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A study on vibration of tapered rectangular plate under non-uniform temperature field

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Nomenclature

$W(x, y)$ - deflection function; x, y - coordinates in the plane of plate; ν - Poisson's ratio; ρ - density of the plate material, kg/m^3; g - plate thickness, m; $T(t)$ - time function; $\tilde{D}$ - visco-elastic operator; E - Young modulus, N/m^2; a - length of the rectangular plate, m; b - breadth of the rectangular plate, m; η - visco-elastic constant, Ns/m^2; G - shear modulus, N/m^2.

1. Introduction

In modern technology, sufficient temperature is produced in most of engineering structures i.e. rockets, submarines etc. Also, it is obvious that plates undergo some vibration due to non-uniform temperature field. Therefore, scientists and engineers are keen interested to know that how non-uniform temperature field affects the vibrational characteristics of non-homogeneous plates of variable thickness due to their utility in constructions of bridges, buildings, wings, tails & fins of rockets & missiles etc.

Non-homogeneous visco-elastic tapered plates are mainly used for two-fold requirements of safety and economy due to their high strength, high temperature resistance characteristics, low cost and high durability. Due to this, vibration of plates had become one of the most interesting research area in last few decades.

Khanna & Kaur [1] worked on thermally induced vibrations of non-homogeneous tapered rectangular pate. Gupta and Khanna [2] studied the effect of linear thickness variations in both directions on vibration of visco-elastic rectangular plate having clamped boundary conditions on all the four edges. Khanna & Sharma [3] calculated frequencies for first two modes of vibration with parabolic thickness variation and bi-parabolic temperature variation. Khanna & Kaur [4] obtained first two modes of frequencies with exponential thickness and temperature variation. Khanna and Sharma [5] investigated free vibrations of non-homogenous square plate with exponential thickness variation. The authors had considered bi-parabolic temperature variations along with linear density variation. Chakraverty [6] introduced new concepts of boundary characteristic orthogonal polynomials on vibration of plates along with a discussion of various plate geometries and boundary conditions. Leissa [7] provided excellent data for vibration of plates of different shapes with different boundary conditions in his monograph. Avalos and Laura [8] discussed transverse vibrations of a simply supported plate of generalized anisotropy with an oblique cutouts. Bambill et. al. [9] carried out an experiment on transverse vibrations of an orthotropic rectangular plate of linearly varying thickness with free edges. Chyanbin et. al. [10] gave results on vibration suppression of composite sandwich beams. Gutirrez et. al. [11] investigated vibrations of rectangular plates of bi-linearly varying thickness with general boundary conditions. R. Lal et. al. [12, 13] evaluated transverse vibrations of non-homogeneous rectangular plates with thickness variation. Liessa [14] discussed vibrations of rectangular plate with general elastic boundary supports.

Before finalizing any mechanical design or structure, researchers and engineers are always in searching of first few modes of vibration so that they would provide more authentic and reliable structures. In order to accomplish this purpose, authors provide a collection of numeric data for first two modes of frequency and deflection at various values of plate parameters i.e. taper constant, thermal gradient, aspect ratio and non-homogeneity constant. Kelvin type model is taken for consideration and plate is assumed clamped on the boundary.

2. Differential equation of motion

Differential equations of motion and time function for visco-elastic rectangular plate are [1]:

