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A Novel Principle Stress Model to Predict the Stress and Load in Sheet Drawing

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1. Introduction

The principle stress model (PSM) is an approximate analytical method widely used in engineering for calculating stress distribution and deformation loads. It plays an important role in the analysis and parameter optimization of material processing techniques such as forging, rolling, drawing, extrusion, and stamping [1-3]. Finite Element Method (FEM), the numerical simulation method, can obtain more accurate and detailed information about plastic deformation behavior and force-energy parameters, making it widely applied in the field of material processing [4-6]. However, compared to the PSM, FEM has disadvantages such as slow solving speed, long computational time, and difficulty in convergence [7-8], making it challenging to achieve rapid solution of mechanical parameters and online applications [9-10]. The sheet drawing is a fundamental stamping process in which a flat blank is formed into various hollow components using a deep drawing die. It is widely used in the manufacturing of various sheet metal components. The stresses and loads during the deep drawing process not only determine the number of forming cycles but also directly affect wrinkling and tearing defects in the drawn parts [11]. Therefore, the stress analysis of sheet metal drawing deformation is not only the core chapter of the application of the principal stress method in elastic or plastic mechanics textbooks [12-13] but also the key and difficult knowledge in the molding process [14-15], design of the dies [16-17] and other relevant professional works.

The deformation of sheet drawing process is simplified to solve plane stress problem conventionally based on the assumption of neglecting blank holder and friction [12]. In order to improve the prediction accuracy, the blank holder and friction is considered and the PSM was established to predict stress in deformation zones [13]. However, the normal stress on the free boundary is set as the function related to the friction in the references, which is not meet with the description of the value is zero in plastic mechanics theory. In order to improve the solution efficiency and accuracy, a new PSM model was proposed and established to obtain the stress distribution and deformation load in sheet drawing, and then the calculated results were compared with those of FEM to validate the reliability of the model.

2. PSM in Sheet Drawing Deformation

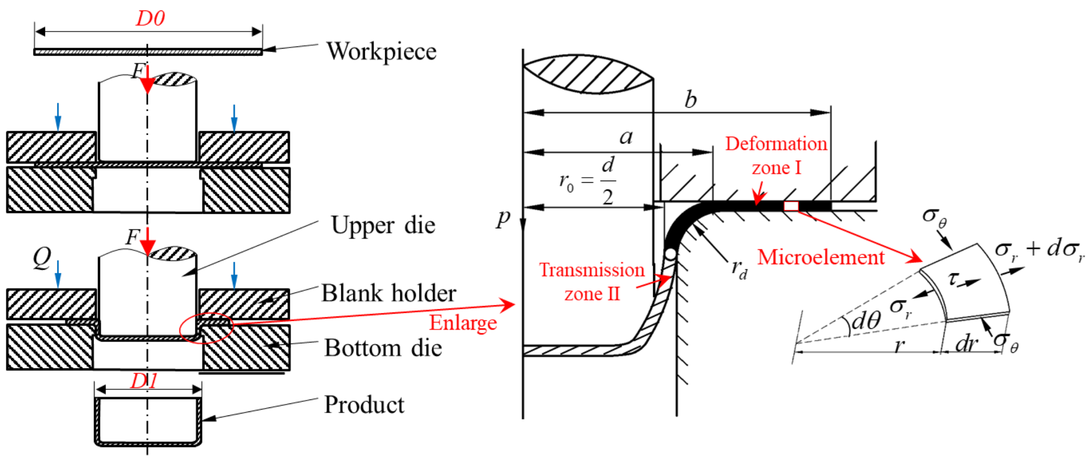
Straight-wall cylindrical drawing is one of the important stamping processes, and the gap between the upper and bottom dies is slightly larger than thickness of the sheet. The workpiece and product is circular sheet with D0 diameter and cylindrical components with D1 outer diameter, respectively (Fig. 1). The blank holder is often used to avoid the wrinkling in the flange deformation zone caused by the thickening instability freely (Fig. 1, a). During the sheet drawing process, the flange part of sheet on the upper die surface is the main deformation zone I, and the bottom and side walls of sheet formed initially are the force transmission zone II [18]. The six unknown stress components and three displacement components need to be simultaneous solved to analyze the sheet drawing process by PSM. However, it is difficult to obtain an analytical solution under rigorously satisfying with these equations and given boundary conditions [12-13]. In order to meet the accuracy requirements when analyzing complex three-dimensional engineering problems, it is often necessary to simplify them to a plane or axisymmetric model as a means of achieving stress or load solution [19].

