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Analysis of nonlinearity of the turbine gas meters time constant during step response

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1. Introduction

Turbine gas meters (TGM) are widely used for natural gas accounting. These meters belong to the tachometric flow meter class. The characteristic particularity of their design is the presence of the rotating parts which rotation speed is associated with the flow rate. Operation under unsteady flow conditions strongly impacts the metrological characteristics of the meter. Due to inertia of the meters, response slows down and the so-called dynamic error of measurement appears. The effect of alternating flow on the behaviour of different type tachometric flow meters is similar, because in all these cases rotation occurs due to flow effects. However, because of certain differences of operation principle, the effect should not be completely identical.

As to TGM, this problem has been studied for the last 80 years. One of the first researches of the TGM dynamic response was published [1]. Further investigation has been continued [2-5]. In [2] authors received analytical solution for TGM behaviour in flow, pulsing by rectangular law, which does not occur in practice. The most known are results of K.N. Atkinson [3], which have been got for flow, pulsing by sine law and are included in [6]. Similar results have been reported in [4]. All these results are obtained from the solution of the differential equation for rotation of the TGM rotor in unsteady flow. Used equations encompass a number of difficult to evaluate parameters, therefore the results are fragmentary and often poorly coincide with each other.

In practice, according to [5], flow pulses by complex laws. This factor complicates the boundary conditions of the differential equation of TGM rotation and impedes the numerical simulation. So far, such attempts have not been successful.

Recently new method for mathematical modelling of TGM behaviour in pulsing flow has been proposed and implemented in [7]. The method is applicable for any arbitrary given law of flow pulsation. The method defines TGM rotor's rotation inertia time constant T as the main and the only parameter which characterizes TGM behaviour in pulsing flow.

Time constant T sometimes is also called the inertia index. It characterizes response of systems, which operates in alternating mode and the input parameter is changing. This change is described by the first order linear differential equation [8, 9] with such expression:

$$T\frac{d\,y(t)}{d\,t}+y(t)=k\times u(t), \tag{1}$$

where $y(t)$ is response; $u(t)$ input; k is conversion factor.

There are many systems which correspond to Eq. (1), e. g:

- RC or RL circuits, where $u(t)$ is input voltage, $y(t)$ is output voltage;
- mechanical rotation system, $u(t)$ is the applied force, $y(t)$ is speed;
- thermal system, $u(t)$ is ambient temperature, $y(t)$ is solid body temperature.

TGM also belongs to such systems as many other meters of various physical quantities. For TGM input is a gas flow rate Q and response is the rotor rotation frequency ω.

Experiment is the most accurate and convenient way to evaluate the time constant. The prevalent experimental method for meters is based on the measurement of the meter response to step (sharp) input change.

In this case, the response of the meter changes exponentially:

$$y(t)=y(0)\times e^{-\frac{t}{T}}. \tag{2}$$

For TGM, as well as in many other cases, it is convenient to use the excess response, i.e. difference between the values of the current and the initial value of the response [7]:

$$\Omega=\frac{\omega-\omega_f}{\omega_{in}-\omega_f}=e^{-\frac{t}{T}}, \tag{3}$$

where Ω is dimensionless response, ω_{in}, ω_f are respectively initial and final TGM rotation frequency, s^{-1}, t is time, s; T is a time constant, s.

From the dependence (3), obtained experimentally, it is not difficult to evaluate a time constant value of TGM. T is equal to time t at which dimensionless response reaches 36.8% of its initial value [8]. Actually, this definition of time constant stays valid only when T value remains constant for all duration of the process under consideration. Otherwise, equation (1) becomes nonlinear, and equation (3) is applicable only at a sufficiently small time interval within which the value of T can be considered unchanged [7]. Exactly such a situation occurs considering TGM. When TGM operate in pulsing flow, time constant changes substantially during the response, and, therefore, this process is essentially nonlinear.

