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Application of strut-and-tie model on eccentric columns

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1. Introduction

Mastering the design by organizing the flow of internal forces is the principal role of an engineer. By selecting the most appropriate structural and material models engineers reach the ultimate goal, which is a finely designed structure of optimal cost and acceptable safety. Due to the ever increasing complexity of structural and material models, engineers often fall for the seemingly easy solution of using computers with the black box software packages. At that moment, engineer's role is reduced to mere keyboard entering of geometrical and load data and printing the offered results. Unfortunately, only a few can recognize an error in the flood of digits or in the blur of colours representing stress distributions.

This issue can be addressed by the use of strut-and-tie models. On one hand, they may seem too simplistic for regular use but on the other hand they provide engineers with full control over the analysis. They call on engineer's active participation in the definition of structural problem by construction of an appropriate strut-and-tie model. In the end, they leave engineers with the feeling of full responsibility for all the results, which in turn will always yield a safe structure.

The strut-and-tie models (STM) have been used for design of reinforced concrete structures since the beginning of the last century, when Ritter [1] was among the first to report an attempt to apply the strut-and-tie models as a design concept for the application to structural problems. He was followed by Mörsch [2] in 1909. Since then, numerous researchers have reported their efforts.

The strut-and-tie models (STM) are widely employed in the design of reinforced concrete structural elements subject to shear and torsion. But, the commonly used methods for the definition of the STMs can hardly offer applicable models for elements with complex loading and geometry conditions, however, STM can be used even for construction analyses during the stage, e.g. [3], when concrete is yet hardening and thus it affect the construction duration, e.g. [4] and [5]. Also Kwak and Noh [6] describe the basic idea of the evolutionary structural optimization method, which determines the optimal topology of the structure, usually represented by a truss-like structure. Hence, the optimization can determine more rational strut-and-tie models. Examples of topology optimization for truss structures are shown in [7] and [8].

This paper explores the applicability of STMs to the elements of two eccentric columns, e.g. in Fig. 1. Even though this is a common problem, its solution using STMs has not yet been published in open literature. The paper applies the STM on structures with different offsets between the columns. The impact of the offset on the maximum load bearing capacity and the optimum STM topology is studied. The results are also used to illustrate the importance of the STM topology's selection on the required reinforcement cross-section area. In other words, the paper promotes the applicability of strut-and-tie models, which are already incorporated in several design codes [9-11].

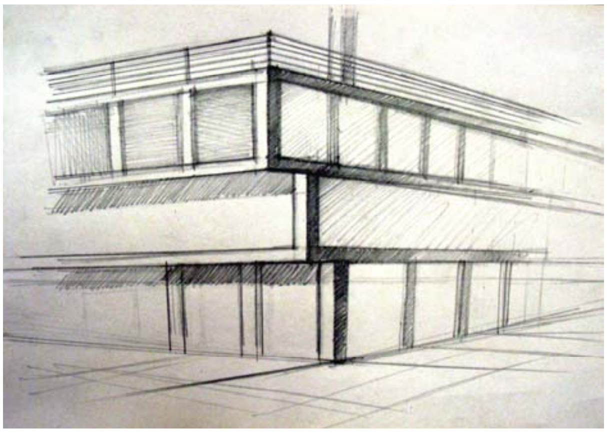


Fig. 1 Example of building with noncontinuous columns

2. Strut and tie models

Design of structural elements from reinforced concrete is generally approached as a design of significant cross-sections, examples of some analyses of cross-sections reinforced concrete members are given in [12, 13]. From this point of view, the structural elements can be divided into separate regions: regions where the Bernoulli's hypothesis holds and regions where it does not hold. In the regions where Bernoulli's hypothesis holds, it facilitates the flexural design of reinforced concrete structures by allowing a linear strain distribution for even the ultimate flexural capacity. Hence, the entire flexural behaviour can be predicted by a simple calculation. These regions are also called beam-like regions (B-regions) [14].

The latter group are the regions represented by joints and locations near applied load and supports, where disturbances and discontinuities occur. Thus, Bernoulli's hypothesis cannot be assumed. Typical examples of structural members containing primarily the discontinuities regions (D-regions) are corbels, short cantilevers, and deep beam. Currently, the STMs for simple structures are recommended in EC2 [9]. In the discontinuities regions, the macroscopic shear strength of concrete contributes to the ultimate shear resistance of elements, comprises the dowel action of longitudinal reinforcement, and aggregates the interlocking across tension-shear crack and tensile stress field that becomes mobilized in concrete through the bond between concrete and reinforcement.

