Dynamic Research on Low-frequency Vibration Isolation Tables

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1. Introduction

In creation of low-frequency vibration isolation systems, a considerable attention is paid currently to active and passive vibration isolation systems [1]. Noise and vibration control is well described in [2, 3]. Vibration isolation systems may be passive, semi-active [4, 5] and active [6]. Usual passive vibration isolation systems (optical tables) are often used for isolation of high-frequency and low-frequency (4–5 Hz) vibrations as well as control of vibration resonance frequencies when high-frequency vibration isolation is required [7, 8, 9, 10]. The said compromise is typical for passive vibration isolation systems; however, passive vibration isolation systems are distinguished for certain operating limitations [1]. Some authors [11, 12] state that active systems in certain cases may ensure better vibration isolation. In cases when better isolation results (that cannot be provided by passive systems) are required, active vibration isolation systems are developed. In some cases, active vibration isolation systems showed better isolation characteristics, as compared to passive ones [13]. However, it is also known that active systems are more expensive and complicated and their reliability is lower, as compared to passive systems. The main limitation of applying the active systems is a necessity of external power supply for vibration isolation. Therefore, application of active systems may be limited only in cases when expenses, complexity and mass increase dramatically. Upon striving to reduce the expenses and to improve the isolation characteristics, passive systems of a new type – negative and quasi-zero stiffness passive systems – have been created [14, 15].

A passive system has present properties that cannot be corrected until the system is operating. Transmission of vibrations depends on the excitation frequency. There are two types of passive vibration isolation systems: they include isolation of the base and isolation of the usable means. One of the most widely accomplished and applied seismic protection system is base isolation. Seismic base isolation [16, 17] is a technology that reduces the influence of earthquake by isolating the building and its contents from potentially dangerous ground, in particular when the frequency range impacts the building considerably. Research on effectiveness of base isolation at seismic excitations of various types encouraged to carry out theoretical and experimental analysis of base isolation and to examine it [18, 19, 20, 21, 22, and 23]. Base isolation supplemented with active or smart devices ensures much better results [24].

Using the base isolation for precise measuring and research appliances is expensive and is not always possible. Therefore, systems able to tackle the problem of vibrations of the base (such systems are based on passive vibration and noise control) are most frequently used. The said systems reduce vibrations and noise by dissipating the energy as heat [25]. Such a system consists of a spring (elastic element) and an energy damper [26, 27]. Elastomers, liquids or negative stiffness elements may be used as well. The springs resist a displacement of the vibrations by stressing opposite forces proportional to their displacement. The damper formed of a piston moving in a viscous liquid or a conductor moving in a magnetic field that resolves kinetic energy as heat [28, 29]. Nevertheless, the spring has its natural resonance frequency that depends on its stiffness, and when the frequency of the vibrations approaches to the natural resonance frequency of the system, the spring turns into an amplifier. Such a simple system does not operate well at vibrations of frequencies below 10 Hz [30]. So, the damping level of such systems is poor enough. However, they are cheap and simple, therefore they are widely used. A passive system has present properties that cannot be corrected until the system is operating [13].

Vibration isolation tables are passive vibration isolation systems formed of a honeycomb panel and pneumatic isolators of vibrations. The most important characteristics of such a table include the displacement compliance of the panel, the transmissibility of the pneumatic isolators of vibrations and the resonance frequency of the table. We can see that if the panel and the pneumatic support are designed in a certain way and new materials and modern technologies are applied, the required damping and the relevant resonance frequencies can be obtained. Therefore, an improvement of such passive vibration isolation systems upon using new materials is relevant and necessary.

2. The object of research

The system usable for isolation of precise equipment from external vibrations is shown below (Fig. 1).

The upper and the lower planes of the table top of table 10 under research (Fig. 1) are made of cold-rolled ferromagnetic steel sheets connected by a light honeycomb structure of a corrosion-resistant steel that ensures exclusive stiffness properties to the table. Usually, the table is mounted on special vibration-isolating supports 9. The resistances of the said structures of the table to impact of static
and dynamic forces both in the vertical and horizontal direction are considered very important quality factors.