$$\begin{bmatrix} D_1\left(\frac{\partial^4 W}{\partial x^4}+2\frac{\partial^4 W}{\partial x^2\partial y^2}+\frac{\partial^4 W}{\partial y^4}\right)+2\frac{\partial D_1}{\partial x}\left(\frac{\partial^3 W}{\partial x^3}+\frac{\partial^3 W}{\partial x\partial y^2}\right)+2\frac{\partial D_1}{\partial y}\left(\frac{\partial^3 W}{\partial y^3}+\frac{\partial^3 W}{\partial y\partial x^2}\right)+ \\ +2\frac{\partial^2 D_1}{\partial x^2}\left(\frac{\partial^2 W}{\partial x^2}+\nu\frac{\partial^2 W}{\partial y^2}\right)+2\frac{\partial^2 D_1}{\partial y^2}\left(\frac{\partial^2 W}{\partial y^2}+\nu\frac{\partial^2 W}{\partial x^2}\right)+2(1-\nu)\frac{\partial^2 D_1}{\partial x\partial y}\frac{\partial^2 W}{\partial x\partial y}\end{bmatrix}-\rho p^2 gW=0 \qquad (1)$$

and

$$\ddot{T}+p^2\tilde{D}T=0, \qquad (2)$$

here D_1 is flexural rigidity of rectangular plate i.e.

$$D_1=\frac{Eg^3}{12\left(1-\nu^2\right)}. \qquad (3)$$

Deflection $w(x, y, t)$ is expressed as the product of deflection function $W(x, y)$ and time function $T(t)$ [2]:

$$w(x,y,t) = W(x,y)T(t). \quad (4)$$

3. Assumptions

Few assumptions are taken by authors to justify practical applications of the present study and to make calculations easy and convenient as well.

Assumption 1: It is assumed that variation in temperature field is bi-linear i.e.:

$$\tau = \tau_0\left(1-\frac{x}{a}\right)\left(1-\frac{y}{b}\right),$$

where τ denotes the temperature excess above the reference temperature at any point on the plate and τ_0 denotes the temperature excess above the reference temperature at $x = y = 0$.

The temperature dependence of the modulus of elasticity for most of engineering materials can be expressed as [3]:

$$E = E_0\left(1-\tau\gamma\right), \quad (5)$$

where E_0 is the value of the Young's modulus at reference temperature and γ is the slope of the variation of E with τ. After substituting the value of τ in Eq. (5), it becomes:

$$E = E_0\left(1-\alpha\left(1-\frac{x}{a}\right)\left(1-\frac{y}{b}\right)\right), \quad (6)$$

where $\alpha = \gamma\tau_0$ $(0 \le \alpha < 1)$, is thermal gradient.

Assumption 2: It is assumed that thickness varies linearly in $x-$direction i.e.:

$$g = g_0\left(1+\beta\frac{x}{a}\right), \quad (7)$$

where β is taper constant in $x-$direction and $g = g_0$ at $x = 0$.

Assumption 3: Also, It is assumed that poisson ratio of plate's material varies exponentially in $x-$direction as [4]:

$$v = v_0 e^{\alpha_1\frac{x}{a}}, \quad (8)$$

where, v_0 denotes poisson ratio at reference temperature and α_1 is non-homogeneity constant.

Since maximum value of poisson ratio is less than equal to 1/2, numeric value of α_1 (as it varies exponentially in this paper) can not be greater than 0.16 (approximately). Hence, variation in poisson ratio is taken from 0.0 to 0.15 (at most) in calculation.

After substituting the values of E, g, and v from Eqs. (6)-(8) in Eq. (3), one obtains:

$$D_1 = \frac{\left(E_0\left(1-\alpha\left(1-\frac{x}{a}\right)\left(1-\frac{y}{b}\right)\right)\right)\left(g_0\left(1+\beta\frac{x}{a}\right)\right)^3}{12\left(1-v_0^2 e^{2\alpha_1\frac{x}{a}}\right)}. \quad (9)$$

Assumption 4: Plate is assumed clamped on the boundary. Hence boundary conditions are [5]:

$$\left.\begin{aligned} W = W_{,x} = 0, x = 0, a; \\ W = W_{,y} = 0, y = 0, b. \end{aligned}\right\} \quad (10)$$

