SSN 1392 - 1207. MECHANIKA. 2013 Volume 19(6): 665-670

Analytical study of Bingham fluid flow through a conical tube

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crossref http://dx.doi.org/10.5755/j01.mech.19.6.5988

Nomenclature

D_a - Darcy number; L - length of conical tube, mm; Q - Darcy velocity, ms^{-1} ; R_0 - entrance radius, mm; R_L - exit radius, mm; R_p - radius of the plug flow region, mm; p - pressure, N/m^2; r - radius, mm; u_B - slip velocity, ms^{-1}; u_{av} - average velocity, ms^{-1}; v_z - velocity component in z - direction, ms^{-1}; k - permeability; z - flow axis; α - slip parameter; μ_0 - coefficient of viscosity at the interface, kgm^{-1}s^{-1}; τ_0 - yield stress, N/m^2 ; τ_{rz} - shear stress, N/m^2.

1. Introduction

The blood vessel can be idealized as a tube with tissue space as circular porous bed (Vide Guha and Chuadhury, 1985).The rotating viscometer data of Rand et al. [1], Bugliarello et al. [2] and Chien et al. [3] suggest the non-Newtonian behaviour of blood. Lew et al. [4] suggested chyme as a non-Newtonian material having plastic-like properties. In view of this the biofluid flow in a living body can possess the non-Newtonian behaviour in general. In order to have a better understanding of blood flow in arteries and veins and chyme flow in stomach. It is necessary to consider the biofluid to be an yield stress fluid. One of the models for yield stress fluids is Bingham model. Hence the study of Bingham fluid flow through a conical tube with permeable wall is of considerable importance in medicine.

Bird et al [5] investigated the Bingham fluid flow in a rigid circular tube. Rathy [6] studied the flow of a Bingham fluid in a channel and in an annulus with impermeable walls. Vajravelu et al. [7] made a study on the Bingham fluid in a circular tube with permeable wall. The velocity field is obtained using Beavers and Joseph [8] slip condition at the permeable wall. Buckingham-Reiner equation for the flow is obtained.

The Bingham fluid flow between two permeable beds is discussed by Goverdhan et al. [9].The flow in the channel is assumed to be governed by Bingham model. The flow in the permeable beds is governed by Darcy's law. The velocity distribution is obtained. Some results are deduced and discussed. Comparini [10] discussed a one-dimensional model for the time dependent flow of a Bingham fluid between two parallel plates. The global existence and uniqueness of classical solution to the problem is proved.

Ravana et al. [11] studied the free surface flow of a Bingham fluid in an inclined channel over a permeable bed. The flow in the channel is described by Bingham model, whereas the flow in the permeable bed is according to Darcy's law. The velocity field, the shear stress, the mass flow rate and its fractional increase are obtained.

The problem of rotational motion of a Bingham fluid in the gap between two coaxial cylinders, the outer one being at rest and the inner one moving at given angular velocity is solved by Comparini [12].

Narahari [13] discussed unsteady flow of a Bingham fluid between two permeable beds having different permeabilities. The velocity distribution in the porous and non-porous regions are obtained. Some deductions are made and the results are discussed.

Sankara Reddy et al. [14] made a detailed study on the Bingham fluid flow in an inclined channel bounded by two permeable beds. The velocity distribution in the porous and non-porous regions are obtained. The temperature variation with Ec.Pr is discussed.

Viswanatha Reddy et al. [15] made a study on Bingham fluid flow in an annulus. The velocity field, the mass flow rate and its fractional increase are obtained. The results are deduced and discussed. Helical flow of a power-law fluid in a thin annulus with permeable walls is investigated by Vajravelu et al. [16]. It is observed that velocity increases due to permeable nature of the annulus.

In this paper, Bingham fluid flow through a conical tube with permeable wall is investigated. The velocity distribution, the volume rate of flow and its fractional increase are obtained. The results are deduced and discussed through graphs.

2. Mathematical formulation

Consider the flow of a Bingham fluid through a conical tube of length L with permeable wall. The flow takes place due to pressure gradient, and the porous medium is homogeneous with permeability k. The flow surrounded by the porous medium is governed by the Bingham model, and the flow in the porous medium is governed by the Darcy's law.