D-regions represent a very complex mechanism. However, in terms of, e.g. internal shear force, the regions need to be quantified very accurately so that structural safety is ensured. Prior to powerful computers and advanced numerical methods, this complex mechanism was analysed with the strut-and-tie, or truss, models and for their simple mathematical description and clear physical interpretation they became very popular with practicing engineers.

3. Application of STMs on eccentric columns

3.1. Problem definition

In order to investigate the applicability of STM on eccentrically connected columns a section of a frame was selected (Fig. 2). The force F is a parameter. The loading force and the boundary conditions are represented by multiples of the force F. The multiplication by the factor 5 represents five floors which are supported by the column. The load imposed by the beam, or slab, is represented by the combination of shear force (equal to F) and bending moment, which is approximately given by

$$M = \frac{1}{12}\frac{F}{3x}\left(6x\right)^{2} = F\,x \tag{1}$$

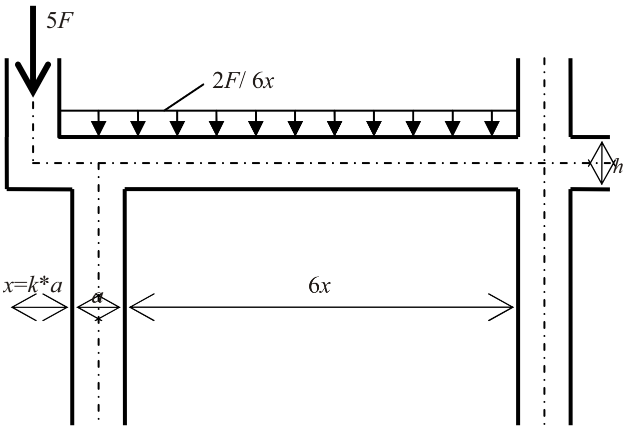


Fig. 2 Selected section frame

Owing to this, the moment of force couple on the right hand side of the structure is quantified with Fx. Assuming that the height of horizontal beam is $h = a$ the problem is further simplified as can be seen on the loading of segment of interest in Fig. 3. Regardless the simplification, the region of interest roughly corresponds with the common span and sizes of beam cross-sections used in ordinary RC frames [15]. For the calculations, the width of the columns and the height of the beams (parameter a) was selected 400 mm.

The length of the cantilever (measured from the outer surface of the lower column) is given in multiples of a, the effect of this multiplicative factor on the resulting maximum load $5F$ and on the STM optimal topology is studied in subsequent sections.

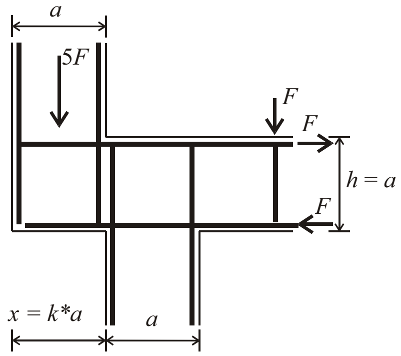


Fig. 3 Region of interest with loading forces and main reinforcement of columns and beam

3.2. Effect of cantilever length on the maximum bearing capacity

To explore the impact of cantilever length on the maximum bearing capacity, three cases of the cantilever length were selected. The used multiplicative factors k are: 1, 1.5, and 2, which correspond to the lengths of 400, 600 and 800 mm, respectively. Cantilever lengths beyond 800 mm (multiplicative factor 2) are not taken into account because that would only result in the development of a B-region between two D-regions of the two columns. Cantilever lengths below 400 mm were not considered because the compressive force would be transferred directly from the upper column to the lower one and no significant shear stress would occur.

In the first case, when $x = a$ (Fig. 4), the leftmost diagonal of strut-and-tie model can transfer the entire load from the upper column to the lower one, provided that the longitudinal reinforcement at the upper surface can take the enormous tensile force.