To satisfy Eq. (10), corresponding two-term deflection function is taken as [6]:

$$W = \left\{\begin{aligned} &\left[\left(\frac{x}{a}\right)\left(\frac{y}{b}\right)\left(1-\frac{x}{a}\right)\left(1-\frac{y}{b}\right)\right]^2 \times \\ &\times\left[A_1 + A_2\left(\frac{x}{a}\right)\left(\frac{y}{b}\right)\left(1-\frac{x}{a}\right)\left(1-\frac{y}{b}\right)\right] \end{aligned}\right\}. \quad (11)$$

4. Methodology

To obtain frequency equation for vibration of rectangular plate, authors used Rayleigh Ritz method. This method is based on principle of conservation of energy i.e. maximum strain energy (E_P) must be equal to the maximum kinetic energy (E_K). So it is necessary for the problem under consideration that [6]:

$$\delta\left(E_P - E_K\right) = 0, \quad (12)$$

where

$$E_K = \frac{1}{2}\rho p^2 \int_0^a\int_0^b g W^2 dy dx \quad (13)$$

and

$$E_P = \frac{1}{2}\int_0^a\int_0^b D_1\left\{\left(\frac{\partial^2 W}{\partial x^2}\right)^2 + \left(\frac{\partial^2 W}{\partial y^2}\right)^2 + 2v\left(\frac{\partial^2 W}{\partial x^2}\right)\left(\frac{\partial^2 W}{\partial y^2}\right) + 2(1-v)\left(\frac{\partial^2 W}{\partial x \partial y}\right)^2\right\} dy dx. \quad (14)$$

Here, authors introduced two non-dimensional variables X and Y i.e.:

$$X = \frac{x}{a};\ Y = \frac{y}{a}. \quad (15)$$

After using Eq. (15) in Eq. (13) and Eq. (14), one gets:

$$E_K^* = \frac{1}{2}\rho p^2 a^2 g_0 \int_0^1\int_0^{\frac{b}{a}} (1+\beta X) W^2 dY dX \quad (16)$$

and

$$E_P{}^* = Q\int_0^1\int_0^{\frac{b}{a}} \left[\begin{array}{l} \left\{ \dfrac{\left[1-\alpha(1-X)\left(1-\frac{a}{b}Y\right)\right](1+\beta X)^3}{\left[1-\nu_0{}^2 e^{2\alpha_1 X}\right]} \right\} \times \\ \times \left\{ \left(\dfrac{\partial^2 W}{\partial X^2}\right)^2 + \left(\dfrac{\partial^2 W}{\partial Y^2}\right)^2 + 2\nu_0 e^{\alpha_1 X}\left(\dfrac{\partial^2 W}{\partial X^2}\right)\left(\dfrac{\partial^2 W}{\partial Y^2}\right) + 2\left(1-\nu_0 e^{\alpha_1 X}\right)\left(\dfrac{\partial^2 W}{\partial X \partial Y}\right)^2 \right\} \end{array} \right] dYdX, \quad (17)$$

where, $Q = \dfrac{E_0 g_0^3}{24a^2}$.

After substituting $E_K{}^*$ and $E_P{}^*$ from Eq. (16) and Eq. (17) in Eq. (12), one obtains:

$$\left(E_P{}^* - \lambda^2 E_K{}^*\right) = 0, \quad (18)$$

where $\lambda^2 = \dfrac{12\rho p^2 a^4}{E_0 g_0^2}$ is frequency parameter.

Eq. (18) consists two unknown constants i.e. A_1 and A_2 arising due to the substitution of W. These two constants are to be determined as follows:

$$\frac{\delta\left(E_P{}^* - \lambda^2 E_K{}^*\right)}{\delta A_n} = 0,\ n = 1, 2. \quad (19)$$

On simplifying Eq.(19), one gets:

$$C_{n1}A_1 + C_{n2}A_2 = 0,\ n = 1, 2, \quad (20)$$

where C_{n1}, C_{n2} for $n = 1, 2$ involve plate parameters and frequency parameter.

