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Analytical solutions for crack initiation angle of mixed mode crack in solid material

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1. Introduction

There are a lot of cracks in solids; a crack will propagate under biaxial loading conditions, forming wing cracks [1]. As the crack slips under shear stress, the slip itself causes wing cracks to grow at the crack tip. The study of mixed mode crack initiation and propagation is very important for understanding the mechanical characteristics of solids [2, 3]. The initiation and propagation problem of mixed mode cracks is of great interest in the field of fracture mechanics. To accurately predict and simulate crack propagation paths, a valid criterion needs to be defined for the crack initiation angle. The crack propagation path is influenced by the value of the crack initiation angle [4]. Because the actual loading condition at the crack tip in solid material is very complex, the crack initiation angle and propagation under mixed-mode loading conditions cannot be simplified to either pure mode I or pure mode II analysis.

Erdogan and Sih [5] were the first to study the problem of mixed mode crack initiation angles. They presented the maximum tangential stress (MTS) criterion based on the tangential stress for crack initiation under mixed mode stress conditions. Since then, several criteria have been proposed for the mixed mode crack initiation problem, such as strain energy, stress triaxiality and strain fields [6-8]. These fracture criteria suppose that a plastic core zone exists at the crack tip, where in simplified form most of the plastic core region is a circle with a constant radius and some non-uniform regions are defined based on the mechanical characteristics of the materials. For the MTS criterion [5] and the minimum strain energy density criterion [9], a circle plastic region is assumed to exist at the crack tip. However, the maximum dilatational strain energy criterion (T-criterion) [7] and the modified MTS criterion [10] use a variable plastic region. In fact, the shape of plastic core region is influenced by the microstructure of the materials, because most solid material have nonlinear, discontinuous, anisotropic mechanical characteristics. The shape of the plastic core region computed by an empirical formula cannot reflect the real situation, and numerical methods need to be adopted to simulate the plastic region at the crack tip [11-15].

The main objective of this study is to present a method for mixed mode crack initiation based on a failure criterion. Key issues include: the stress state at the crack tip, computation of stress intensity factors based on crack tip stress and the crack initiation angle. The remainder of the paper is organized as follows. In Section 2, a computational method for determining the stress intensity factors at the crack tip is established based on the crack tip elastic stresses. In Section 3, the minimum J_2 (second invariant variable of the stress deviation) yield function is used to define the variable radius of the plastic core region and crack initiation angle. In Section 4, the crack initiation angle of mixed mode cracks under different loading conditions are simulated, and compared with the simulated results for the MTS and T criteria. Conclusions are presented in Section 5.

2. Stress intensity factors at the crack tip

In this section, the mixed mode cracks are divided into two types: tension cracks and compression-shear cracks. A computational method for determining the stress intensity factors at the crack tip is established based on the crack tip elastic stresses.

2.1. Stress components at the crack tip

The existing research data show that the stress state at a mixed mode crack tip is very complex, depending on the different loading conditions. Mixed mode cracks are divided into two types, combined with the external loading conditions: tension cracks and compression-shear cracks. The crack propagation paths are assumed to be wing cracks. Fig. 1 shows the mixed mode cracks under biaxial loading conditions.

As shown in Fig. 1, a it is assumed that for a crack with an inclination φ under biaxial tensile loading conditions, there is no friction effect at the crack surface. As shown in Fig. 1, b a crack with an inclination φ under biaxial compressive loading condition and the friction effect at the crack surface is considered in this study. The crack length is $2c$, the crack initiation angle is θ, the axial stress is σ_1 and the confining stress is σ_2. r is the radius of one element far from the crack tip, and θ is the crack initiation angle, the stress components are described by σ_r and σ_θ. If the external loading is a tensile stress, it has a positive value; and if the external loading is a compressive stress, it has a negative value.

For a crack under a mixed mode condition, simplifying assumptions are used to analyze the stress state at the crack tip. The linear elastic stress field at the crack tip under biaxial loading condition can be expressed as follows (plane stress problem) [3]:

$$\begin{cases} \sigma_r = \dfrac{1}{2\sqrt{2\pi r}}\left[K_\text{I}(3-cos\theta)cos\dfrac{\theta}{2}+K_\text{II}(3cos\theta-1)sin\dfrac{\theta}{2}\right]; \\ \sigma_\theta = \dfrac{1}{2\sqrt{2\pi r}}cos\dfrac{\theta}{2}\left[K_\text{I}(1+cos\theta)-3K_\text{II}sin\theta\right]; \\ \tau_{r\theta} = \dfrac{1}{2\sqrt{2\pi r}}cos\dfrac{\theta}{2}\left[K_\text{I}sin\theta+K_\text{II}(3cos\theta-1)\right], \end{cases} \quad (1)$$

where σ_r, σ_θ, and $\tau_{r\theta}$ are the radial, circumferential and shear stresses; K_I and K_II are the stress intensity factors of mode I and mode II cracks.