The thickness of sheet in the flange zone often keeps constant and the strain in the thickness direction is zero during the sheet drawing processes. According to the plastic mechanics theory, the solution of deformation problem can be simplified to plane strain analysis. Considering the friction between the blank holder and bottom die, an equilibrium differential equation along the radial direction $\sum F_r = 0$ can be established as follows as

$$\left(\sigma_r + d\sigma_r\right)\left(r + dr\right)d\theta \cdot t + 2\sigma_\theta dr\sin\left(\frac{d\theta}{2}\right) \cdot t - \sigma_r r d\theta \cdot t + 2\tau \cdot r d\theta \cdot dr = 0 \,, \tag{1}$$

where the σ_r is radial stress, r is radius, t is thickness of sheet, σ_θ is circumferential stress and τ is friction.

The Eq. (1) after simplify derivation of is written as follow:



a b

Fig. 1 Deformation of sheet drawing: a – sheet drawing process, b – stress state

$$\frac{d\sigma_r}{dr}+\frac{\sigma_r+\sigma_\theta}{r}+\frac{2\tau}{t}=0\,. \tag{2}$$

The circumferential σ_θ and radial stress of element the stress state (Fig. 1) is assumed as compressive and tensile stress, respectively. Therefore, compared with radial stress the circumferential stress is needed to add a negative sign before incorporating into the yield criterion according to the rule [20]. Subsequently, the simplified condition of von Mises yield criterion is written as

$$\sigma_r-(-\sigma_\theta)=\beta\sigma_s\Rightarrow\sigma_r+\sigma_\theta=\beta\sigma_s\,, \tag{3}$$

where, β is the stress state coefficient and the value equals to $\beta = 2/\sqrt{3}$ for plane strain problem.

The friction force exhibits a relationship with the blank holder force, friction coefficient and contact area, and then the shear stress of friction is described as

$$\tau=\frac{fQ}{\pi\left(b^2-a^2\right)}\,. \tag{4}$$

And then, substituting the Eqs. (3) and (4) into the Eq. (2), the equilibrium differential equation is written as

$$\frac{d\sigma_r}{dr}=-\frac{\beta\sigma_s}{r}-\frac{2fQ}{\pi\left(b^2-a^2\right)t}\,. \tag{5}$$

Integrating both sides of the Eq. (5), and then the equation is obtained as follows as

$$\sigma_r=-\beta\sigma_s\ln r-\frac{2fQ}{\pi\left(b^2-a^2\right)t}r+C\,. \tag{6}$$

On the free surface of the workpiece, both the normal and shear stress is zero [21], and consequently, the redial stress conditions at the outer free boundary of the flange deformation zone is described as follows

$$\sigma_r\big|_{r=b}=0\,. \tag{7}$$

Using the stress boundary condition, the r equals to b, the radial stress σ_r on the free surface equals to zero. Substituting the boundary condition Eq. (7) into Eq. (6), and then the integration constant is obtained as is written as follows

$$C=\beta\sigma_s\ln b+\frac{2fQ}{\pi\left(b^2-a^2\right)t}b\,. \tag{8}$$

Substituting the integration constant C into the Eq. (6), and then the radial stress distribution σ_r in the flange deformation zone is described as follows as

$$\sigma_r=\beta\sigma_s\ln\left(\frac{b}{r}\right)+\frac{2fQ}{\pi\left(b^2-a^2\right)t}(b-r)\,. \tag{9}$$

The value of radial stress is greater than zero according to the Eq. (9), and it has the same direction as that of assumption so that it is tensile stress actually. Substituting the radial stress equation into the Eq. (3), and then the distribution of circumferential stress σ_θ is written as

$$\sigma_\theta=\beta\sigma_s\left[1+\ln\left(\frac{r}{b}\right)\right]+\frac{2fQ}{\pi\left(b^2-a^2\right)t}(b-r)\,. \tag{10}$$