In [3] expression of the following type for evaluation of TGM time constant has been suggested:

$$T = \frac{C}{Q}, \tag{4}$$

where C is factor, which depends on the rotor inertia, m^3, Q is the actual value of the flow rate, m^3/s. The constant C is evaluated from the following dependence:

$$C = \frac{(1+\eta)J}{\rho r^2}, \tag{5}$$

where ρ is gas (fluid) density, kg/m^3; r is equivalent radius of the rotor, m; J is moment of inertia of the rotor, $kg\ m^2$; η is a dimensionless flow rate factor.

Similar expression has been proposed in [7].

In [10, 11] the value of tachometric flow meters' time constant is considered as an unchangeable during the step response process.

However, the experiments showed that the time constant depends not only from the actual flow rate, but also on other factors, first of all, from current TGM rotor's rotation frequency.

Considering the utmost importance of time constant for evaluation dynamic error of tachometric meters, first of all, TGM, this study was aimed at clarification of TGM time constant dependence on all influencing factors

2. Theoretical basis of the experimental method

A special numerical method has been developed to evaluate the TGM time constant dependence on the current rotation parameters. The main features of the method are the following:

1. The time constant is determined in accordance with Eq. (3). Eq. (3) is submitted in logarithmic form:

$$ln(\Omega) = -\frac{t}{T}. \tag{6}$$

2. The experimentally determined dimensionless rotation frequency changing in time during the step response is being approximated by 6th degree polynomial (the approximation by power polynomial of any other degree is possible, if it agrees better with the experimental data).

$$ln(\Omega) = a_1 t + a_2 t^2 + \ldots + a_6 t^6, \tag{7}$$

where a^n is an approximation coefficient, s^{-n}; t^n is time, s^n.

3. Since the left sides of Eqs. (6) and (7) are the same, their right sides are equal. This yields the expression for time constant:

$$T = -\frac{1}{a_1 + a_2 t + \ldots + a_n t^{n-1}}. \tag{8}$$

Using Eq. (8) time constant of TGM can be calculated for any moment of the response process.

This method can be used for evaluation of time constant of any type tachometric flow meters including cup anemometer, as well as in many other cases.

3. Experimental technique

The experimental part of the work consisted of measuring the turbine gas meter response to step flow rate change. The facility which in sufficiently short time enables to transfer the meter from the measuring line with the initial flow rate value into the line with the other, final flow rate value, has been used. The facility was earlier developed and described in [7]. The improved method of determination of the current meter's rotor rotation frequency has been applied. The frequency has been evaluated by measuring the time interval between two consecutive high frequency pulses from the meter under test.

Turbine gas meter with measurement range (0.014 - 0.278) m^3/s has been used for the tests.

4. Results and discussion

The dependence of the dimensionless meter's response in time in logarithm form for the case of the frequency decaying is presented in Fig. 1. This kind of representation of the results enables to understand the nature of response. A straight line means that the response varies with time exponentially with the constant exponent, and the time constant remains unchangeable. The emergence of line curvature means that the exponent and time constant are changing. Thus, if in the certain part of the curve the curvature has a bulge down the exponent in this area is decreasing, and the time constant is increasing, in the case of bulge upwards - on the contrary.

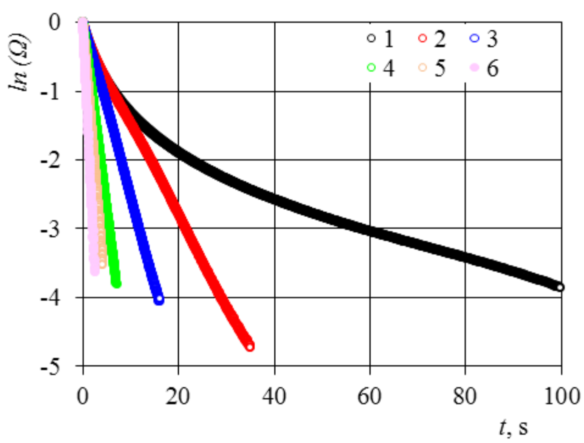
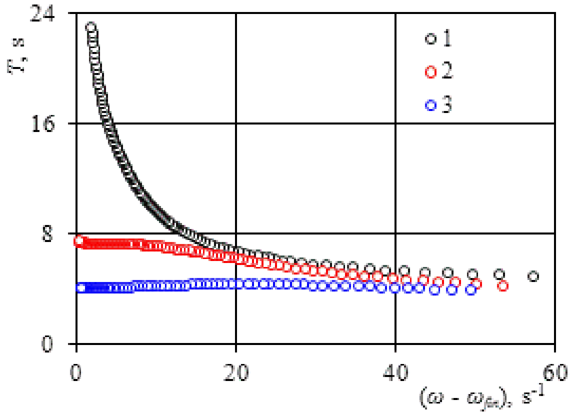