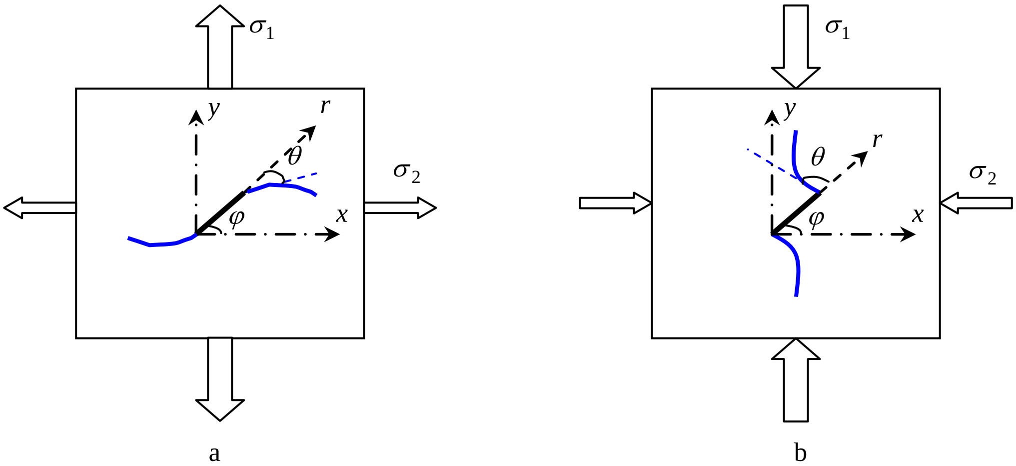


Fig. 1 Mixed mode crack initiation and propagation under biaxial loading conditions: a) a tension crack and b) a compression-shear crack

For a crack in the center of an infinite plane, the crack length is $2c$ and the crack inclination is φ. The stress intensity factors at the crack tip can be represented in a unified form as:

$$K_i = \sigma\sqrt{\pi c}\, f_{K_i}(\varphi) \quad (i = \text{I}, \text{II}), \quad (2)$$

where σ is the external stress, which can be a tensile or compressive stress, and $f_{K_i}(\varphi)$ is a function of the different loading conditions and inclinations φ of the crack.

The stress intensity factors of mode I and mode II cracks for the plane stress problem are [6]:

$$\begin{cases} K_\text{I} = \sigma\sqrt{\pi c}\, f_{K_\text{I}}(\varphi); \\ K_\text{II} = \sigma\sqrt{\pi c}\, f_{K_\text{II}}(\varphi). \end{cases} \quad (3)$$

Combining Eqs. (1)-(3), the stress components can be rewritten as:

$$\begin{cases} \sigma_x = \dfrac{\sigma\sqrt{c}}{\sqrt{2r}} f_x(\theta, f_{K_i}); \sigma_y = \dfrac{\sigma\sqrt{c}}{\sqrt{2r}} f_y(\theta, f_{K_i}); \\ \sigma_\theta = \dfrac{\sigma\sqrt{c}}{\sqrt{2r}} f_\theta(\theta, f_{K_i}); \tau_{xy} = \dfrac{\sigma\sqrt{c}}{\sqrt{2r}} f_{xy}(\theta, f_{K_i}), \end{cases} \quad (4)$$

where $f_x(\theta, f_{K_i})$, $f_y(\theta, f_{K_i})$, $f_\theta(\theta, f_{K_i})$, and $f_{xy}(\theta, f_{K_i})$ are the function of crack inclination and stress intensity factors.

The stress state at the crack tip is very complex and a plastic core region exists at the crack tip where crack initiation and propagation takes place. In the crack initiation and propagation process, the stress intensity factors are key fracture mechanical parameters. The crack is propagated step by step and results in variation of stress intensity factors at the crack tip.

2.2. Stress intensity factor at the tension crack tip

For the mixed mode crack under tensile stress, as shown in Fig. 1, a the tensile loading conditions can be divided into three types: uniaxial tension; biaxial tension and tensile-compressive stress.

When the crack is under the uniaxial tension condition, there is only axial tensile stress σ_1, and the stress intensity factors can be computed as:

$$\begin{cases} K_\text{I} = \sigma_1 cos^2\varphi\sqrt{\pi c}\,; \\ K_\text{II} = \sigma_1 sin\,\varphi\, cos\,\varphi\,, \end{cases} \quad (5)$$

where K_I is the stress intensity factor of a mode I crack, K_II is the stress intensity factor of a mode II crack, φ is the inclination of the crack and c is the half length of the crack.

When the crack is under biaxial tension, the axial stress is σ_1, the confining stress is σ_2, and the relationship between them is assumed to be:

$$\sigma_2 = \lambda\sigma_1 \quad (6)$$

where λ is the ratio of the confining and axial stresses with a range from 0 to 1.

