechanika* 5(85): 61-65.
3. **De Boor, C.A.** 1987. *A Practical guide to spline, applied mathematical sciences*, Springer – Verlag, 54-57.
4. **Gray, A.** 1997. Normal curvature, §18.2 in *modern differential geometry of curves and surfaces with mathematica*, 2nd ed. Boca Raton, FL: CRC Press, 363-367.
5. **Hamlin, G.; Sanderson, A.C.** 1998. *Tetrobot: A Modular Approach to Reconfigurable Parallel Robotics*, Kluwer Academic Publishers, Boston.

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## SKAITMENINĖS INFORMACIJOS GENERAVIMAS TETROBOT MODULIO PAGALBA

### Re z i u m ė

Tam tikru masteliu atgaminant paviršių, jų apskaičiavimui taikomi matematiniai metodai, šiuo atveju „kubinė interpoliacija“.

Paviršiaus modelis apdorojamas panaudojant ketursienės struktūras. Straipsnio antroje dalyje aprašomi ketursienių struktūrų geometrijos tyrimo aspektai.

Straipsnio gale aprašoma modeliavimui skirta programinė įranga, paviršiaus modeliavimas ir roboto prototipas.

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## GENERATING OF DIGITISED WITH THE AID OF TETROBOT MODULE

### S u m m a r y

For reproducing a surface at a certain scale it is necessary to digitize it by mathematical methods, in this case “cubic interpolation”.

The surface model will be processed using a tetrahedral structure. In the second part of the paper there are presented aspects regarding the geometry of tetrahedral structures.

The end of the paper presents a software modeling, simulation of surface and the prototype robot.

**Keywords:** reproduction of a surface, mathematical methods, tetrahedral structure.

Received May 20, 2011

Accepted November 15, 2012