Fig. 1 Dimensionless meter's step response. Initial flow rate 0.194 m^3/s; *1 ÷ 6* final flow rate respectively 0; 0.014; 0.028; 0.055; 0.083; 0.139 m^3/s

More detailed analysis of the meter behaviour during response process is possible by evaluating of time constant changing with procedure, described above.

More detailed analysis of the meter behaviour during response process is possible by evaluating time constant changing with the procedure, described above. The dependence of time constant on current excessive meter's rotor rotation frequency is presented in Fig. 2, a for small values of the final flow rate and in Fig. 2, b for the bigger values. In the second case, the results are presented both for increasing and decreasing of rotation frequency.

Time constant significantly changes its value during the response process. Dependence of time constant is essentially nonlinear. This fact does not correspond substantially to the majority of existing views on the behaviour of this parameter.



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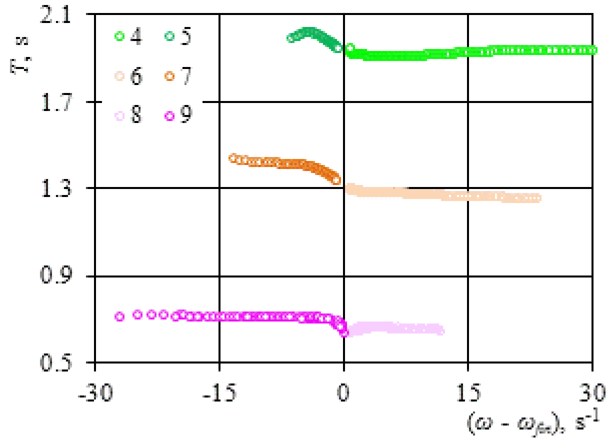


b

Fig. 2 Dependence of time constant on current excessive meter's rotor rotation frequency with different final flow rates, initial flow rate 0.194 m^3/s. a) *1 ÷ 3* - final flow rates 0; 0.014; 0.028 m^3/s respectively; b) *4 ÷ 6* - final flow rates 0.055; 0.083; 0.139 m^3/s respectively, increasing of rotation frequency; *7 ÷ 9* - the same final flow rates, decreasing of rotation frequency, initial flow rate 0.014 m^3/s

With the decrease of final flow rate with the same value of this parameter the average time constant value is greater in the case of increasing rotation frequency than in the case of decreasing frequency.

Rate of time constant change grows when final flow rate value is nearby the meter's lower measurement limit, especially with the current rotation frequency decreasing. In this case time constant value increases many times.

Impermanence of time constant should substantially affect the value of the dynamic error, especially when under significant flow pulsing amplitude, actual flow rate value falls below lower limit of measurements.

Of course, details of the behaviour of the meter in the process of response depend on the features of the meter design.

In Fig. 3 the dependence of time constant on current excessive meter's rotor rotation frequency with different initial flow rates is presented.