![Scheme of research](image)

**Fig. 1 Scheme of research on the dynamic characteristics of a table:**
1, 2, 3, 4, 5, 6 – vibration transducers; 7 – platform (base); 8 – vibrator; 9 – supports of vibration isolation; 10 – experimental table for vibration isolation; 11 – impulse generator for shock impact; 12 – measuring amplifier; 13 – generator; 14 – computer with analyser program. Directions of vibration excitation: Z – vertical; Y – horizontal transverse; X – horizontal longitudinal

An idealized “seismic” mounting system is a rigid table mounted on a massive base or vibration-damping supports. In the world’s practice, vibration-isolating supports of various structures with pneumatic shock-absorbers are used. Such supports should ensure the table’s stability in the vertical and horizontal direction. The impact of the horizontal environmental vibrations is particularly glaring when laboratories are arranged on upper floors of a building.

For manufacturing objects resistant to impact of dynamic and static forces, light honeycomb structures are frequently used. The properties of a honeycomb are predetermined by the size of its cell, the thickness of the walls, its material and so on. The typical properties of honeycomb structures are a light weight as well as compression and flexing strength. The said properties are especially important when an optimal ratio of the mass and stiffness characteristics is required. Namely for this reason, honeycomb structures are used in manufacturing equipment for airplanes, helicopters and so on.

Structures of the said type are widely used for vibration-isolating laboratory tables. Honeycomb tables are distinguished for good vibration damping properties, their weight is much less, as compared to massive tables produced of granite. In a majority of cases, a light honeycomb structure, not heavy granite table resistant to mechanical loads, is chosen.

### 3. Experimental details

For the research, tools and equipment for measuring and analysis of vibrations and other dynamic characteristics were used. They included: an impact hammer with a force measuring converter from „Brüel&Kjær” company, an amplifier and movable measurement results processing equipment „Machine Diagnostics Toolbox” as well as vibration sensors.

Any solid body affected by external static or dynamic forces deforms. The said phenomena are particularly important for tables, if various optical elements are put or fixed on the working plane of the table top and precise measurements are carried out. The tables should not lose their most important mechanical properties, such as a stability of shape, flatness of the table top and other parameters not only because of static loads. Undesirable phenomena very often are caused by dynamic forces as well. The influence of vibrations caused by external forces or other factors is most intensively expressed at the resonance frequencies typical for each structure. The harmful vibrations of machines, vehicles and mechanisms are transferred via the foundation and the communications even to more distant objects. It is important to design a product in a way that minimizes the influence of forces acting in the vertical & horizontal direction and transfers the resonance zone as far as possible from the most frequency occurring and most typical in the work time vibrations. A vibration isolation system should filter the vibrations of the floor (foundation) before they achieve the surface of the table. In the world practice, a resistivity of such tables to vibrations is considered nearly the most important technical parameter.

### 4. Dynamic research

The following dynamic characteristics of honeycomb structure panels of the table were explored:
- the vibration acceleration / time response;
- the dynamic displacement in the frequency range (10–1000) Hz;
- the first resonance frequencies were established;
- Dynamic Deflection Coefficient (DDC) was calculated;
- The Maximum Relative Table Top Motion (MRTM).

The table top is divided to sectors and the measurement points are marked in the said sectors. The goal of the research: to establish the most sensitive points of the table top and to carry out measurements in them.

It was found that the highest amplitudes of vibrations and the highest number of resonance frequencies are typical for angles of the table. The measurement points were chosen in the distance of about 150 mm from an angle of the table. If a less distance is chosen, unstable results of repeated measurements are observed.

The dependence of dynamic displacement on frequency may be established in several ways. One of them is applying a harmonic force of constant amplitude and variable frequency to a chosen point of the table top surface and measuring the vibrations of the table top. This method is hard to accomplish because of technical difficulties.

Another way is excitation of the table’s vibrations by shock disturbances (delta function) and measuring the vibrations.

The dependence of dynamic displacement of the honeycomb structure table tops on frequency was established upon excitation of vibrations by shock disturbances. Each marked point of the table top is beaten with a special hammer for 10 times (Fig. 2). Close to the point of impact, a vibration sensor is fixed. The answer of the object under research to the impact in the specified frequency range is
fixed by the measurement results processing equipment “Machine Diagnostics Toolbox”.