The stress intensity factors under biaxial tension can be computed as:

$$\begin{cases} K_{\mathrm{I}} = \sigma_1 \left(cos^2\varphi + \lambda sin^2\varphi \right) \sqrt{\pi c}\ ; \\ K_{\mathrm{II}} = \sigma_1 \left(1-\lambda \right) sin\varphi cos\varphi \sqrt{\pi c}\ . \end{cases} \quad (7)$$

When the crack is under tensile-compressive stress, it is assumed that the axial stress is a tensile stress, while the confining stress is a compressive stress. The relationship between the axial and confining stresses is assumed to be:

$$\sigma_2 = -\lambda \sigma_1 , \quad (8)$$

where λ is the ratio of the confining and axial stresses also with a range from 0 to 1.

The stress intensity factors under tensile-compressive stress can be computed as:

$$\begin{cases} K_{\mathrm{I}} = \sigma_1 \left(cos^2\varphi - \lambda sin^2\varphi \right) \sqrt{\pi c}\ ; \\ K_{\mathrm{II}} = \sigma_1 \left(1+\lambda \right) sin\varphi cos\varphi \sqrt{\pi c}\ . \end{cases} \quad (9)$$

For the stress intensity factors at the crack tip, we defined a scale factor η between mode I and model II stress intensity factors as:

$$\eta = K_{\mathrm{I}} / K_{\mathrm{II}} \quad (10)$$

Then the scale factor under different external loading condition can be computed as:

$$\eta = cos\varphi / sin\varphi \qquad \text{(Uniaxial tension)} \qquad (11,\ a)$$

$$\eta = \left(cos^2\varphi + \lambda sin^2\varphi \right) \Big/ \left[\left(1-\lambda \right) sin\varphi cos\varphi \right] \qquad \text{(Biaxial tension)} \qquad (11,\ b)$$

$$\eta = \left(cos^2\varphi - \lambda sin^2\varphi \right) \Big/ \left[\left(1+\lambda \right) sin\varphi cos\varphi \right] \qquad \text{(Tensile-compressive stress)} \qquad (11,\ c)$$

The scale factor for different external loading condition will be used for the crack initiation analysis process. It has an important impact on the crack propagation path.

2.3. Stress intensity factor at compression-shear crack tip

According to the mixed mode crack under compressive stress, as shown in Fig. 1, b the compressive loading conditions can be divided into two types: uniaxial compression and biaxial compression. Here, the friction effect at the crack surface is considered for the crack initiation and propagation problem for a compression-shear crack.

When the crack is under uniaxial compression, there is only an axial compressive stress σ_1, and the normal and shear stresses at the crack surface are computed as follows [16]:

$$\begin{cases} \sigma = \sigma_1 cos^2\varphi\ ; \\ \tau = \sigma_1 sin\varphi cos\varphi\ , \end{cases} \quad (12)$$

where σ is the normal stress at the crack surface and τ is the shear stress at the crack surface. The effective shear stress is influenced by the mechanical characteristics of the crack surface, can be computed as:

$$\tau^{eff} = \sigma_1 sin\varphi cos\varphi + \tau_c - \mu \sigma_1 cos^2\varphi , \quad (13)$$

where τ_c is the cohesion of the crack surface and μ is its friction coefficient.

Here, the cohesion of the crack surface τ_c is assumed equal to 0 and is not considered for the shear stress at the crack surface, and the effective shear stress is then:

$$\tau^{eff} = \sigma_1 sin\varphi cos\varphi - \mu \sigma_1 cos^2\varphi . \quad (14)$$

The stress intensity factors can be computed as:

$$\begin{cases} K_{\mathrm{I}} = \sigma_1 cos^2\varphi \sqrt{\pi c}\ ; \\ K_{\mathrm{II}} = \left(\sigma_1 sin\varphi cos\varphi - \mu \sigma_1 cos^2\varphi \right) \sqrt{\pi c}\ . \end{cases} \quad (15)$$

When the crack is under biaxial compression, the axial compressive stress is σ_1 and the confining stress is σ_2, and the normal and shear stresses at the crack surface are computed as:

$$\begin{cases} \sigma = \sigma_1 cos^2\varphi + \sigma_2 sin^2\varphi\ ; \\ \tau = \left(\sigma_1 - \sigma_2 \right) sin\varphi cos\varphi\ . \end{cases} \quad (16)$$

Let the cohesion of the crack surface also be 0, the effective shear stress at the crack surface is:

$$\tau^{eff} = \left(\sigma_1 - \sigma_2 \right) sin\varphi cos\varphi - \mu \left(\sigma_1 cos^2\varphi + \sigma_2 sin^2\varphi \right) . \quad (17)$$

The ratio of confining and axial stresses is introduced so that:

$$\sigma_2 = \lambda \sigma_1 , \quad (18)$$

where λ is the ratio between the confining and axial stresses with the range from 0 to 1.