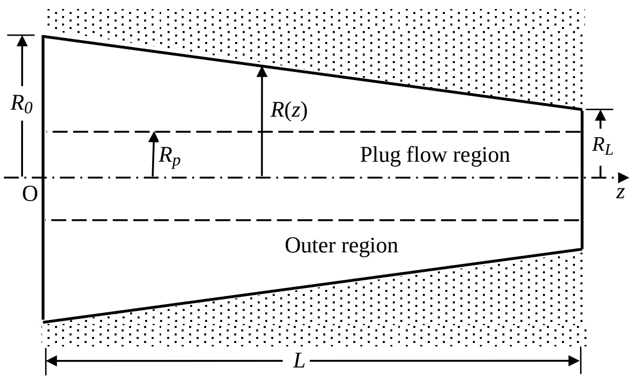

Fig. 1 Physical model: conical tube

The tube has a radius R_0 at the entrance and radius R_L at the exit (Fig. 1). The tube radius at any distance z from the inlet is given by

$$R(z) = R_0 + \left(\frac{R_L - R_0}{L}\right) z \,. \tag{1}$$

The flow is axi-symmetric. Cylindrical polar coordinate system is used. The following assumptions are made in deriving the governing equations:

- the flow is steady and incompressible;
- the flow is in axial direction;
- all physical quantities except the pressure are function of z only;
- the body forces are negligible.

In view of the above assumptions, the governing equations and boundary conditions of the flow take the following form:

2.1. Governing equation

$$\frac{1}{r}\frac{d}{dr}(r\tau_{rz}) = -\left(\frac{dp}{dz}\right), \tag{2}$$

where

$$\tau_{rz} = -\mu_0 \frac{dv_z}{dz} + \tau_0 \,. \tag{3}$$

2.2. Boundary conditions

$$\tau_{rz} = \text{ finite at } r = 0 \,; \tag{4}$$

$$\frac{dv_z}{dr} = 0 \text{ at } r = R_p \,; \tag{5}$$

$$v_z = u_B \text{ at } r = R \,; \tag{6}$$

$$\frac{dv_z}{dr} = \frac{\alpha}{\sqrt{k}}(u_B - Q) \text{ at } r = R \,, \tag{7}$$

where

$$Q = -\frac{k}{\mu}\frac{dp}{dz} \,. \tag{8}$$

By integrating equation Eq. 2 and using the boundary condition (4), we obtain:

$$\tau_{rz} = -\left(\frac{dp}{dz}\right)\frac{r}{2} a \,. \tag{9}$$

Substituting (9) in (3), we get:

$$\tau_0 - \mu_0 \frac{dv_z}{dz} = -\left(\frac{dp}{dz}\right)\frac{r}{2}, \tag{10}$$

where τ_0, τ_{rz}, α, k, u_B, Q, μ_0, u_{av}, v_z, p and R_p are the yield stress, Shear stress, slip parameter, permeability, slip velocity, Darcy velocity, coefficient of viscosity at the interface $r = 0$, average velocity, velocity component in z- direction, pressure and radius of the plug flow region.

2.3. Dimensionless equations

The flow physical parameters are made dimensionless as follows:

$$v_z^* = \frac{v_z}{u_{av}}; \quad v_z^* = \frac{v_B}{u_{av}}; \quad \tau_0^* = \frac{\tau_0}{\frac{\mu_0 u_{av}}{R_0}}; \quad \tau_{rz}^* = \frac{\tau_{rz}}{\frac{\mu_0 u_{av}}{R_0}}; \quad z^* = \frac{z}{L};$$

$$p^* = \frac{p}{\frac{L\mu_0 u_{av}}{R_0^2}}; \quad R_p^* = \frac{R_p}{R_0}; \quad R^*(z) = \frac{R_p(z)}{R_0}; \quad R_L^* = \frac{R_L}{R_0};$$

$$Q^* = \frac{Q}{u_{av}} \,.$$

2.3.1. Radius equation

Eq. (1) takes the form (* are omitted here after):

$$R = 1 + (R_L - 1) z \,. \tag{11}$$

2.3.2. Governing equation

Eq. (10) takes the form (* are omitted here after):

$$\frac{\tau_0}{r} - \frac{1}{r}\frac{dv_z}{dr} = \frac{P}{2}, \tag{12}$$

where $P = -\frac{dp}{dz}$.