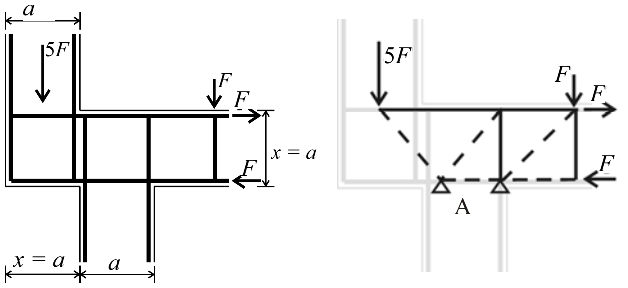


Fig. 4 Reinforcement arrangement (left), corresponding strut-and-tie model (right) for $x = 400$ mm

The maximum bearing capacity is limited by the stress in the concrete in the joint A (Fig. 4), where three struts join. This joint must transfer the load to the column. The maximum load at the joint edges is given by the hydrostatic law [9]

$$\sigma_{Rd,max} = 1.0\ v' f_{cd} \tag{2}$$

where $v' = 1 - f_{ck}/250$ (f_{ck} is in MPa).

The following assumptions were made: the lower edge of the joint must not exceed the edge of the frame and that concrete C 30/37 is used, the maximum bearing capacity yields $5F \approx 900$ kN.

The STM calculation was also performed in software CAST [16]. The detailed results from CAST for load $5F = 900$ kN are shown in the Fig. 5.

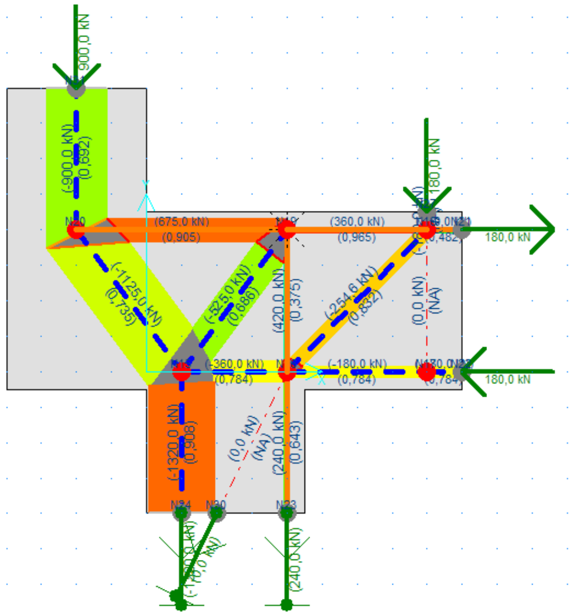


Fig. 5 Strut-and-tie model by software CAST, detailed results for $x = 400$ mm

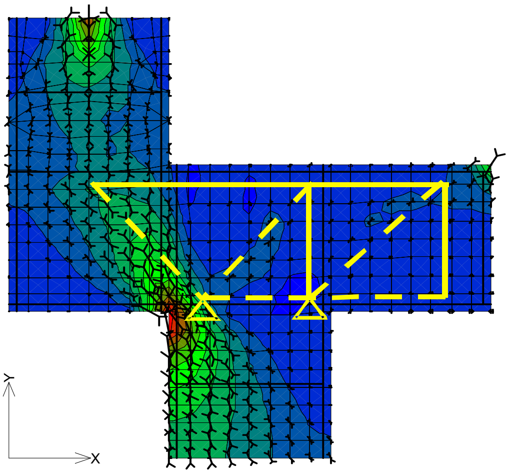


Fig. 6 Numerical model results and corresponding strut-and-tie model for $x = 400$ mm

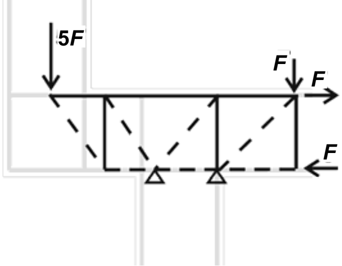


Fig. 7 The basic STM topology for $x = 600$ mm

To confirm the applicability of the STM, the structure was also modeled in commercial software ATENA [17] whose results are presented in Fig. 6. The results and the estimated maximum bearing capacity are in good agreement. In the second case, the cantilever length was 600 mm. The corresponding basic STM topology is presented in Fig. 7. The maximum bearing capacity is, similarly to the preceding case, limited by the joint at the edge of the lower column. Using the same assumptions as above, the maximum load calculated using the STM yields $5F = 600$ kN. The detailed results of the STM calculations are again presented on the output from the software CAST in Fig. 8. These results agree with the numerical modeling from ATENA which are in Fig. 9.