Eq. (20) is a set of two simultaneous homogeneous equations of variables A_1 and A_2 which has infinite numbers of solutions. Choosing $A_1 = 1$, one can easily evaluate A_2 i.e. $A_2 = \left(\dfrac{-C_{n1}}{C_{n2}}\right)$.

For a non-trivial solution, the determinant of the coefficients of Eq. (20) must be zero i.e.:

$$\begin{vmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{vmatrix} = 0. \quad (21)$$

Equation (21) is a quadratic equation in λ^2 from which two values of λ^2 can be found.

Time period of the vibration of visco-elastic plate is given by:

$$K = \frac{2\pi}{p}. \quad (22)$$

5. Formulation of deflection

With the help of the values of A_1 and A_2, one can obtain deflection function W as:

$$W = \left[\begin{array}{l} \left[XY\left(\dfrac{a}{b}\right)(1-X)\left(1-\dfrac{a}{b}Y\right)\right]^2 \times \\ \times \left[1 + \left(\dfrac{-C_{11}}{C_{12}}\right)XY\left(\dfrac{a}{b}\right)(1-X)\left(1-\dfrac{a}{b}Y\right)\right] \end{array} \right]. \quad (23)$$

Time function for non-homogeneous rectangular plate can be obtained by solving Eq. (2) as [3]:

$$T(t) = e^{a_1 t}\left[\cos b_1 t + \left(-\frac{a_1}{b_1}\right)\sin b_1 t\right], \quad (24)$$

where $a_1 = -\dfrac{p^2\eta}{2G}$ and $b_1 = p\sqrt{1 - \left(\dfrac{p\eta}{2G}\right)^2}$.

Thus, deflection w can be expressed, by using Eqs. (23)-(24) in Eq. (4), as:

$$w = \left[\left[\begin{array}{l} \left[XY\left(\dfrac{a}{b}\right)(1-X)\left(1-\dfrac{a}{b}Y\right)\right]^2 \\ \times\left[1 + \left(\dfrac{-C_{11}}{C_{12}}\right)XY\left(\dfrac{a}{b}\right)(1-X)\left(1-\dfrac{a}{b}Y\right)\right] \end{array} \right] \right] \times \quad (25)$$
$$\times\left\{e^{a_1 t}\left[\cos b_1 t + \left(-\frac{a_1}{b_1}\right)\sin b_1 t\right]\right\}.$$

6. Results and discussion

In calculations, following parameters are used: $E_0 = 7.08\times10^{10}\ \mathrm{N/m^2}$; $G = 2.632\times10^{10}\ \mathrm{N/m^2}$; $\upsilon_0 = 0.345$; $\eta = 14.612\times10^5\ \mathrm{Ns/m^2}$; $\rho = 2.80\times10^{10}\ \mathrm{kg/m^2}$; $g_0 = 0.01\ \mathrm{m}$.

In Table 1, frequency for first two modes of vibration is reported at fixed aspect ratio $\dfrac{a}{b} = 1.5$ for different values of non-homogeneity constant α_1 for the following combinations of thermal gradient α and taper constant β: $\alpha = \beta = 0.0$; $\alpha = \beta = 0.2$; $\alpha = \beta = 0.6$.

It is obvious to note that frequency increases for both the modes of vibration with increasing α_1 as well as increasing values of α and β (from 0.0 to 0.6).

Table 1
Frequency Vs non-homogeneity constant at fixed aspect ratio $\dfrac{a}{b} = 1.5$

α_1	$\alpha = \beta = 0.0$		$\alpha = \beta = 0.2$		$\alpha = \beta = 0.6$	
	Mode 1	Mode 2	Mode 1	Mode 2	Mode 1	Mode 2
0.00	64.77	255.98	69.67	275.29	79.47	313.50
0.05	64.99	256.84	69.91	276.25	79.75	314.68
0.10	65.22	257.76	70.17	277.29	80.05	315.93
0.15	65.47	258.74	70.45	278.39	80.38	317.29

Again first two modes of frequency at different values of aspect ratio are shown in Table 2 for the following combinations of thermal gradient α, taper constant β and non-homogeneity constant α_1: $\alpha = \beta = \alpha_1$ and $\alpha = \beta = 0.2$, $\alpha_1 = 0.1$.