2.3.3. Dimensionless boundary conditions

Eqs. (4)-(8) take the form (* are omitted here after):

$$\frac{dv_z}{dr} = 0 \text{ at } r = R_p \,; \tag{13}$$

$$v_z = u_B \text{ at } r = R \,; \tag{14}$$

$$\frac{dv_z}{dr} = \frac{\alpha}{\sqrt{k}}(u_B - D_a P) \text{ at } r = R \,, \tag{15}$$

where $D_a = \frac{k}{R_0^2}$.

3. Solution of problem

Solving (12) and using the boundary condition (14) we obtain the velocity field around the plug flow region as:

$$v_1 = -\frac{P}{4}\left(r^2 - R^2\right) + \tau_0 (r - R) + u_B \,, \tag{16}$$

when $R_p \le r \le -R$.

Using (15) in (16), we get the slip velocity at the porous wall as:

$$u_B = \frac{P\sqrt{D_a}}{\alpha}\left[\alpha\sqrt{D_a} - \frac{R}{2} + \frac{\tau_0}{P}\right]. \tag{17}$$

Using (13) in (16), we obtain the relation between τ_p and R_p as:

$$R_p = \frac{2\tau_0}{P}. \tag{18}$$

Taking $r = R_p$ in Eq. (16) and using the relation (18), we get the velocity field in the plug flow region as:

$$v_2 = -\frac{P}{4}\left(R - R_p\right)^2 + u_B, \tag{19}$$

when $0 \le r \le R_p$.

The flow in the porous region ($r \ge R_p$) is governed by the Darcy law which is given by:

$$Q = PD_a. \tag{20}$$

4. Deductions

Case (a): when $R(z) = 1$ and the permeability parameter $k \to 1$ in the Eqs. (16), (17) and (19) we get the velocity field outside the plug flow region $r \ge R_p$:

$$v_1 = -\frac{P}{4}\left(r^2 - 1\right) + \tau_0\left(r - 1\right) + u_B, \tag{21}$$

and that in the plug flow region $r \le R_p$.

$$v_2 = -\frac{P}{4}\left(1 - R_p\right)^2, \tag{22}$$

which are in agreement with those of Bird et al (1960).

Case (b): when $R(z) = 1$, the Eqs. (16), (17) and (19) we get the velocity field for the flow of a Bingham fluid flow through a circular pipe with permeable wall as follows:

$$v_1 = -\frac{P}{4}r^2 + \tau_0 r + u_B - \tau_0, \tag{23}$$

when $R_p \le r \le 1$;

$$u_B = \frac{P\sqrt{D_a}}{\alpha}\left[\alpha\sqrt{D_a} - \frac{1}{2} + \frac{\tau_0}{P}\right]; \tag{24}$$

$$v_2 = \frac{P}{4}\left(1 - R_p\right)^2 + u_B, \tag{25}$$

when $r \ge R_p$. These equations agree with those of Vajravelu and Sreenadh (1987).

Case (c): when $R(z) = 1$ as $\tau_0 \to 0$, the velocity field for the flow with permeable wall is given by:

$$v_z = -\frac{P}{4}r^2 + u_B + \frac{P}{4}, \tag{26}$$

when $R_p \le r \le 1$.

$$u_B = \frac{P\sqrt{D_a}}{\alpha}\left[\alpha\sqrt{D_a} - \frac{1}{2}\right]. \tag{27}$$

These results are in a good agreement with those of Sreenadh and Arunachalam (1983).

5. Volume rate flow

The Volume rate of flow for the flow of a Bingham fluid flow through a conical tube is:

$$Q^1 = \int_0^{2\pi}\int_0^{R_p} v_2 r dr d\theta + \int_0^{2\pi}\int_0^{R} v_1 r dr d\theta =$$

$$= \frac{p\pi}{8}\left(B_1\right) - \frac{\pi\tau_0}{8}\left(B_2\right) + u_B \pi R^2, \tag{28}$$

where $B_1 = R^4 + 3R_p^4 - 4RR_p^3$ and $B_2 = R^3 - 2R_p^3 + 3RR_p^2$.

When the permeability parameter $k \to 0$ (i.e., $D_a \to 0$) (28) reduces to:

$$Q^1 = \frac{p\pi}{8}\left(B_1\right) - \frac{\pi\tau_0}{3}\left(B_2\right), \tag{29}$$

where $B_1 = R^4 + 3R_p^4 - 4RR_p^3$ and $B_2 = R^3 - 2R_p^3 + 3RR_p^2$.