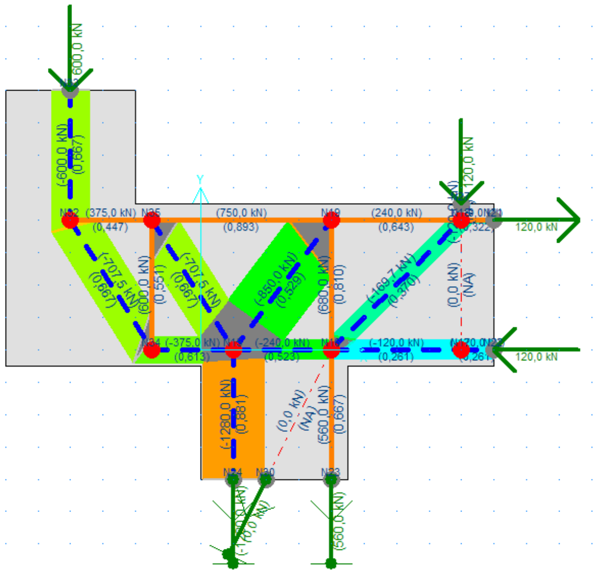


Fig. 8 Strut-and-tie model by software CAST, detailed results for $x = 600$ mm

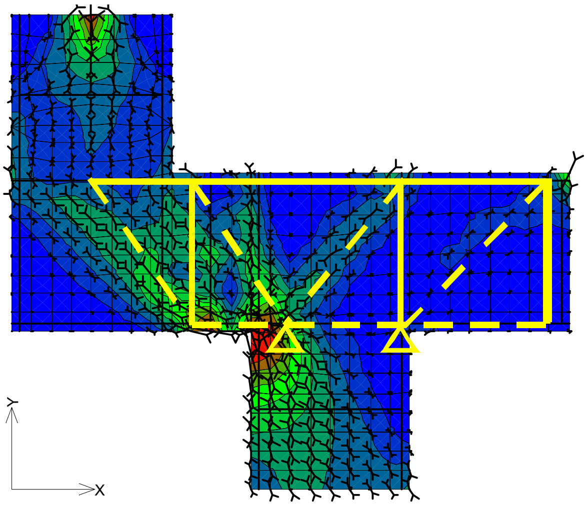


Fig. 9 Numerical model results and corresponding strut-and-tie model for $x = 600$ mm

The last case is for the cantilever length of 800 mm. The basic STM topology for this case is depicted in Fig. 10. The maximum bearing capacity in this case is calculated analogically to the preceding cases. It yields 500 kN. The detailed results obtained using the CAST software are presented in Fig. 11 and the verification by full numerical simulation using ATENA is in Fig. 12. From these three cases it can be concluded that STM models can be used as valid alternative tool for analyses of eccentric columns.