Authors noticed that both the modes of frequency increase with increasing value of aspect ratio for both combinations of α, β and α_1. Also, a small increment is found in both the modes of frequency when combined values of α, β and α_1 increases.

Table 2

Frequency *Vs* Aspect Ratio

$\frac{a}{b}$	$\alpha = \beta = \alpha_1 = 0.0$		$\alpha = \beta = 0.2$, $\alpha_1 = 0.1$	
	Mode 1	Mode 2	Mode 1	Mode 2
0.25	24.30	99.00	26.51	108.13
0.50	26.22	104.86	28.58	114.41
0.75	30.58	120.33	33.28	131.03
1.00	38.32	149.97	41.62	162.92
1.25	49.75	195.35	53.95	211.85
1.50	64.77	255.98	70.17	277.29

Since deflection function assumed in Eq. (11) is symmetrical for X and Y, it shows same values for $X = 0.2$ and $X = 0.8$ as well as $X = 0.4$ and $X = 0.6$. Same case is valid for Y. Also for $X = 0.0$ and $X = 1.0$ or $Y = 0.0$ and $Y = 1.0$, deflection becomes zero. Therefore, in calculation of deflection, authors reported the values of deflection only at $X = 0.2$, 0.4 and $Y = 0.2$, 0.4.

At different values of time function i.e. $T = 0K$ and $T = 5K$, deflection for both modes of vibration with different α_1 i.e. $\alpha_1 = 0.0, 0.1$ are calculated for different values of X and Y for the following cases:

Table 3: $\alpha = \beta = 0.0;\ \frac{a}{b} = 1.5;\ X = 0.2, 0.4;\ Y = 0.2, 0.4$;

Table 4: $\alpha = \beta = 0.2;\ \frac{a}{b} = 1.5;\ X = 0.2, 0.4;\ Y = 0.2, 0.4$;

Table 5: $\alpha = \beta = 0.6;\ \frac{a}{b} = 1.5;\ X = 0.2, 0.4;\ Y = 0.2, 0.4$.

In Tables 3-5, an acute increment is noticed in both modes of deflection at each paired value of X and Y along with different values of α_1 i.e. 0.0 and 0.1 for $T = 0$K. In Table 3, for $T = 5$K, an acute decrement is noticed in both modes of deflection at each paired value of X and Y with $\alpha_1 = 0.0$ and 0.1. At $T = 5$K, variation in both modes of deflection is different in Tables 4 and 5 as compared to Table 3. Here, very small decrement (but not negligible) is found in first mode of deflection and again very sensitive increment is found in second mode of deflection at each paired value of X and Y with $\alpha_1 = 0.0$ and 0.1.

For $T = 0$K and $T = 5$K, first two modes of deflection corresponding to increasing aspect ratio are tabulated in Table 6 at fixed $\alpha = \beta = 0.2$, $\alpha_1 = 0.1$ at two paired values of X and Y.

At $X = Y = 0.2$ & $T = 0$K, deflection increases for both the modes of vibration as aspect ratio increases from 0.5 to 1.5.

First mode of deflection increases continuously with increasing aspect ratio but second mode first increases and then decreases with increasing aspect ratio for the following cases:

i) $X = Y = 0.2$ and $T = 5$K;
ii) $X = 0.4$, $Y = 0.2$ and $T = 0$K;
iii) $X = 0.4$, $Y = 0.2$ and $T = 5$K.