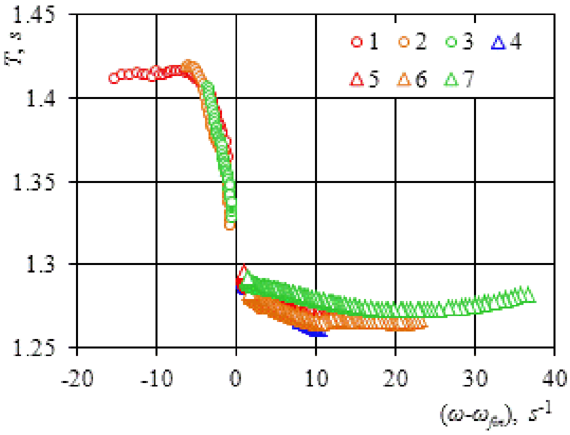


Fig. 3 Time constant dependence on current excessive meter's rotor rotation frequency with different initial flow rates, final flow rate 0.055 m^3/s. *1 ÷ 7* – initial flow rates 0,0139; 0.0147; 0.056; 0.139; 0.167; 0.194; 0.222 m^3/s respectively

Curves of time constant change during response for various initial flow rates coincide with each other within the accuracy of the experiment. Thus, the initial flow rate does not affect the time constant.

Dependence of time constant for sufficiently small values of excess rotation frequency, or frequency, which correspond to the final flow rate, is presented in Fig. 4.

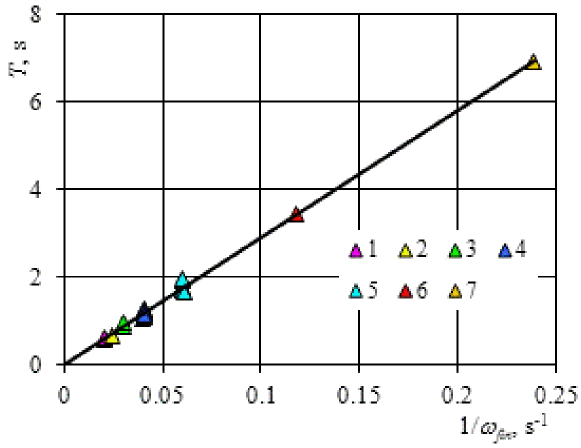


Fig. 4 Dependence of TGM time constant to small enough current excessive meter's rotor rotation frequency values. *1 ÷ 7* –final flow rates 0.167; 0.139; 0.111; 0.083; 0.056; 0.0278; 0,0139 m^3/s respectively

All experimental points agree as well on the line passing through the zero line of time constant depending on the reciprocal of the frequency.

Therefore, the time constant obeys to (3), but only in one particular case when current meter rotation frequency and frequency which corresponds the current flow rate.

Similar results earlier have been obtained by us for another tachometric meter - cup anemometer [11]. The results are presented in Fig. 5.

Newly obtained data on time constant provide opportunity to refine the results of calculation tachometric meters dynamic error.

Up to now, such calculations using method, proposed in [7] have been performed for cup anemometer

[12]. Some results are presented in Fig. 6, a (taking into consideration the variability of the time constant in the process of step response) and Fig. 6, b (time constant is unchangeable during step response).

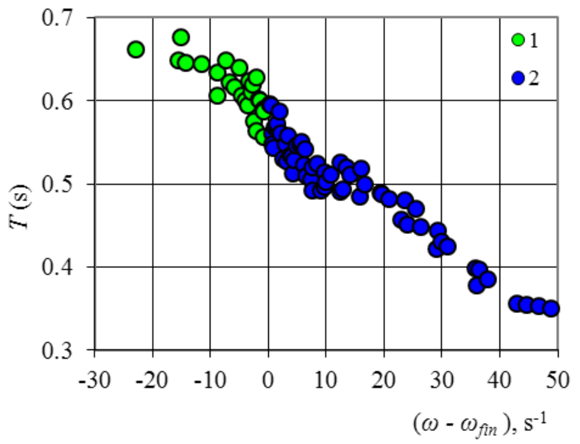
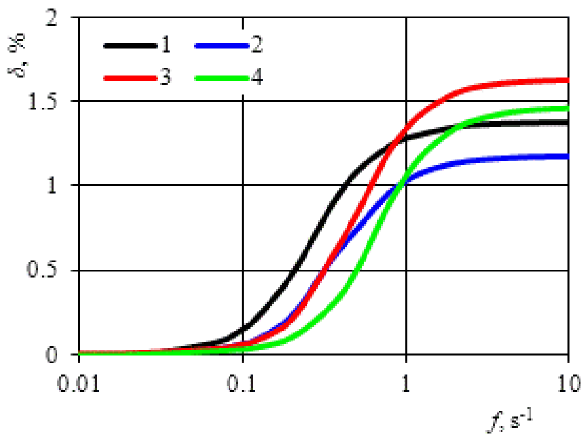