![Image of a person holding a device]

**Fig. 2** Definition of dynamic characteristics of an optical plate of honeycomb design at marked points

![Graph showing vibration acceleration amplitude/time response]

**Fig. 3** The vibration acceleration amplitude/time response of an optical honeycomb design panel in vibration transducer 1 after impact with hammer

In such a way, the curve of the table top dynamic displacement and the vibration acceleration/time response are obtained. An example of a vibration acceleration / time response of a honeycomb structure from vibration transducer 1 is provided in Fig. 3 and an example of a dynamic displacement curve – in Fig. 4.

![Graph showing dynamic displacement and resonance frequencies]

**Fig. 4** Curve of dynamic displacement and resonance frequencies of the honeycomb structure panel in vibration transducer 1

Up to 80 Hz, the table can be considered an ideally rigid body when displacement decreases inversely as square of frequency \((\omega = 2\pi f)\). This expression reflects a straight (Fig. 4) and is usable as a base for calculating the amplifying factor \(Q\). When frequencies exceed 80 Hz, the displacement curve starts deflecting from the straight typical for an ideally rigid body. The table cannot be considered an ideally rigid body, because it starts deforming while affected by the vibrations. The first resonance frequency equals to 199 Hz.

The table 10 with pneumatic vibration isolators 9 (Fig. 1) placed on the vibrating platform may be analysed as a six degree of freedom system, or, when the parameters of the isolators are identical, as a single degree of freedom system (Fig. 5, a, b).

For establishing the dynamic displacement of the table and the resonance frequencies of the system, we affect the mass \(m\) by the force \(F(t)\). If the system is analysed as a six degree of freedom system (Fig. 5, a), the differential equation of its motion when the excitation force affects the table top shall be expressed as follows:

\[
M\ddot{x}(t) + H\dot{x}(t) + Cx(t) = F(t),
\]

where: \(M\), \(H\) and \(C\) are \(6 \times 6\) matrices of mass, damping factors and stiffness factors; \(F(t)\) is the column matrix of summarized external forces.

When the platform (Fig. 5, b) is vibrating upon the influence of \(X_0(t) = X_{exp\text{iao}}\), the differential Eq. (1) will be expressed as follows:
\[ M\ddot{x}(t) + H\dot{x}(t) + Cx(t) = (C + iH\omega)x(t) \text{exp}(i\omega t). \]  

(2)

When the same system is analysed as a single freedom of degree system (i.e. the vibration isolators are identical), the differential Eq. (1) of the system will be expressed as follows:

\[ m\ddot{x}(t) + h\dot{x}(t) + cx(t) = F(t). \]  

(3)

If the excitation force is described by harmonic expression \(|x|\), the resonance frequencies of the system obtained from partial solution of the Eq. (3) will be expressed as follows:

\[ X_d = F_0 \cos(\omega t - \varphi) / \sqrt{(c - m\omega^2)^2 + \omega^2 h^2}, \]  

(4)

where: \( \varphi = \arctg \left[ h\omega / (c - m\omega^2) \right] \).

In respect of displacement, velocity and acceleration, the resonance frequencies will be expressed as follows. When the displacement is measured, the resonance frequency:

\[ \omega_r = \sqrt{\frac{\omega_0^2(1 - 2\xi^2)}{(1 - 2\xi^2)^2}}, \]  

(5)

when the velocity is measured, the resonance frequency:

\[ \omega_{r,v} = \omega_0, \]  

(6)

when the acceleration is measured, the resonance frequency:

\[ \omega_{r,a} = \sqrt{\frac{1}{1 - 2\xi^2}} = \omega_0 \sqrt{\frac{1}{1 - 2\xi^2}}. \]  

(7)

When the damping factor \( \xi \) varies between 0 and 0.7, the difference between these resonance frequencies is inconsiderable. The resonance frequencies of the tables under research often vary between 3 and 4 Hz. The properties of a table are predetermined not only by its resonance frequencies; they also depend on the sizes of the table, the place of its putting on the supports, the properties of the upper and lower surfaces of the table top and the lateral walls, the structure and the material of the honeycomb, the size of the load and its distribution and so on. The maximum bending deflection upon affecting of a static load is considered a measure of stiffness of the table.

The dynamic characteristics of the table and the platform are assessed in the best way by the dynamic displacement – the parameter that’s value is inversely proportional to the value of dynamic stiffness. The curves of displacement show the amplitude of the displacement in the point affected by the unit force (as a function of frequency).