The circumferential stress is greater than zero, which is contrary with the assumed direction, and it is compressive stress actually according to the solution rule of PSM [20]. The actual distributions of main stress components in the deformation zone can be described by these Eqs. (9) and (10) added the negative sign. For the plane strain problem, the stress σ_z in the thickness direction with zero strain equals to half of the sum value of the circumferential and radial stress

$$\sigma_z = \beta\sigma_s\left(ln\left(\frac{b}{r}\right)-\frac{1}{2}\right). \quad (11)$$

It is shown in the Eq. (11) that the value of stress in thickness direction changes gradually along the radius of the flange in the deformation zone, and it is compressive and tensile close to the edge of flange and the fillet of bottom die, respectively. Furthermore, the radial stress has the maximum value on the corner of the fillet of bottom die according to the Eq. (9), and the value with radius $r = a$ is described as follows as

$$\sigma_r\big|_{r=a} = \beta\sigma_s\, ln\left(\frac{b}{a}\right)+\frac{2fQ}{\pi(b+a)t}. \quad (12)$$

Ignoring the influence of sheet bending on the stress in the fillet zone of bottom die, the radial stress with the r equals to a can be equivalent to the unidirectional tensile stress in the transmission region between upper and bottom dies. Subsequently, the deformation load during sheet drawing processes can be obtained according to the force balance and Eq. (12)

$$F = \pi d\cdot t\cdot\left[\beta\sigma_s\, ln\left(\frac{b}{a}\right)+\frac{2fQ}{\pi(b+a)t}\right], \quad (13)$$

where d is the inner diameter of the bottom die (mm); t is the thickness of the sheet (mm); σ_s is the yield strength (MPa); b is the outer radius of the flange (mm); a is the inner radius of the flange (mm); f is the friction coefficient, and Q is the force of the blank holder (N).

3. Results and Discussion

3.1. FEM model

To verify the reliability of the PSM in this study, the FEM software DEFORM is used to analyze the stress distribution and deformation load in sheet drawing processes. The initial thickness and diameter of sheet is 2 mm and 100 mm, respectively. The diameter of upper die is 40 mm, and the gap between dies is 2.5 mm. The fillet of upper and bottom die is 4 mm. The speed of upper die is set to 0.1 mm/s within the displacement of 10 mm, and both the blank holder and bottom die is fixed. The material used to analysis is 1100 aluminum alloy and the stress-strain curve [22] is shown in (Fig. 2, b). The quadrilateral mesh is used, and the size in thickness and radial direction is divided into 4 layers and 130 layer elements to ensure solution accuracy. Total number of elements and nodes is 520 and 655, respectively (Fig. 2, a). The friction coefficient is set to 0.05 and the dies are set to rigid materials.

3.2. FEM analysis

It can be seen from the Fig.3 that the sheet thickness variation can be divided into five zones during drawing processes. The thickness has an increased trendy in the flange deformation zone I, but the value keeps a stable value