The stress intensity factors can be computed as:

$$\begin{cases} K_{\mathrm{I}} = \sigma_1 \left(cos^2\varphi + \lambda sin^2\varphi \right) \sqrt{\pi c}\ ; \\ K_{\mathrm{II}} = \begin{pmatrix} \sigma_1 \left(1-\lambda \right) sin\varphi cos\varphi - \\ -\mu \sigma_1 \left(cos^2\varphi + \lambda sin^2\varphi \right) \end{pmatrix} \sqrt{\pi c}\ . \end{cases} \quad (19)$$

For a mixed mode crack under compression, the friction effect is considered in this study, and the crack initiation and propagation process is influenced by the shear strength of the crack surface. The crack propagation is limited by the shear strength of the crack surface where the crack closes under compression, but opens under tension. The crack propagates more easily under tension than under compression.

3. Analytical solution for crack initiation of mixed mode crack

For a mixed mode crack under biaxial loading, there is a plastic region around the crack tip and crack propagation will occur with the minimum elastic fracture energy, as shown in Fig. 2. For the plane stress problem, the minimum J_2 (second invariant variable of the stress deviation) yield function is used to define the variable radius of the plastic core region.

In the plastic core region, crack propagation will occur when the fracture energy is at a minimum. For the crack initiation angle, it is assumed that the minimum angle which will allow the plastic core radius to form is when the minimum J_2 (second invariant variable of the stress deviation) yield function is used to define the crack initiation angle.

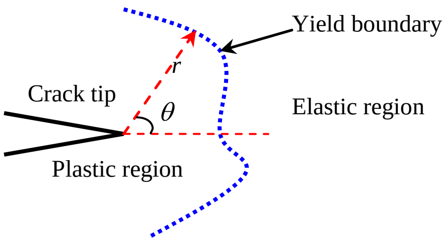


Fig. 2 Plastic core region at the crack tip related to crack initiation and propagation

The crack initiation will happen when J_2 is in the minimum direction, because J_2 can reflect the real stress state at the crack tip, and the volume strain energy density is larger than the shape change energy density. The crack initiation angle is determined by:

$$\frac{\partial J_2}{\partial \theta} = 0, \tag{20}$$

where the second invariant variable of the stress deviation can be computed as follow,

$$J_2 = \frac{1}{6}\left[\left(\sigma_x - \sigma_y\right)^2 + \left(\sigma_y - \sigma_z\right)^2 + \left(\sigma_z - \sigma_x\right)^2\right] + \\ + \left(\tau_{xy}^2 + \tau_{yz}^2 + \tau_{zx}^2\right). \tag{21}$$

Moreover, the second invariant variable of the stress deviation J_2 should be at the minimum value:

$$\frac{\partial^2 J_2}{\partial \theta^2} > 0 \quad \text{and} \quad J_{2min} = J_2\left(\theta_0\right). \tag{22}$$

Combining Eqs. (20) and (21), we then obtain:

$$12K_{\mathrm{I}}K_{\mathrm{II}}cos\left(2\theta\right) + 3\left(K_{\mathrm{I}}^2 - 3K_{\mathrm{II}}^2\right)sin\left(2\theta\right) - \\ -4K_{\mathrm{I}}K_{\mathrm{II}}cos\,\theta - 2\left(K_{\mathrm{I}}^2 - K_{\mathrm{II}}^2\right)sin\,\theta = 0. \tag{23}$$

The crack initiation angle θ_0 can be solved by Eq. (23), and the computed results for the crack initiation angle should also satisfy Eq. (22).

If we know the stress intensity factors at the tip of mixed mode crack, the crack initiation angle can be determined based on the failure criterion of solid material. The presented method can compute the crack initiation angle for the simple mode I or mode II crack problem.

When the crack is a mode I fracture, the stress intensity factor of K_{II} is equal to 0, and the Eq. (23) can be simplified as:

$$3K_{\mathrm{I}}^2 sin\left(2\theta\right) - 2K_{\mathrm{I}}^2 sin\theta = 0. \tag{24}$$

The solution results of Eq. (32) are:

$$\theta_0 = 0 \quad \text{and} \quad \theta_0 = \pm arccos\,\frac{1}{3}. \tag{25}$$

The crack initiation angle θ_0 should also satisfy Eq. (22), then:

$$\begin{cases} \left.\dfrac{\partial^2 J_2}{\partial \theta^2}\right|_{\theta_0 = \pm arccos\frac{1}{3}} = -\dfrac{16K_I^{\,2}}{3} < 0; \\ \left.\dfrac{\partial^2 J_2}{\partial \theta^2}\right|_{\theta_0 = 0} = 4K_I^{\,2} > 0. \end{cases} \tag{26}$$

So the crack initiation angle for the mode I fracture is:

$$\theta_0 = 0 \tag{27}$$

The analysis result is the same for several classic fracture criterions, such as MTS, T and other criteria.