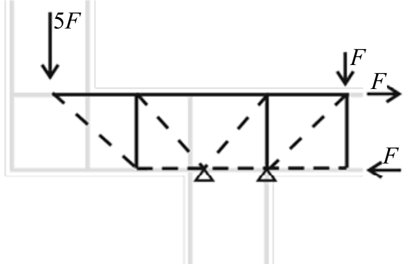


Fig. 10 The basic STM topology for $x = 800$ mm

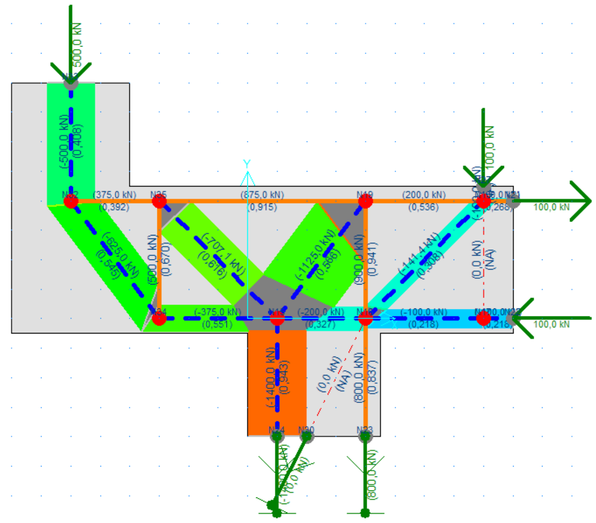


Fig. 11 Strut-and-tie model by software CAST, detailed results for $x = 800$ mm

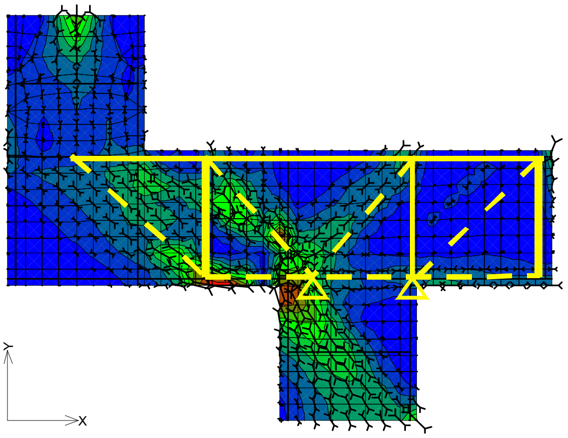
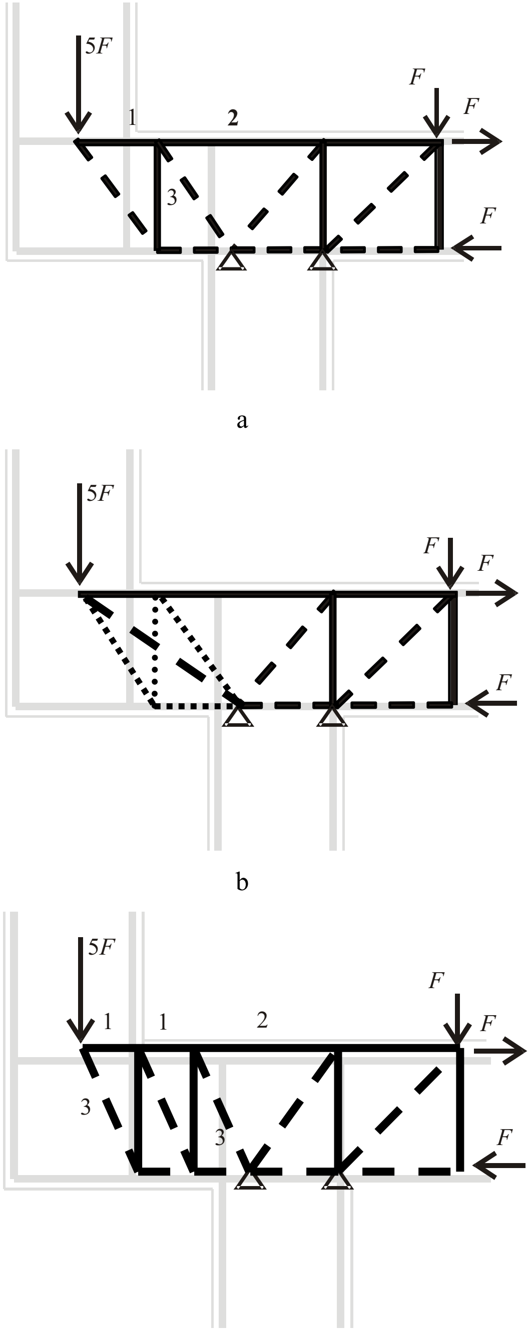


Fig. 12 Numerical model results and corresponding strut-and-tie model for $x = 800$ mm

3.3. The impact of STM topology on reinforcement area

The reinforcement cross-section area is one of the most important characteristics of a design because it directly influences the cost of the structure and its safety. The objective of efficient design is reinforcement of minimum cost with sufficient safety.