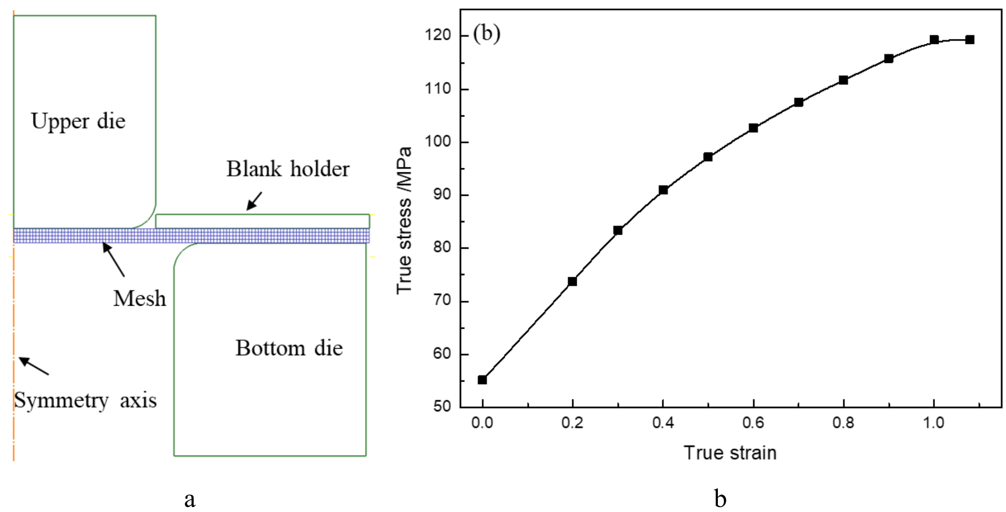


Fig. 2 FEM and material model: a – material model, b – true stress-strain curve

because of the limit of blank holder. Therefore, the engineering problem of drawing processes simplified to the plane strain is reasonable. The thickness decreases about 15% seriously in the deformation zone II closed to the fillet of bottom die. The bending effect weakens gradually leads to the slight thickness increment of the sheet just entered the force transmission zone III. However, the thickness continuous decreases obviously because of the unidirectional drawing deformation along the axial direction. The most severe thickness reduction occurs in the fillet zone IV of upper die leads to the crack defect, and the maximum reduction rate is over to 50%. In the bottom zone V of upper die the sheet thickness reduces slightly under the effect of radial tensile stress and friction, and the reduction rate is about 14%.

The radial stress at the outer surface of flange equals to zero approximately (Fig. 4, a) consistent with the plastic mechanics theory of free boundary condition. Both three directional tensile stress state and maximum thinning easily result in the crack occurred in the fillet of upper die. In the flange deformation zone, σ_r and σ_θ is tensile and compressive stresses, respectively, and more noticeable thickening occurred closer the outer of the flange. The σ_z in the