When the crack is a mode II fracture, the stress intensity factor of K_{I} is equal to 0, and Eq. (23) can then be simplified as:

$$2K_{\mathrm{II}}^2 sin\,\theta - 9K_{\mathrm{II}}^2 sin\left(2\theta\right) = 0. \tag{28}$$

The solution results for Eq. (36) are:

$$\theta_0 = 0 \quad \text{and} \quad \theta_0 = \pm arccos\,\frac{1}{9}. \tag{29}$$

The crack initiation angle θ_0 should also satisfy Eq. (22), then:

$$\begin{cases} \left.\dfrac{\partial^2 J_2}{\partial \theta^2}\right|_{\theta_0 = \pm arccos\frac{1}{9}} = \dfrac{160K_{\mathrm{II}}^2}{9} > 0; \\ \left.\dfrac{\partial^2 J_2}{\partial \theta^2}\right|_{\theta_0 = 0} = -16K_{\mathrm{II}}^2 < 0. \end{cases} \tag{30}$$

So the crack initiation angle for the mode II fracture is:

$$\theta_0 = \pm 83.602\ 6^{\circ}. \tag{31}$$

The crack initiation angle of the mode II fracture is 70.53° computed using the maximum circumferential stress fracture criterion, and the crack initiation angle of the mode II fracture is -70.53° computed using the maximum tensile stress fracture criterion II. The simulation results computed by this method are larger than for other classic fracture criteria, but the computing error is very small.

4. Simulated results of crack initiation angle

In this section, the crack initiation angle of a mixed mode crack under different loading conditions is simulated, and compared with the simulated results for the MTS and T criteria. The crack initiation angle influenced by the loading ratio between the confining and axial stresses, and the friction effect at the crack surface under compression, are analyzed. The crack propagation characteristics under different loading conditions are presented in the end.

4.1. Tension crack

According to the mixed mode crack under tensile stress, for the plane stress problem, the scale factor η between mode I and mode II stress intensity factors is introduced in Eq. (23), giving:

$$12\eta\cos(2\theta)+3(\eta^2-3)\sin(2\theta)-$$
$$-4\eta\cos\theta-2(\eta^2-1)\sin\theta=0\,. \qquad (32)$$

For the mixed mode crack under uniaxial tensile loading, the loading ratio λ is 0 and Eq. (11, a) is substituted into Eq. (32), and a function for the crack inclination and crack initiation angle can then be obtained. Table 1 shows the computed results for the crack initiation angle for different crack inclinations under uniaxial tensile loading.

Table 1

Compute results of the crack initiation angle for different crack inclinations under uniaxial tensile loading

No.	1	2	3	4	5	6	7	8	9	10
Crack inclination, °	0.000	10.000	20.000	30.000	40.000	50.000	60.000	70.000	80.000	90.000
Crack initiation angle, °	0.000	-18.486	-31.798	-41.490	-49.470	-59.660	-63.486	-69.839	-76.857	-83.621
K_I / K_{II}	none	5.6713	2.7475	1.7321	1.1918	0.8391	0.5774	0.3740	0.1763	0.0000

Fig. 3 shows the crack initiation angle for a mixed mode crack under uniaxial tensile loading using different fracture criteria.

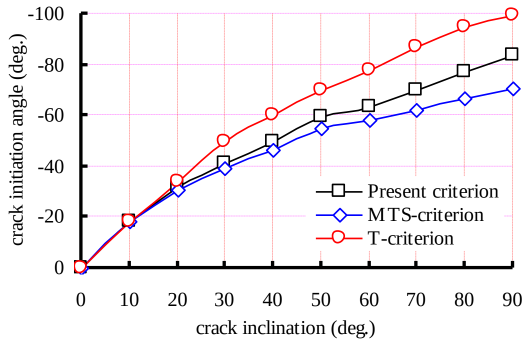


Fig. 3 Crack initiation angle for a mixed mode crack under uniaxial tensile loading

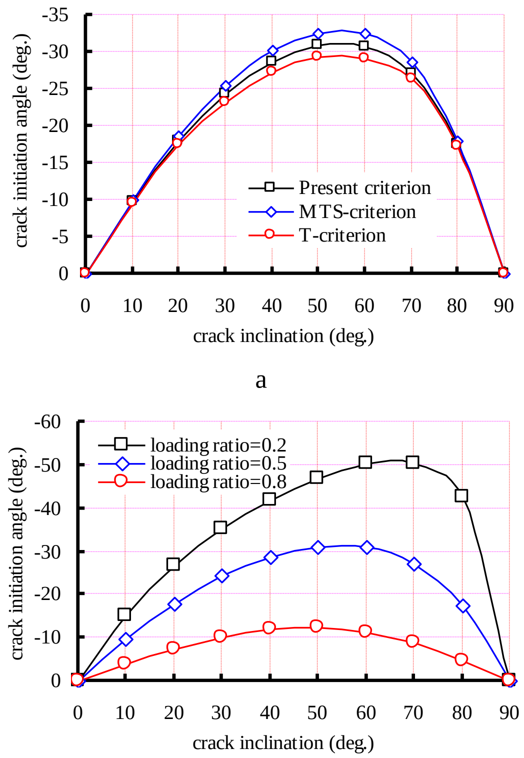
As shown in Fig. 3, the computed results for the crack initiation angle under uniaxial tensile loading by the presented criterion is less than the T-criterion and larger than the MTS-criterion. The crack initiation angle increases with increasing crack inclination. The relationship between the crack initiation angle and crack inclination is a nonlinear function, and can be simplified into a linear relationship as:

$$\theta_0+\varphi\cong 0°, \qquad (33)$$

where φ is the crack inclination and θ_0 is the crack initiation angle.

From Fig. 1, a we observe that for the mixed mode crack under uniaxial tensile loading, the crack initiation angle is perpendicular to the axial loading direction.

For the mixed mode crack under biaxial tensile loading, the loading ratio λ is given as 0.2, 0.5 and 0.8, and substituting Eq. (11, b) into Eq. (32), we then obtain a function for the crack inclination and crack initiation angle. Fig. 4, a shows the compute results for the crack initiation angle for different crack inclinations under biaxial tensile loading when the loading ratio is 0.5. Fig. 4, b shows the compute results of crack initiation angle for different crack inclinations under different biaxial tensile loading.

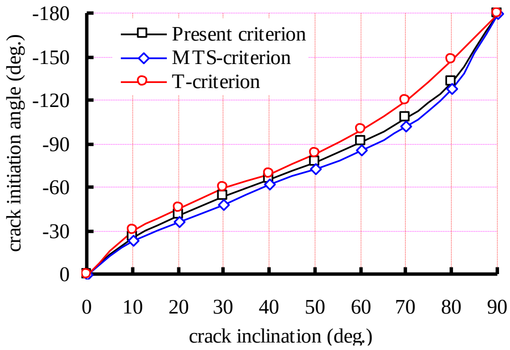


b

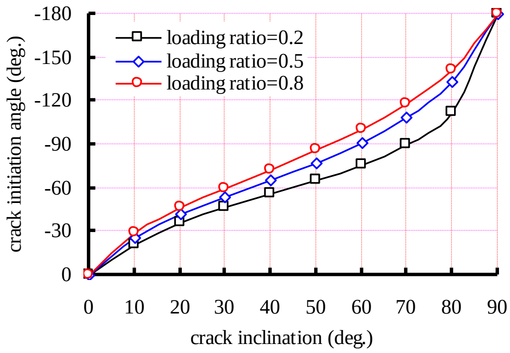
Fig. 4 Computed results for the crack initiation angle for different crack inclinations under biaxial tensile loading: a) loading ratio is 0.5 and b) loading ratio impact on the crack initiation angle

As shown in Fig. 4, a for the mixed mode crack under the loading ratio of 0.5 for biaxial tensile stress, the crack initiation angle first increases and then decreases with the increasing of crack inclination, and the error between the presented criterion and the MTS and T criteria is very small. The crack initiation angle is influenced by the external loading pattern. As shown in Fig. 4, b the crack initiation angle decreases with increasing loading ratio, and the crack propagation is restrained by the lateral tensile stress. The crack is easy to propagate under biaxial tensile loading, while with increasing lateral tensile stress, the crack propagation is progressively more difficult. If the lateral tensile stress is equal to the axial tensile stress, the crack ceases to propagate.

For the mixed mode crack under tensile-compressive loading, the loading ratio λ is set at 0.2, 0.5 and 0.8, and substituting Eq. (11, c) into Eq. (32), we then obtain a function for the crack inclination and the crack initiation angle. Fig. 5, a shows the compute results for the crack initiation angle for different crack inclinations under tensile-compressive loading when the loading ratio is 0.5. Fig. 5, b shows the compute results for the crack initiation angle for different crack inclinations under different tensile-compressive loading conditions.



a



b

Fig. 5 Computed results for the crack initiation angle for different crack inclinations under tensile-compressive loading: a) loading ratio is 0.5 and b) loading ratio impact on the crack initiation angle

As shown in Fig. 5, a for the mixed mode crack under a loading ratio of 0.5 for the tensile-compressive stress condition, the crack initiation angle increases with increasing crack inclination. The error between the presented criterion and the MTS and T criteria is very small. As shown in Fig. 5, b the crack initiation angle increases with increasing loading ratio, and the crack propagation is accelerated by the lateral compressive stress. With increasing lateral tensile compression, the crack propagation accelerates.