The curves of displacement of the table and the platform show the dynamic response of the table top in a free space as well as the distortions of the plane of the surface when the table and the platform are affected by the vibrations. They outline the frequency range where the behaviour of the table and the platform is identical with behaviour of an absolutely rigid body and provides information on two most important parameters that affect the dynamic characteristics – the minimum resonance frequency and the maximum resonance amplification (they are usable for calculating the real relative motion between two points on the surface of the structure.

The displacement shall be expressed by the following formula:

\[ s = \frac{x}{|F|}. \]  

(8)

where: \(|x|\) is vibration displacement amplitude \(|F|\) is force amplitude. A measurement unit of displacement is mm/N.

A mathematical model of an ideally rigid body will be expressed as follows:

\[ M\ddot{x} = F. \]  

(9)

When a rigid body of mass \( M \) is affected by an external harmonic force, the solution of this equation will be following:

\[ x = x_0 \sin(\omega t), \]  

(10)

where: \( x_0 = -\frac{F_0}{M\omega^2} \).

This means that motion of the body is harmonic and the amplitude of the said motion is inversely as the square of the angular rate. In this example, the displacement \( s \) is expressed as follows:

\[ s = \frac{x_0}{F_0} = \frac{1}{M\omega^2}, \]  

(11)

i.e. the displacement of an ideally rigid body is directly proportional to \( 1/\omega^2 \) and it is depicted as a straight line on a logarithmic scale.

For non-rigid bodies, the displacement curves show the resonance frequencies of the structure and the maximum amplification at a resonance. A displacement is a reliable characteristic of a specific mechanical system.

A dynamic deflection coefficient (DDC) assesses a relative dynamic characteristic of the table top. The said coefficient may be established from the displacement curve by calculating it according to the following formula:

\[ DDC = \sqrt{\frac{Q}{f_0^2}}, \]  

(12)

where: \( f_0 \) is the resonance frequency, \( Q = A/B \) is the amplification factor at the resonance frequency.

The coefficients \( A \) and \( B \) are found from the dynamic displacement curve (Fig. 4). A good table top is defined by a low value of the dynamic deflection coefficient. In this case, the dynamic deflection coefficient (DDC) varies between 0.003 and 0.0033 in the poorest angles and in the centre of the plate.

The Maximum Relative Table Motion (MRTM) is a relative motion between two points of the table top. The higher value of motion of different points is, the poorer stability of the table takes place, so a possibility to accomplish
tuning of the components mounted on the surface of the table top is less expectable. In addition to the dynamic characteristics of the table top, the relative motion depends also on the characteristics of the isolation system and the vibrations of the environment.

The maximum relative table top motion:

\[ MRTM = CT \sqrt{\frac{Q}{f_0^2}} (PSD) \]  

(13)

here \( T \) is the transmissibility of the isolator; \( Q \) is the amplification factor calculated from the displacement curve; \( PSD \) is the power spectral density of the environmental vibration intensity level; \( C \) is a constant that establishes units of acceleration and doubles the value in order to assess the poorest case of the relative motion between any two points:

\[ C = 2g \sqrt{\frac{1}{32\pi^2}} = 0.623 \text{m/s}^2. \]  

(14)

In the point 1 of the table under research, the value of the calculated maximum relative (MRTM) is 0.59 nm and in other points it varies in the range of 0.5 – 0.7 nm. This dimension is explained by the fact that the relative motion of the point of the table top is measured while affected by acceleration of 1 m/s². The resonance frequencies vary between 199 and 220 Hz (in a single point – up to 370 Hz). The values of dynamic parameters of the panel and the platform are very close.

In a case of a single degree of freedom system, the platform (Fig. 5, b) vibrates according to:

\[ x(t) = x_0 \exp(i\omega t), \]  

the differential Eq. (2) of the system’s motion will be expressed as follows:

\[ m\ddot{x}(t) + h\dot{x}(t) + cx(t) = (c + ih\omega)X_0 \exp(i\omega t). \]  

(15)

If vibrations of the platform are described by the harmonic expression, the values of transmissibility ratios are easily found from the equation (15) above [31]:

the absolute transmissibility:

\[ T_s = \frac{\ddot{x}}{\ddot{x}_0} = \left[ 1 + \frac{(2\xi\omega / \omega_0)^2}{(1 - \omega^2 / \omega_0^2) + (2\xi\omega / \omega_0)^2} \right]^{1/2}, \]  

(16)

the relative transmissibility:

\[ T_{r-s} = \frac{x - x_0}{x_0} = \frac{\omega^2}{\left[ 1 - \left( \frac{\omega}{\omega_0} \right)^2 \right]^2 + 4\xi^2 \left( \frac{\omega}{\omega_0} \right)^2}. \]  

(17)

The transmissibility of the acceleration and the relative displacement is shown in Fig. 6, a, b. We can see that vibration damping starts from 5 Hz.