The main advantage of the STM is its simplicity and the possibility of direct application to the selection of reinforcement area calculation. However, for the optimum reinforcement area, the optimum STM topology must be selected [7, 8]. This section explores the dependency of the reinforcement area in concrete section of eccentric columns on the topology of the STM. The objective is to illustrate that the optimization of the STM topology remains simple even for complex structures such as eccentric columns.



a

b

c

Fig. 13 Three possible STM topologies for $x = 600$ mm

To explore the dependency of the reinforcement area on the topology, the case with cantilever length of 600 mm was selected. Three alternative STM topologies were investigated. These topologies are presented in Fig. 13, where the three topologies that can be used for the STM model of the studied element are presented. Fig. 13, a presents the basic intuitive topology used in preceding section. Fig. 13, b represents a statically indeterminate modification of the basic topology inspired by the combination with the case of cantilever length $x = 400$ mm (Fig. 4). Fig. 13, c then represents a case when additional reinforcement is included in the structure. All three cases in Fig. 13 were evaluated in terms of maximum bearing capacity.

The maximum bearing capacity is limited by the concrete and in all it yields 600 kN regardless the topology, in other words regardless the reinforcement. To illustrate this, Fig. 14 presents the dependency of the reinforcement area on the maximum load 5*F*. The reinforcement areas from Fig. 14 are calculated using EC2 [9].

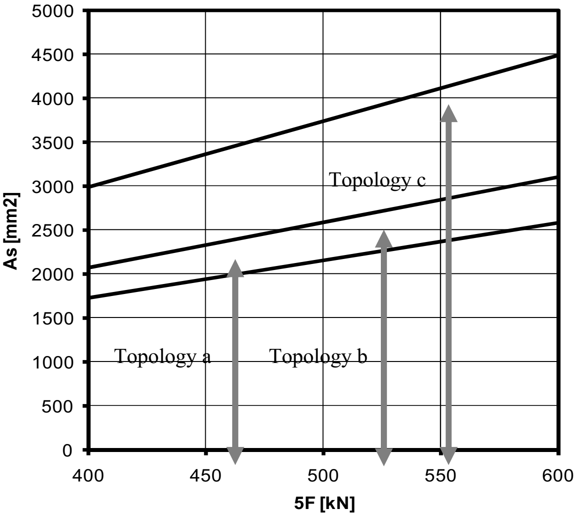


Fig. 14 Reinforcement area in dependency on the maximum load 5*F*

Fig. 14 shows that the statically indeterminate topology b) requires the lowest reinforcement area, thus it maximizes the cost reduction. As can be seen, the topology c) uses unnecessarily large reinforcement area, thus is highly unfeasible.

To conclude, this section illustrates that the optimization of STM topology, which can significantly reduce the cost of reinforcement, is relatively simple even in the case of complex structures such as eccentric columns.

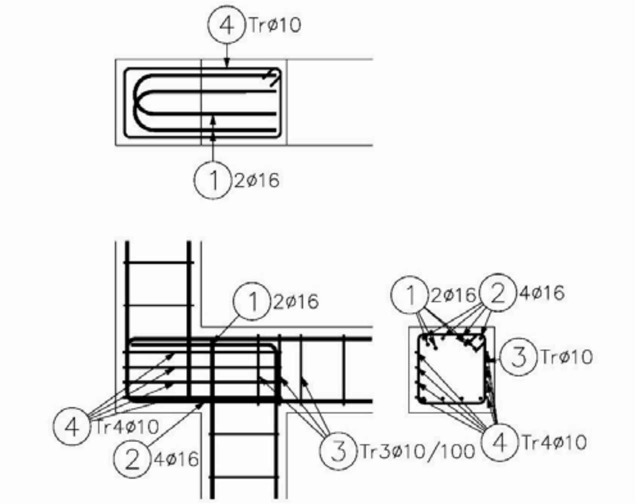


Fig. 15 Optimum reinforcement design for x = 400 mm

3.4. Examples of optimum reinforcement arrangement

This section uses the results from section 3.2 and 3.3 to present the final design of the optimum reinforcement (in terms of cost and safety) for the cantilever lengths x = 400 mm and x = 600 mm. The reinforcement for the cantilever length x = 400 mm is presented in Fig. 15. The main longitudinal reinforcement consists of horizontal loops – Nr. 1 and 2. The transversal tension is held by horizontal stirrups – Nr. 4.

Fig. 16 represents the optimum reinforcement design for the cantilever length of 600 mm using topology b according to section 3.3. The main longitudinal reinforcement is represented by horizontal loops – Nr. 1 and Nr. 2. The vertical forces in ties are covered by vertical stirrups – Nr. 3.