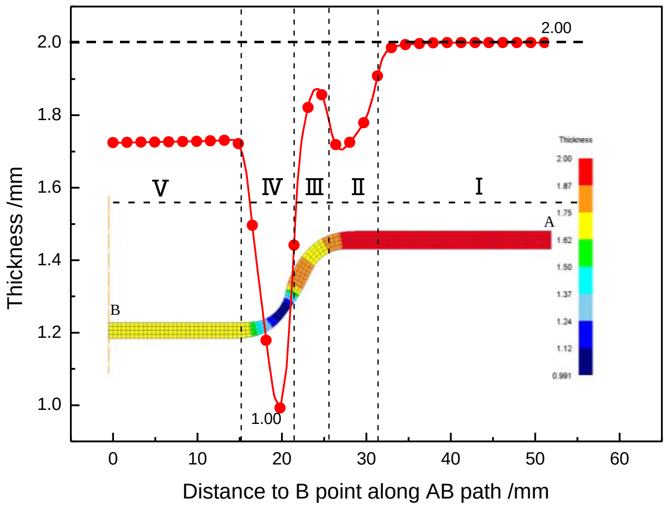


Fig. 3 Change of the thickness

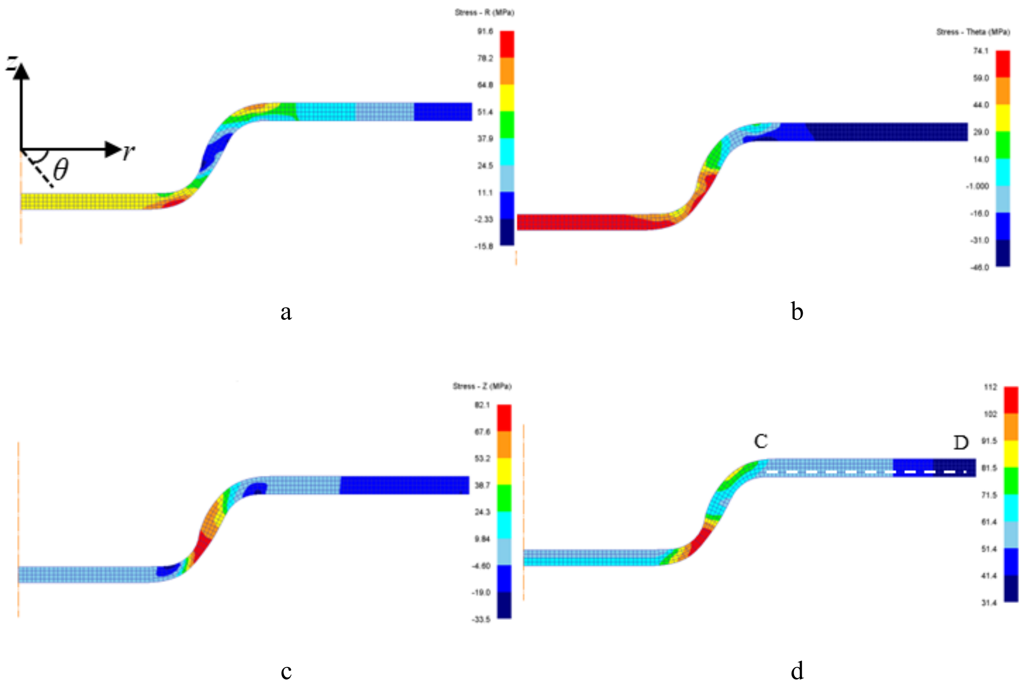


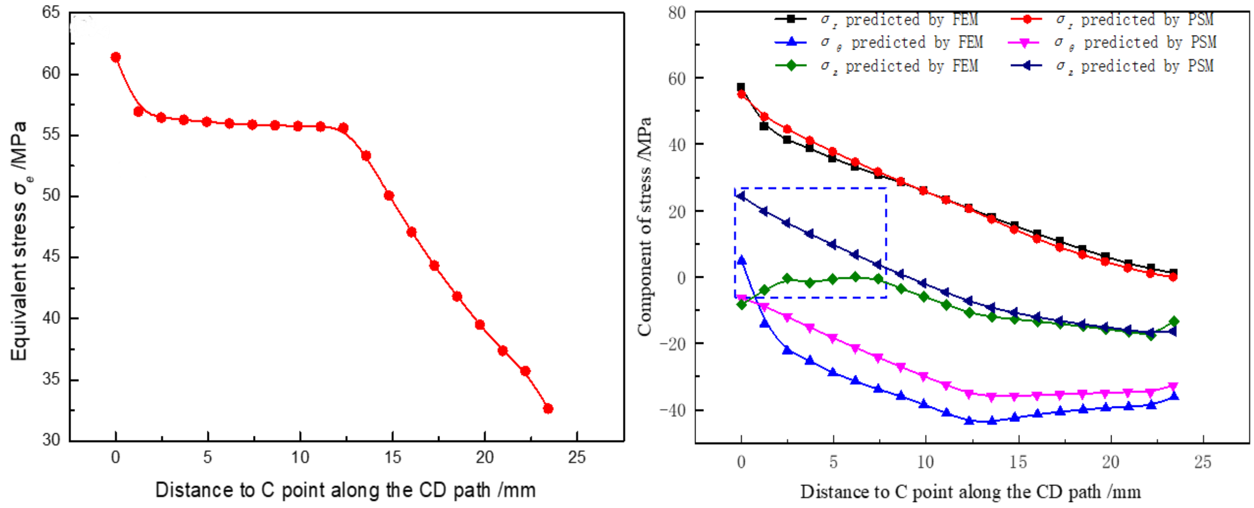
Fig. 4 Distribution of component of stress：a – radial stress σ_r, b – circumferential stress σ_θ, c – thickness stress σ_z, b – equivalent stress σ_e

flange deformation zone is compressive stress and the value becomes tension, and the calculated stress state is in good agreement with the PSM. Resulting from the bending in the fillet of bottom die, the stress state is compressive inside of sheet, but the σ_r and σ_z are greater than zero with σ_θ less than zero outside of sheet. In the force transmission zone, both the σ_r and σ_θ are approximately equals to zero and the σ_z exhibits tensile stress, indicates a unidirectional tensile stress state. In the deformation zone of the bottom of upper die, the radial and circumferential stress is tensile and the σ_z equals to zero. The equivalent stress gradually increases from the end of the flange to the fillet of bottom die due to the working hardening, and the maximum value equals to about 112 MPa (Fig. 4, d).