Table 3

Deflection ($\times 10^{-5}$) *Vs* Non-Homogeneity constant at $\alpha = \beta = 0.0;\ \frac{a}{b} = 1.5$ for $T = 0$K and $T = 5$K*

α_1 ↓	Y ↓	$X = 0.2$		$X = 0.4$	
		Mode 1	Mode2	Mode 1	Mode2
0.0	0.2	114.6210 **{50.4459}**	39.5098 **{1.8086}**	259.8370 **{114.3570}**	6.3376 **{0.2901}**
0.1		114.6360 **{50.1875}**	39.5100 **{1.7987}**	259.8900 **{113.7790}**	6.3383 **{0.2885}**
0.0	0.4	20.8718 **{9.1859}**	14.9592 **{0.6847}**	47.1142 **{20.7335}**	27.1595 **{1.2435}**
0.1		20.8730 **{9.1381}**	14.9593 **{0.6810}**	47.1184 **{20.6283}**	27.1595 **{1.2364}**

*Values in bold and {} brackets show deflection for both the modes of vibration for T = 5K

Table 4

Deflection ($\times 10^{-5}$) *Vs* Non-Homogeneity constant at $\alpha = \beta = 0.2;\ \frac{a}{b} = 1.5$ for $T = 0$K and $T = 5$K*

α_1 ↓	Y	$X = 0.2$		$X = 0.4$	
		Mode 1	Mode2	Mode 1	Mode2
0.0	0.2	114.9010 **{47.5410}**	39.5181 **{1.8370}**	260.7830 **{107.9010}**	6.3654 **{0.2960}**
0.1		114.9440 **{47.2598}**	39.5195 **{1.8614}**	260.9270 **{107.2820}**	6.3702 **{0.3000}**
0.0	0.4	20.8938 **{8.6449}**	14.9599 **{0.6957}**	47.1887 **{19.5247}**	27.1616 **{1.2630}**
0.1		20.8972 **{8.3882}**	**14.9600** **{0.7046}**	47.2000 **{18.9589}**	27.1620 **{1.2793}**

*Values written in bold and {} brackets show deflection for both the modes of vibrations for T = 5K

Table 5

Deflection ($\times 10^{-5}$) *Vs* Non-Homogeneity constant at $\alpha = \beta = 0.6;\ \frac{a}{b} = 1.5$ for $T = 0$K and $T = 5$K*

α_1 ↓	Y ↓	$X = 0.2$		$X = 0.4$	
		Mode 1	Mode2	Mode 1	Mode2
0.0	0.2	116.6170 **{42.6163}**	39.5673 **{3.3710}**	266.5740 **{97.4164}**	6.5316 **{0.5564}**
0.1		116.7410 **{42.3458}**	39.5707 **{3.5744}**	266.9930 **{96.8471}**	6.5430 **{0.5910}**
0.0	0.6	21.0289 **{7.6847}**	14.9638 **{1.2748}**	47.6445 **{17.4111}**	27.1747 **{2.3152}**
0.1		21.0387 **{7.6314}**	14.9640 **{1.3517}**	47.6775 **{17.2942}**	27.1756 **{2.4548}**

*Values written in bold and {} brackets show deflection for both the modes of vibrations for T = 5K

Table 6

Deflection ($\times 10^{-5}$) *Vs* Aspect Ratio at $\alpha = \beta = 0.2$, $\alpha_1 = 0.1$ for $T = 0$K and T = 5K*

a/b ↓	Y ↓	$X = 0.2$		$X = 0.4$	
		Mode 1	Mode2	Mode 1	Mode2
0.5	0.2	21.1747 **{14.7462}**	14.9677 **{3.5092}**	48.1365 **{33.5225}**	27.1881 **{6.3743}**
1.0		65.6752 **{38.7730}**	33.0597 **{4.1768}**	147.9260 **{87.3318}**	37.8484 **{4.7818}**
1.5		114.9440 **{47.2598}**	39.5195 **{1.8614}**	260.9270 **{107.2820}**	6.3702 **{0.3000}**

*Values written in bold and {} brackets show deflection for both the modes of vibrations for $T = 5$K.