4.2. Compression-shear crack

For the mixed mode crack under a compressive loading condition, the fracture type is a compressive-shear crack. A scale factor η between mode I and mode II stress intensity factor is defined for the compressive-shear crack:

$$\eta = \frac{f_{K_I}(\varphi,\mu)}{f_{K_I}(\varphi,\mu)} = \frac{K_I}{K_{II}}. \quad (34)$$

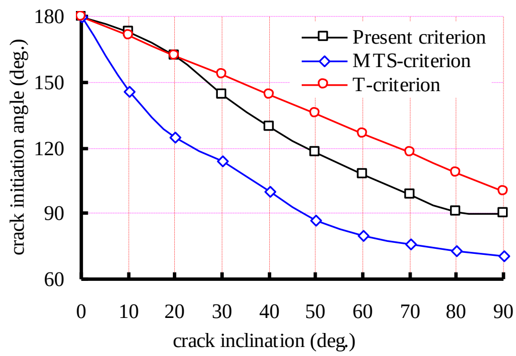
When the compressive-shear cracks under uniaxial compression, the stress intensity factors at the crack tip are:

$$\begin{cases} f_{K_I}(\varphi,\mu) = cos^2\varphi; \\ f_{K_I}(\varphi,\mu) = sin\varphi cos\varphi - \mu cos^2\varphi. \end{cases} \quad (35)$$

When the compressive-shear cracks under biaxial compression, the stress intensity factors at the crack tip are:

$$\begin{cases} f_{K_I}(\varphi,\mu) = cos^2\varphi + \lambda sin^2\varphi; \\ f_{K_I}(\varphi,\mu) = (1-\lambda) sin\varphi cos\varphi - \mu\left(cos^2\varphi + \lambda sin^2\varphi\right). \end{cases} \quad (36)$$

For the mixed mode crack under uniaxial compression, substituting Eq. (35) into Eq. (32), we then obtain a function for the crack inclination and crack initiation angle. Fig. 6, a shows the compute results for the crack initiation angle for different crack inclinations under uniaxial compression when the friction coefficient of the crack surface is 0.2. Fig. 6, b shows the compute results for the crack initiation angle for different crack inclinations under different friction coefficients for the crack surface.



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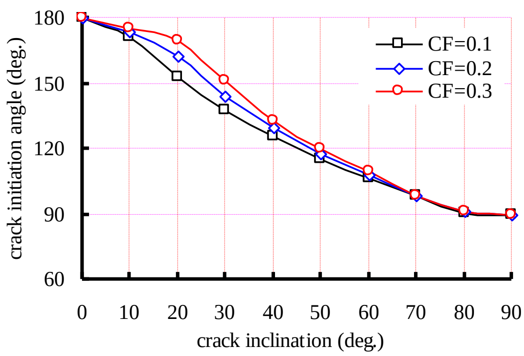


b

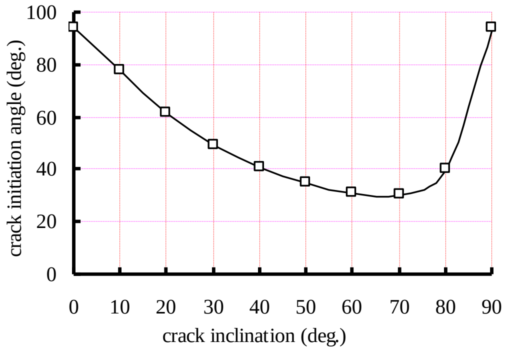
Fig. 6 Computed results for the crack initiation angle for different crack inclinations under uniaxial compression: a) friction coefficient of the crack surface is 0.2 and b) friction coefficient of the crack surface impact on the crack initiation angle (CF is the friction coefficient of the crack surface)

As shown in Fig. 6, a there is a great difference between different fracture criteria for mixed mode cracks, and the crack initiation angle decreases with increasing crack inclination, following a nonlinear relationship. As shown in Fig. 6, b the crack initiation angle is obviously influenced by the crack friction coefficient. The crack surface friction prevents the crack propagating, and it increases as the crack friction coefficient increases while the other parameters remain constant.

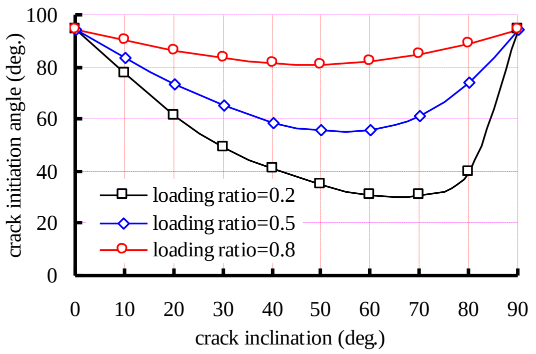
The relationship between the crack initiation angle and crack inclination can be described by a linear function:

$$\theta_0 + \varphi \cong 180°. \tag{37}$$

For the mixed mode crack under biaxial compression, substituting Eq. (36) into Eq. (32), we then obtain a function for the crack inclination and the crack initiation angle. Fig. 7, a shows the computed results for the crack initiation angle for different crack inclinations under biaxial compression when the friction coefficient of the crack surface is 0.2, and the loading ratio is also 0.2. Fig. 7, b shows the computed results for the crack initiation angle for different crack inclinations under different loading ratios, λ of 0.2, 0.5 and 0.8.