6. Fractional increase

The fractional increase in the volume rate of flow of the Bingham fluid through a conical tube with permeable wall over what it would be if the wall of the tube were impermeable is:

$$F = \frac{\left(Q^1 - Q_0^1\right)}{Q_0^1} = \frac{u_B R^2}{\frac{PB_1}{8} - \frac{\tau_0 B_2}{3}} + 3R_p^4 - 4RR_p^3, \tag{30}$$

where $B_1 = R^4 + 3R_p^4 - 4RR_p^3$ and $B_2 = R^3 - 2R_p^3 + 3RR_p^2$.

7. Discussion of the results

The velocity profiles are shown in Figs. 2 - 12 for different values of τ_0, Darcy number D_a, $P = 10$ and $\alpha = 1$. It is observed that the velocity attains maximum value at $r = 0$ and decreases with the increment in r.

For fixed r and D_a, the velocity decreases due to increase in z. The slip velocity at the permeable wall decreases with the increment in the value of D_a. On comparing the velocity profiles for permeable and impermeable conical tubes, it is found that the velocity is enhanced due to the permeability of the wall of the conical tube. It is also observed that the velocity remains constant from the axis (i.e., $r = 0$) up to some value of r (which is the Plug flow region) and then decreases to a value at the permeable wall (which is the non Plug flow region).

For fixed r and τ_0, the velocity increases with decrement in D_a. As D_a decreases the gap between the velocity curves becomes smaller for any fixed τ_0. For larger τ_0, there is an increase in plug flow region. From Fig. 13 it is observed that, for the fixed Darcy number D_a, the velocity decreases along the axis with the increase in τ_0. It is the other way at the permeable wall.