If the table stands on the floor in the laboratory, the environmental vibrations shall be assessed. For assessing the intensity level of the environmental vibrations, the concept of Power Spectral Density (PSD) is used.

The power spectral density is a general term usable upon ignoring the physical processes. An existing physical process is specified by the appropriate data. For example, the term “power spectral density of acceleration amplitude”, or “the spectral acceleration amplitude density” is usable instead of the term “power spectral density” when the spectrum of acceleration amplitude is defined. In the international standard (ISO 2041:2009), the following definition of power spectral density is provided: the power spectral density \( S(f) \) of the value \( X_0(t) \) is a root-square-value of a part of the said parameter having passed through a narrow-band filter with the central frequency \( f \) that corresponds to the unit frequency range when it approaches to zero and the time of the averaging approaches to the infinity.

The power spectral density is expressed as follows:

\[ S(f) = \lim_{B \to 0} \frac{1}{BT} \int_{T} X_0^2(f,t,B)dt, \]  

(18)

where: \( X_0^2(f,t,B) \) is a square of the part of the value \( X_0(t) \) having passed through a narrow-band filter with the band width \( b \) and the central frequency \( f; \) \( T \) is the time of the averaging.
Upon applying Fourier transform, \( S(f) \) shall be expressed as follows:

\[
S(f) = \lim_{T \to \infty} \frac{2}{T} |F(f,T)|^2, \\
f \geq 0; \\
F(f,T) = \int_{0}^{T} X_0(t)e^{-i2\pi ft}dt. \\
\]

When the process is steady, the power spectral density equals to double Fourier transform of auto-covariance function and is expressed by the following formula:

\[
S(f) = 2 \int_{-\infty}^{\infty} k(t)e^{-i2\pi f_0t}e^{-i2\pi ft}dt = \\
= 4\int_{0}^{\frac{\pi}{f}} k(t)\cos(2\pi ft)dt \quad (f \geq 0).
\]

The Power Spectral Density (PSD) is usable for definition of the intensity level of external vibrations. In the case under research, PSD = \(10^9\) g\(^2\)/Hz (in the environment close to a road with high intensity traffic).

5. Conclusions

The methodology for experimental research and establishing the dynamic parameters of honeycomb structure panels was proposed and approved. The presented results of the research show that the dynamic deflection curves for honeycomb structure plates and the platform are straight lines up to 80 Hz, this means that they are absolutely rigid and their first resonance appears at 199 Hz: in addition, their dynamic deflection coefficients and the maximum relative plate motions conform to the standards set by the best manufacturers.

The vibration excitation testing equipment that enables establishing dynamic characteristics on the objects under research, such as DDC; MRTM; the absolute transmissibility and the relative transmissibility had been completed and tested.

The following dynamic parameters of honeycomb structure plates had been established: vibration acceleration/time response (Fig. 3); dynamic displacement, (formula (8)); first resonance frequency 199 Hz, (Fig. 4); Dynamic Deflection Coefficient (DDC) in the centre of the table top and its edges vary between 0.003 and 0.0033; the values of Maximum Relative Table Top Motion in the limits of the table top vary between 0.5 and 0.7 nm.

References


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DYNAMIC RESEARCH ON LOW-FREQUENCY VIBRATION ISOLATION TABLES

Summary

In the paper, an establishment of dynamic characteristics of tabletops of the newly-developed optical tables is being discussed upon. Low-frequency vibration isolation systems are reviewed. Theoretical and experimental tests have been performed. Dynamic models of an optical table on a vibrating platform at different excitations have been developed, the dynamic displacement and the resonance frequencies of the system have been established and vibration transmissibility curves have been presented. The obtained dynamic characteristics of the mechanical passive low-frequency vibration isolation system show that such a system is able to isolate the vibrations effectively. The results of the performed experimental tests confirm the data of the theoretical research.

Keywords: active and passive vibration isolation systems, negative stiffness, quasi-zero stiffness, transmissibility.

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