a



b

Fig. 7 Computed results for the crack initiation angle for different crack inclinations under biaxial compression: a) loading ratio is 0.2 and b) loading ratio impact on the crack initiation angle

As shown in Fig. 7, a for the mixed mode crack under a loading ration of 0.2 for the biaxial compression condition, the crack initiation angle first decreases and then increases with the increasing crack inclination. The crack initiation angle is influenced by the external loading pattern. As shown in Fig. 7, b, the crack initiation angle decreases with increasing loading ratio, and the crack propagation is restrained by the lateral compressive stress. The crack propagation is limited by the lateral stress under biaxial compression.

When the crack inclination is less than about 70°, the relationship between the crack initiation angle and crack inclination can be described by a linear function:

$$\theta_0 + \varphi \cong 90°. \tag{38}$$

The crack initiation angle of a mixed mode crack is influenced by the external loading pattern, crack inclination, and the crack friction effect.

5. Conclusions

Cracks propagate forming wing cracks under biaxial loading conditions, and the study of mixed mode crack initiation and propagation is very important for understanding the mechanical characteristics of solids. The stress state at the crack tip is very complex. A plastic core region exists at the crack tip, and the crack initiation and propagation will occur in this region. In the crack initiation and propagation process, the stress intensity factors are key fracture mechanical parameters. A computational method for determining the stress intensity factors at the crack tip is established based on the crack tip elastic stresses. The stress intensity factors at the tension crack tip are computed with three tensile loading conditions: a. uniaxial tension; b. biaxial tension; c. tensile-compressive stress. The stress intensity factors at the compressive-shear crack tip are computed with two tensile loading conditions: a. uniaxial compression; b. biaxial compression. The friction effect at the crack surface is considered for the crack initiation problem for the compression-shear crack.

A plastic core region exists at the crack tip, the minimum J_2 (second invariant variable of the stress deviation) yield function is used to define the variable radius of the plastic core region. The crack initiation angle of a mixed mode crack under different loading condition was simulated. The computed results show that the crack initiation angle under uniaxial tensile loading using the presented criterion is less than for the T criterion and larger than for the MTS criterion. The crack initiation angle is influenced by the loading ratio between the confining and axial stresses, and the friction effect at the crack surface under compression.

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KIETOJO KŪNO MIŠRAUS PAVIDALO PLYŠIO PRADŽIOS KAMPO ANALITINIS NUSTATYMAS

R e z i u m ė

Esant dviejų ašių kryptimis veikiančiai apkrovai plyšys lėktuvo sparne vystosi kaip įtrūkimas su plastiškos šerdies sritimi jo viršūnėje. Šiame straipsnyje skiriami dviejų rūšių mišraus pavidalo plyšiai: tempimo įtrūkimai bei gniuždymo ir šlyties įtrūkimai. Pasiūlytas metodas įtempių intensyvumo koeficientui nustatyti esant skirtingoms tempimo ir gniuždymo apkrovoms, pagrįstas plyšio viršūnės tampriaisiais įtempiais. Apskaičiuoti įtempių intensyvumo, esant skirtingoms tempimo ir gniuždymo apkrovoms, koeficientai. Nagrinėjamas trinties poveikis plyšio radimosi pradžiai bei gniuždymo ir šlyties plyšio plitimui. Valkšnumo funkcija J_2 (įtempio deviacijos antrasis pastovusis kintamasis) naudojama mišraus pavidalo plyšio pradžios kampui nustatyti. Skaičiavimo rezultatai rodo, kad plyšio pradžios kampui įtakos turi plyšio kryptis, apkrovos būdas, dydis ir plyšio paviršiaus trinties koeficientas.

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ANALYTICAL SOLUTIONS FOR CRACK INITIATION ANGLE OF MIXED MODE CRACK IN SOLID MATERIAL

S u m m a r y

A crack will propagate under biaxial loading conditions forming wing cracks, where there is a plastic core region at the crack tip. In this paper, mixed mode cracks are divided into two types: tension cracks and compression-shear cracks. A computational method for determining the stress intensity factors at the crack tip is established based on the crack tip elastic stresses. The stress intensity factors under different tensile and compressive loading conditions are computed. The friction effect at the crack surface is considered for the crack initiation and propagation problem of a compression-shear crack. The minimum J_2 (second invariant variable of the stress deviation) yield function is used to determine the crack initiation angle of a mixed mode crack. Computed results show that the crack initiation angle is influenced by the crack inclination, loading pattern, loading ratio and the friction effect of crack surface.

Keywords: mixed mode crack, fracture, crack initiation, failure criteria, plastic core region.

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