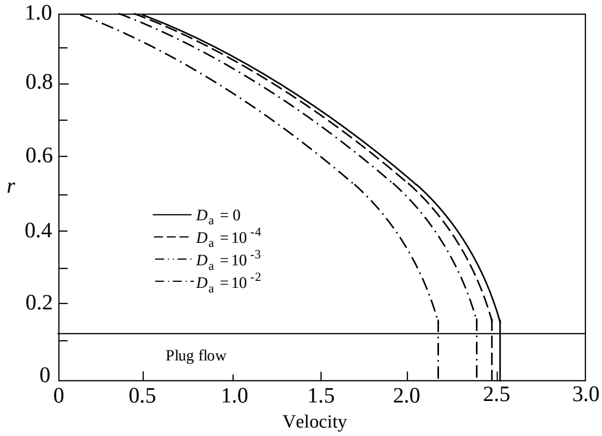

Fig. 2 Velocity profiles for $z = 0$, $\tau_0 = 0.1$

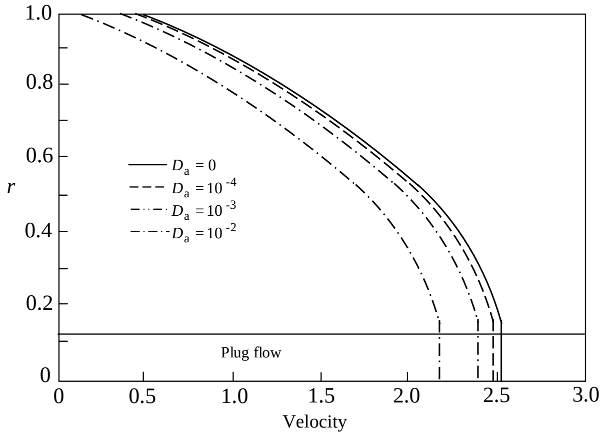

Fig. 3 Velocity profiles for $z = 0$, $\tau_0 = 0.5$

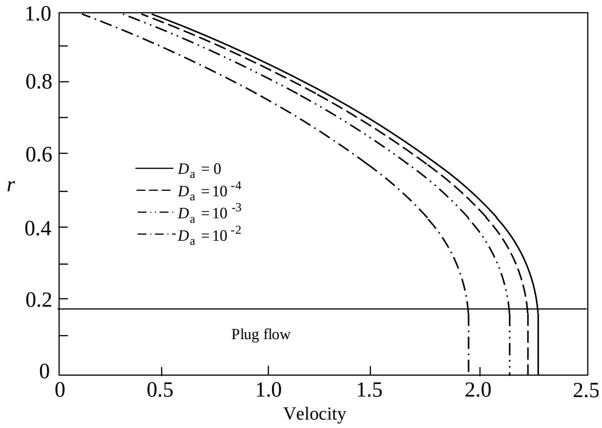

Fig. 4 Velocity profiles for $z = 0$, $\tau_0 = 0.8$

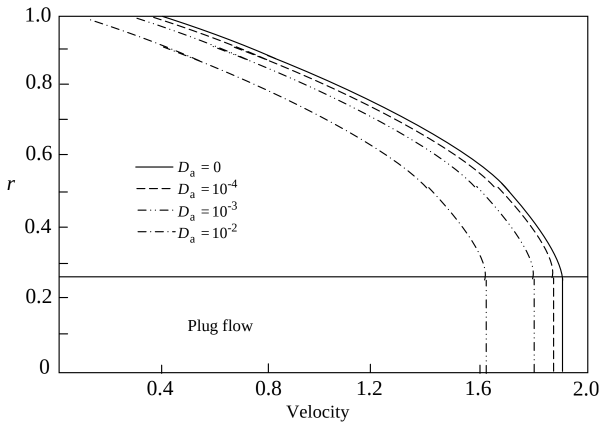

Fig. 5 Velocity profiles for $z = 0$, $\tau_0 = 1.1$

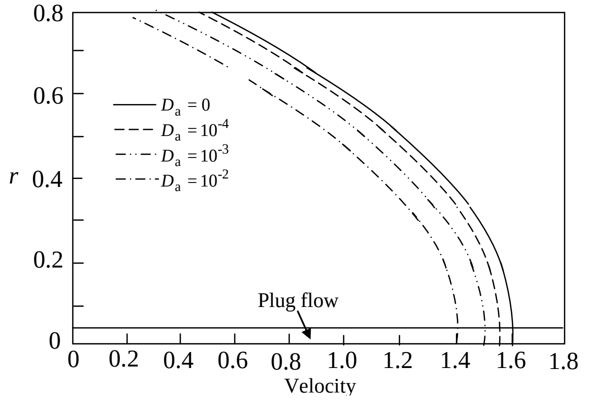

Fig. 6 Velocity profiles for $z = 0.4$, $\tau_0 = 0.1$

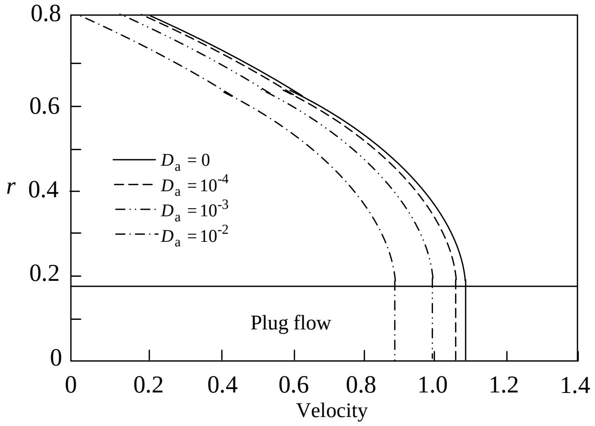

Fig. 7 Velocity profiles for $z = 0.4$, $\tau_0 = 0.5$

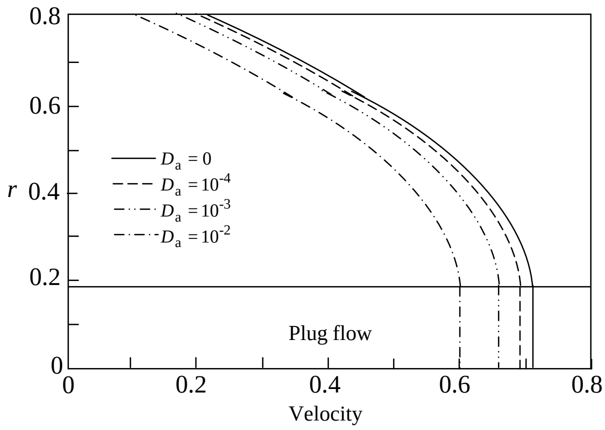

Fig. 8 Velocity profiles for $z = 0.4$, $\tau_0 = 0.8$

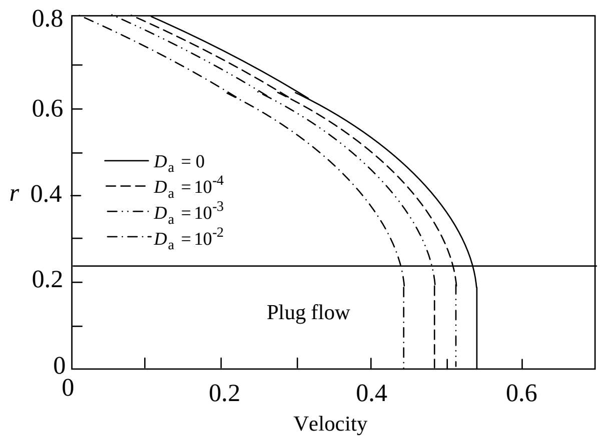

Fig. 9 Velocity profiles for $z = 0.4$, $\tau_0 = 1.1$

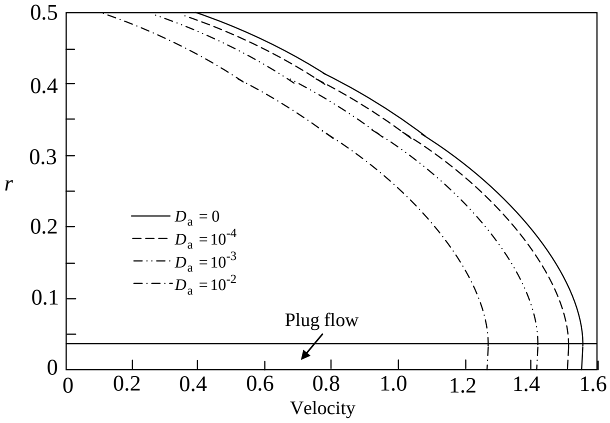

Fig. 10 Velocity profiles for z = 1, τ_0 = 0.1

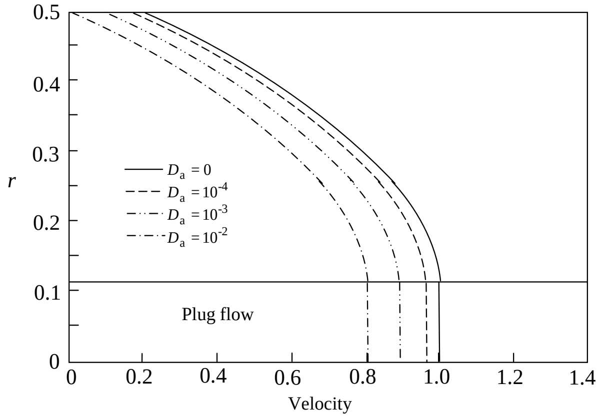

Fig. 11 Velocity profiles for z = 1, τ_0 = 0.5

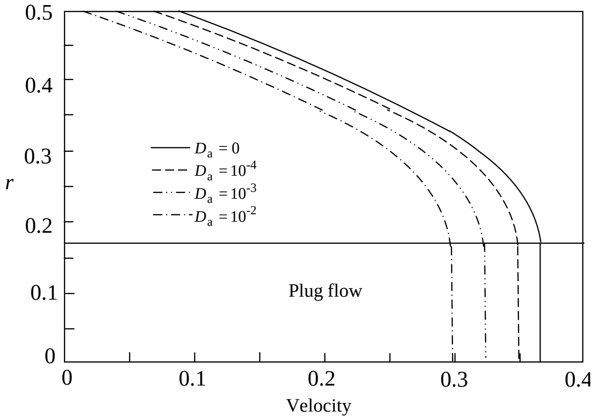

Fig. 12 Velocity profiles for z = 1, τ_0 = 0.8

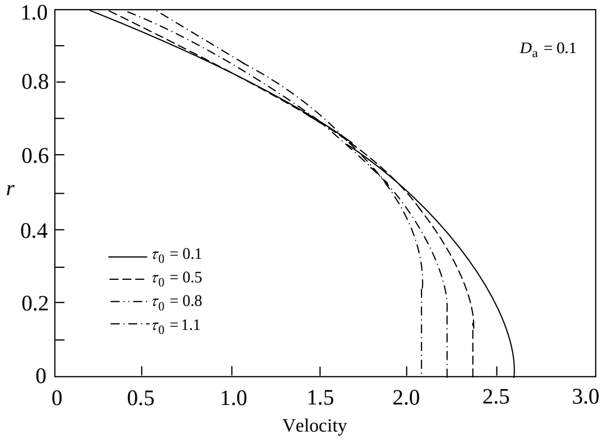

Fig. 13 Velocity profiles for D_a = 0.1

The fractional increase in volume rate of flow is numerically evaluated and is depicted in Fig. 14. It is observed that for a fixed Darcy number D_a, the fractional increases with yield stress τ_0.

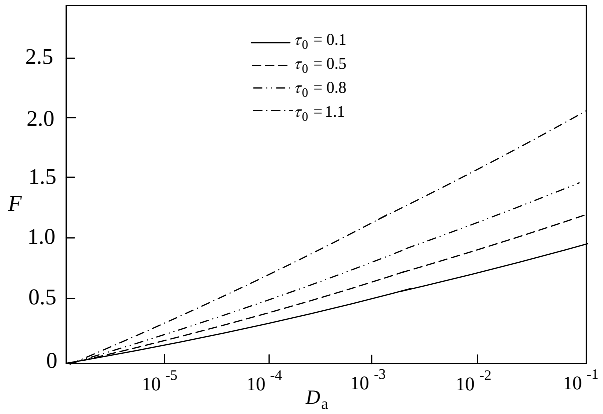

Fig. 14 Fractional increase

6. Conclusion