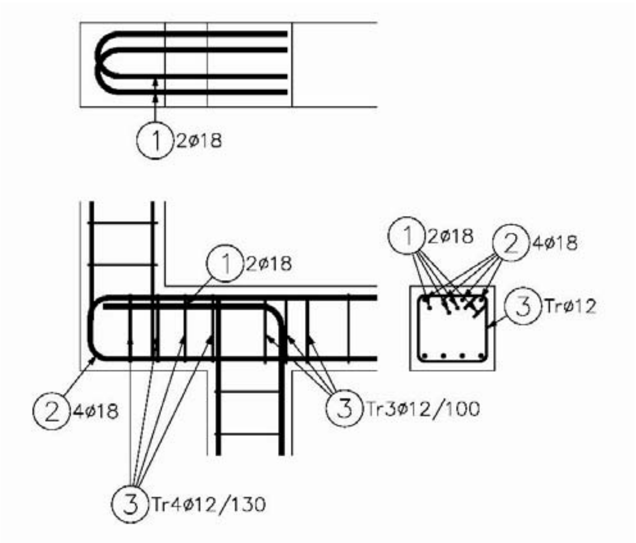


Fig. 16 Optimum reinforcement design for x = 600 mm

4. Conclusion

This paper has explored the possibility of the application of strut-and-tie models on more complex structures, specifically eccentric columns. It has been demonstrated that the STM can be successfully applied and good agreement in the prediction of maximum bearing capacity between the STM and full numerical simulation of the structure has been presented in section 3.2.

Additionally, it has been shown that the optimization of the STM topology, which ultimately reduces the cost of the reinforcement, remains simple even for the relatively complex case studied in this paper.

Finally, the optimum reinforcement design developed based on the STM has been presented to document the practical applicability of the model.

To conclude, the paper proves the potential of the STM's application on more complex structures beyond the specification of EC2 [9].

Acknowledgement

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SANTVAROS MODELIO TAIKYMAS EKSCENTRIŠKOMS KOLONOMS

Reziumė

Populiarus santvaros modelis EC2 dažnai kvalifikuojamas kaip standartinė priemonė nesudėtingoms konstrukcijoms imituoti. Straipsnyje parodoma, kaip šis modelis pritaikomas sudėtingesnėse konstrukcijose, pavyzdžiui, ekscentriškose kolonose. Palyginimas su pilnutiniu skaitiniu imitavimu rodo, kad santvaros modelis leidžia korektiškai prognozuoti maksimalią keliamąją galią. Konstrukcijos optimizavimo kaina, naudojant santvaros modelio topologiją, yra nedidelė, net esant sudėtingoms ekscentriškoms kolonoms. Straipsnyje taip pat aprašomas praktinis optimalus armavimas naudojant santvaros modelį.

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APPLICATION OF STRUT-AND-TIE MODEL ON ECCENTRIC COLUMNS

Summary

The popular strut-and-tie model is currently defined by EC2 as a standard tool for simple structures. This paper explores the possibility of application of the strut-and-tie model on more complex structures, specifically on the eccentric columns. Comparison with full numerical simulation shows that the strut-and-tie model can correctly predict the maximum bearing capacity. The optimization of structure cost associated with the optimization of the strut-and-tie model topology is demonstrated to remain simple even for the complex case of eccentric columns. The practical optimum design of reinforcement using the strut-and-tie model is also presented.

М. Франтова, П. Штемберк, В. Вала

ПРИМЕНЕНИЕ МОДЕЛИ ШПРЕНГЕЛЯ ДЛЯ ЭКСЦЕНТРИЧЕСКИХ КОЛОН

Резюме

Известная модель Шпренгеля часто квалифицируется ЕС2 как стандартное устройство в несложных конструкциях. Настоящая статья рассматривает применение этой модели в более сложных конструкциях, например, в эксцентрических колонах. Сопоставление с полной числовой имитацией показывает, что модель Шпренгеля может хорошо прогнозировать максимальную несущую способность. Цена оптимизации конструкции при использовании топологии модели Шпренгеля является незначительной даже для сложных эксцентрических колон. В статье также приведен пример практического оптимального армирования с использованием модели Шпренгеля.

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