7. Comparison and Conclusions

A comparison between the results of present paper (frequency) with results available in [4] is reported in Table 7 for different values of taper constant and thermal gradient at fixed non-homogeneity constant ($\alpha_1 = 0.0$) From Table 7, one can easily seen that both modes of frequency in the present paper are lesser than [4] at each paired value at α and β except at $\alpha = \beta = 0.0$ where frequency is equal to [4] for both modes of vibration.

On the behalf of above comparison, authors conclude the following:

- frequency can be actively controlled by linear tapering as compared to exponential tapering [4];
- bi-linear variation in temperature field provides much better results as compared to exponential variation in temperature field;
- results of present study may provide more realistic mechanical designs or structures.

Table 7

Comparison of the frequencies of the present problem with [4]** at $\alpha_1 = 0.0$

β ↓	$\alpha = 0.0$		$\alpha = 0.2$		$\alpha = 0.4$		$\alpha = 0.6$	
	Mode 1	Mode 2	Mode 1	Mode 2	Mode 1	Mode 2	Mode 1	Mode 2
0.0	64.77 **{64.77}**	255.98 **{255.98}**	63.13 **{69.06}**	249.50 **{272.86}**	61.45 **{73.10}**	242.84 **{288.75}**	59.71 **{76.92}**	236.00 **{303.82}**
0.2	71.40 **{71.84}**	282.12 **{283.76}**	69.67 **{76.89}**	275.29 **{303.54}**	67.91 **{81.62}**	268.28 **{322.10}**	66.09 **{86.10}**	261.08 **{339.65}**
0.4	78.27 **{80.25}**	309.17 **{316.46}**	76.46 **{86.21}**	301.95 **{339.73}**	74.60 **{91.78}**	294.56 **{361.50}**	72.70 **{97.03}**	286.98 **{382.04}**
0.6	85.34 **{90.21}**	336.90 **{354.87}**	83.43 **{97.27}**	329.28 **{382.35}**	81.47 **{103.85}**	321.49 **{407.99}**	79.47 **{110.02}**	313.50 **{432.10}**

**Values written in bold and {} brackets are from [4].

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KŪGINĖS STAČIAKAMPĖS PLOKŠTELĖS VIRPESIŲ, VEIKIANT NETOLYGIAI PASISKIRSČIUSIAM TEMPERATŪRINIAM LAUKUI, TYRIMAS

R e z i u m ė

Straipsnyje tyrinėjami tampriai elastinės nehomogeninės kūginės stačiakampės plokštelės virpesiai. Plokštelės kūgiškumas yra orientuotas viena kryptimi. Eksponentinis Puasono koeficiento pokytis yra susijęs su plokštelės medžiagos nehomogeniška prigimtimi. Temperatūrinio lauko netolygumas yra bi-linearus t.y. tiesinis x- ir y-kryptimis. Pirmųjų dviejų virpesių modų dažniai ir įlinkiai paskaičiuoti ir suvesti lentelėn įvairiems plokštelės parametrams. Pateiktas šios studijos rezultatų palyginimas su literatūroje duotais.

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A STUDY ON VIBRATION OF TAPERED RECTANGULAR PLATE UNDER NON- UNIFORM TEMPERATURE FIELD

S u m m a r y

The present analysis is about to study the vibration of tapered rectangular plate made up of visco-elastic non homogeneous material. Tapering in the plate is considered in one direction. Exponential variation in poisson ratio is assumed for non homogeneous nature of plate's material. Non uniformity in temperature field is considered bi-linear i.e. linear in x– direction and linear in y– direction. Frequency and deflection for first two modes of vibration are calculated and tabulated for various values of plate parameters. A comparison of the results of present study with those available in literature is given.

Keywords: vibration, frequency, visco-elastic, non-homogeneous, plate parameters, deflection.

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