Fig. 5 Time constant dependence on current excessive rotation frequency for cup anemometer, final flow velocity 7.5 m/s. initial flow velocities: *1* ÷ 2.5; *2* ÷ 20 m/s



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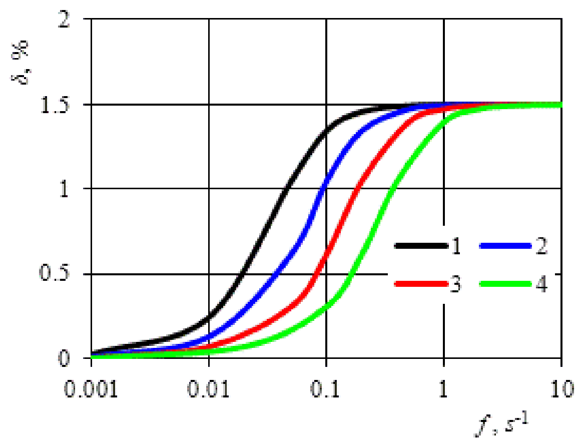

Fig. 6 Dynamic error of the cup anemometer: a) variable time constant; b) fixed time constant. *1 ÷ 4* – average flow velocity 7.5; 10; 12.5; 15 m/s respectively

The results significantly differ both quantitatively and qualitatively. The main differences are the following:

1. Points of transition (the values of flow pulsation frequency) are different.

2. The limit values of dynamic error are different.

3. In the first case, the maximum error depends not only on pulsation amplitude, but also on the flow velocity.

4. In the first case the error curves for different speeds intersect each other.

Non-linearity of time constant is necessary to take into account when calculating the dynamic error of tachometric flow meters.

5. Conclusions

1. Variation of turbine gas meter time constant during step response process has been evaluated using the specially developed method. It has been found that time constant depends not only on the final value of flow rate, but also on the current meter's rotor rotation frequency.

2. Such situation is typical for different types of tachometric flow meters.

3. Time constant non-linearity affects the meters' dynamic error both quantitatively and qualitatively and should be taken into account calculating the dynamic error.

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TURBININIO DUJŲ SKAITIKLIO LAIKO PASTOVIOSIOS NETIESIŠKUMO ANALIZĖ KINTANT ATSAKUI

R e z i u m ė

Eksperimentiniu būdu ištirtas turbininio dujų skaitiklio atsakas į žingsninį debito kitimą. Pagal sukurtą metodiką ir naudojant atsako matavimo rezultatus nustatyta skaitiklio sukimosi inercijos laiko pastoviosios priklausomybę. Nustatyta, kad laiko pastovioji priklauso ne tik nuo galutinės debito vertės, bet taip pat nuo skaitiklio rotoriaus sukimosi dažnio momentinės vertės. Rezultatai gali būti panaudoti analizuojant skaitiklio elgesį esant kintamam debitui.

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ANALYSIS OF NONLINEARITY OF THE TURBINE GAS METERS TIME CONSTANT DURING STEP RESPONSE

S u m m a r y

The step response of the turbine gas meter has been studied experimentally. Using the specially developed method the turbine gas meter rotation inertia time constant during step response has been evaluated. It has been found that the time constant depends not only on the final value of flow rate, but also on the current rotation frequency of the meter rotors. The results can be used for calculating the dynamic error of different types of tachometric meters with pulsing flow rate.

Keywords: turbine, turbine gas meter, response, time constant, dynamic error.

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