In this work, we have investigated a Bingham fluid flow through a conical tube with permeable wall. The velocity distribution, the volume rate of flow and its fractional increase are obtained. The Velocity profiles obtained for different values of the yield stress, and a fixed Darcy number, pressure gradient along the flow equal to 10 and slip parameter equal to 1, show that the velocity attains maximum value on the tube axis and decreases with the increment of the rayon.

For a fixed rayon and fixed pressure gradient along the flow, the velocity decreases along the z-axis (when z increase). The slip velocity at the permeable wall decreases with the increment in the value of the Darcy number. The comparison between the velocity profiles for permeable and impermeable conical tubes shows that the velocity is enhanced due to the permeability of the wall of the conical tube. The velocity remains constant in the Plug flow region and decreases in the non Plug flow region.

For fixed radius and fixed yield stress, the velocity increases with decrement in the Darcy number. As Darcy number decreases the gap between the velocity curves becomes smaller for any fixed yield stress. For larger yield stress, there is an increase in plug flow region. For fixed Darcy number, the velocity decreases along the axis flow with the increase in yield stress. It is shown that for a fixed Darcy number, the fractional increases with yield stress.

Acknowledgements

This work has been supported by Research Laboratory of Industrial Technologies, University Ibn Khaldoun of Tiaret-Algeria.

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N. Sad Chemloul

ANALITINĖ BINGHAMO SKYSČIO SRAUTO TEKĖJIMO KŪGINIU VAMZDŽIU STUDIJA

R e z i u m ė

Tiriamas Binghamo skysčio srauto tekėjimas kūginiu vamzdžiu su pralaidžiomis sienelėmis. Srautas teka dėl slėgių gradiento, kai korėta terpė yra homogeninė, o jos skvarbos koeficientas yra *k*. Srautas, apsuptas korėtos terpės, nusakomas Binghamo modeliu, o srautas korėtoje terpėje – Darsio dėsniu. Nustatytas greičio pasiskirstymas, srauto tūrio koeficientas ir nežymus jo padidėjimas. Rezultatai pateikiami ir aptariami naudojantis grafais.

N. Sad Chemloul

ANALYTICAL STUDY OF BINGHAM FLUID FLOW THROUGH A CONICAL TUBE

S u m m a r y

Bingham fluid flow through a conical tube with permeable wall studied. The flow takes place due to pressure gradient, and the porous medium is homogeneous with permeability *k*. The flow surrounded by the porous medium is governed by the Bingham model, and the flow in the porous medium is governed by the Darcy's law. The velocity distribution, the volume rate of flow and its fractional increase are obtained. The results are deduced and discussed through graphs.

Keywords: Bingham fluid, conical tube, permeable wall Darcy's law.

Received September 19, 2012
Accepted October 10, 2013