3.3. Comparative analysis

A comparison of calculated stress and load by PSM and FEM in drawing is shown in Fig.5. The parameters used to solution as follows: the blank holder force Q is 52.5 kN; the inner diameter a of the flange deformation zone is 26.5 mm; the outer diameter b is 49.5 mm and the friction coefficient is 0.05. The working hardening effect leads to a gradual increment of equivalent force along the CD path (Fig. 5, a) from point C to point D, and the value of point C is about 62 MPa with higher twice than that of point D approximately. According to the Mises criterion and subsequent yield theory, the equivalent force as the yield strength input into equation (10) and (11) to obtain the distribution of different stress components (Fig. 5, b). Along the CD path from the point C to point D both the radial stress σ_r and circumferential stress σ_θ predicted by PSM is in a good agreement with the calculated value by FEM, and the relative error is less than 5%. Moreover, the radial stress continuously increases, and the absolute value of circumferential stress initially increases and then decreases. Because the influence of bending on the stress is not considering the calculated thickness stress σ_z close to the fillet of bottom die by PSM has a higher relative error compared with that of FEM. However, there is a good agreement in the other zones and the relative error is also less than 5%. The radial stress of the point D on the free surface equals to zero, which meets with the mechanics theory of free boundary. Therefore, it is slightly inaccurate to set the boundary stress value as the relationship function with friction of blank holder in the reference [13].

The radial stress σ_r of point C is substituted into the Eq. (13) and the deformation load of the sheet drawing force is obtained to 15.7 kN by the PSM. Compared with the calculated value of 15.2 kN by FEM the relative error is about 3.5%, and the proposed PSM is reliable. In a word, the proposed PSM demonstrates a significant accuracy and reliability to solve the stress in the flange deformation zone and the deformation load during sheet drawing processes. Compared with the FEM, it is certain difficult for the PSM to obtain the detailed information of stress distribution, but the mathematic model has a significant fast computational speed and higher solution efficiency, and it is beneficial for the engineering application of PSM.



a b

Fig. 5 Stress components in the CD path under different conditions: a – equivalent stress σ_e, b – with compression edge ring σ_r and σ_θ

4. Conclusions

1. A novel PSM were proposed and established to predict successfully the stress in radial, circumferential and thickness direction during the sheet drawing with blank holder, and the upper and bottom die, products size and sheet thickness have important influence on the stress and load. Compared with the conventional mathematical models, the radial stress on the free surface used to zero by PSM, which meets with the mechanics theory of free boundary.

2. The thickening instability in deformation can be avoided with the blank holder, and the thickness in the flange keeps about stable value. According to the PSM method, three-dimensional sheet drawing processes simplified to be solution of the plane strain problem is completely reliable.

3. The calculated stress distribution and deformation load by PSM has a good agreement with those of FEM, and the most relative error is less than 5%. The proposed PSM demonstrates a significant accuracy and reliability to solve the stress in the flange deformation zone and the deformation load during sheet drawing processes. Compared with the FEM, the PSM has not more detailed calculated results of plastic deformation but a significant fast computational speed and higher solution efficiency, and it is beneficial for the online engineering application.

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Conflict of Interest

The authors declare that they have no conflict of interest.

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A NOVEL PRINCIPLE STRESS MODEL TO PREDICT THE STRESS AND LOAD IN SHEET DRAWING

S u m m a r y

The solution of stress and load of metal sheet deep drawing serves as a primary basis for guiding production, verifying strength of dies, and optimizing technology parameters. Based on the differential equations, boundary conditions, theory simplification and solution equations, an principle stress model (PSM) to predict the stress distribution and deformation load in drawing processes with a blank holder were proposed, and then the results were compared with the predicted value by finite element method (FEM) under the same conditions. Due to the thickness of the flange in deformation zone remains basically unchanged under the action of blank holder, it is reasonable to simplify the solution of deformation to plane strain problem. The PSM and FEM predict that the radial tensile stress and circumferential compressive stress in the flange zones remain basically consistent trends, and the relative error of most of predicted results is less than 5%. The influence of the die fillet bending on the stress state of inner and outer sides is not consider, leads to the calculated stress and load by PSM slightly higher than those of FEM. The solution of stress and load solution by PSM is reliable and convenient, and the FEM predicts more detailed and accurate results but need consume much larger calculating time. This research is of great significance for improving the solution efficient and engineering applications of plastic mechanics in sheet deep drawing deformation.

Keywords: PSM, FEM, sheet drawing, stress distribution, drawing force.

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